NRCSE

4.2 Statistics, data, and deterministic models

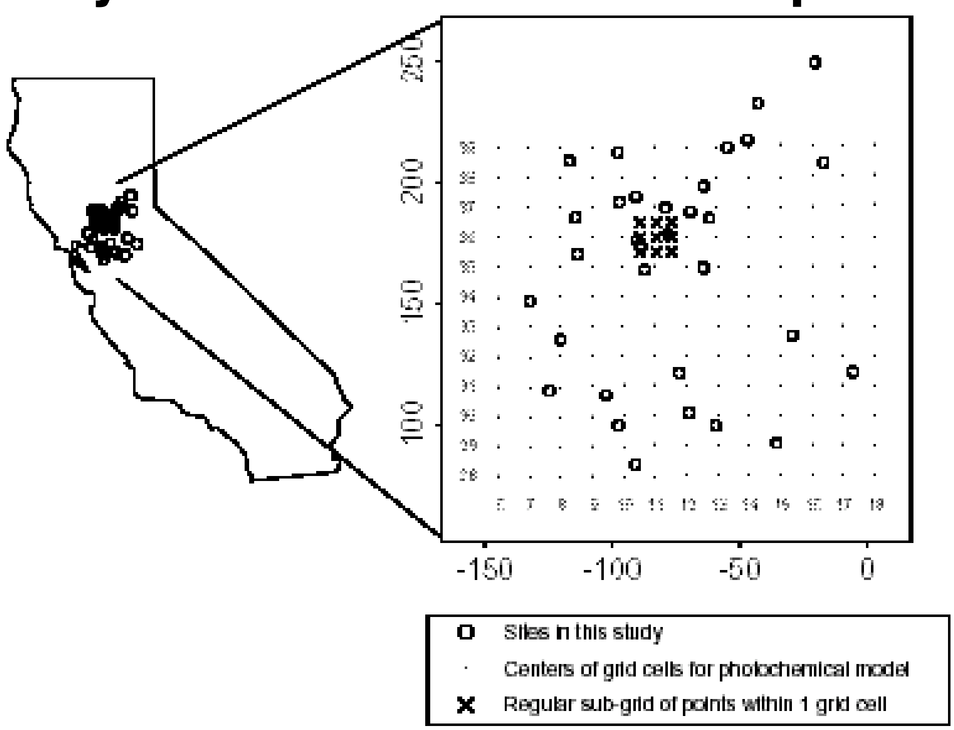
Some issues in model assessment

- Spatiotemporal misalignment
  - Grid boxes vs observations
- Types of error
  - Measurement error and bias
  - Model error
  - Approximation error
- Manipulate data or model output?
- Two case studies:
  - SARMAP – kriging
  - MODELS-3 – Bayesian melding
- Other uses of Bayesian hierarchical models

Assessing the SARMAP model

60 days of hourly observations at 32 sites in Sacramento region

Hourly model runs for three "episodes"



Task

Estimate from data the ozone level at x's in a grid square. Use sum to estimate integral over grid square.

Issues:

- **Transformation**
- **Diurnal cycle**
- **Temporal dependence**
- **Spatial dependence**
- **Space-time interaction**

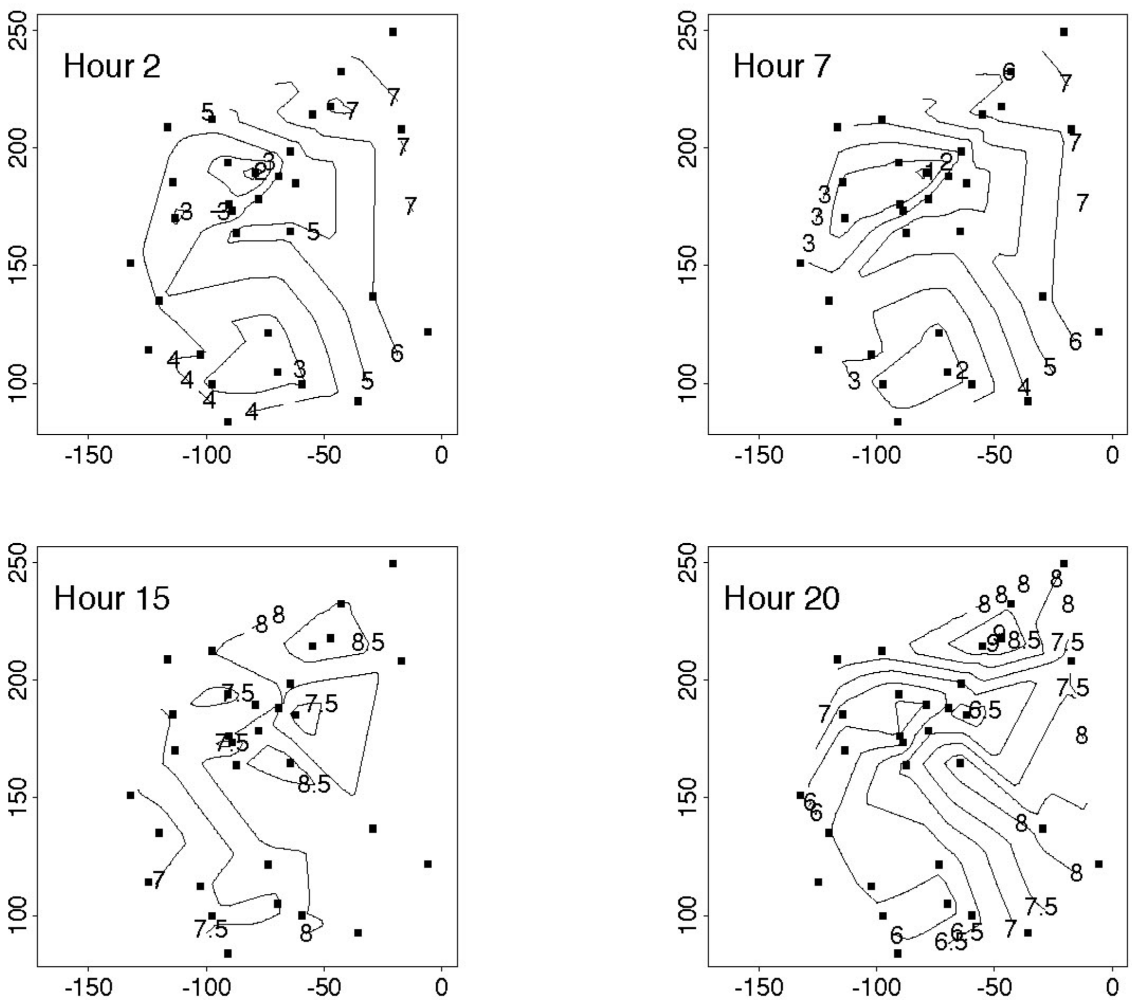
Transformation

Heterogeneous variability–mean and variance positively related

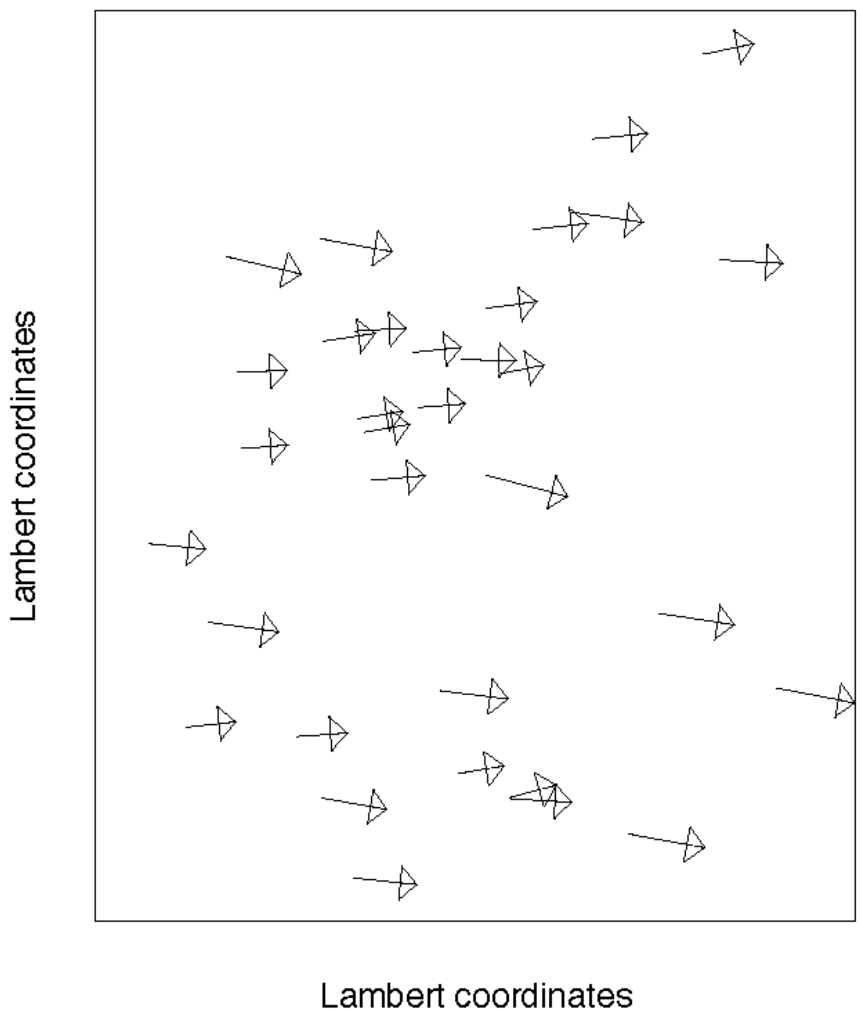
Square root transformation

All modeling now on square root scale–approximately normal

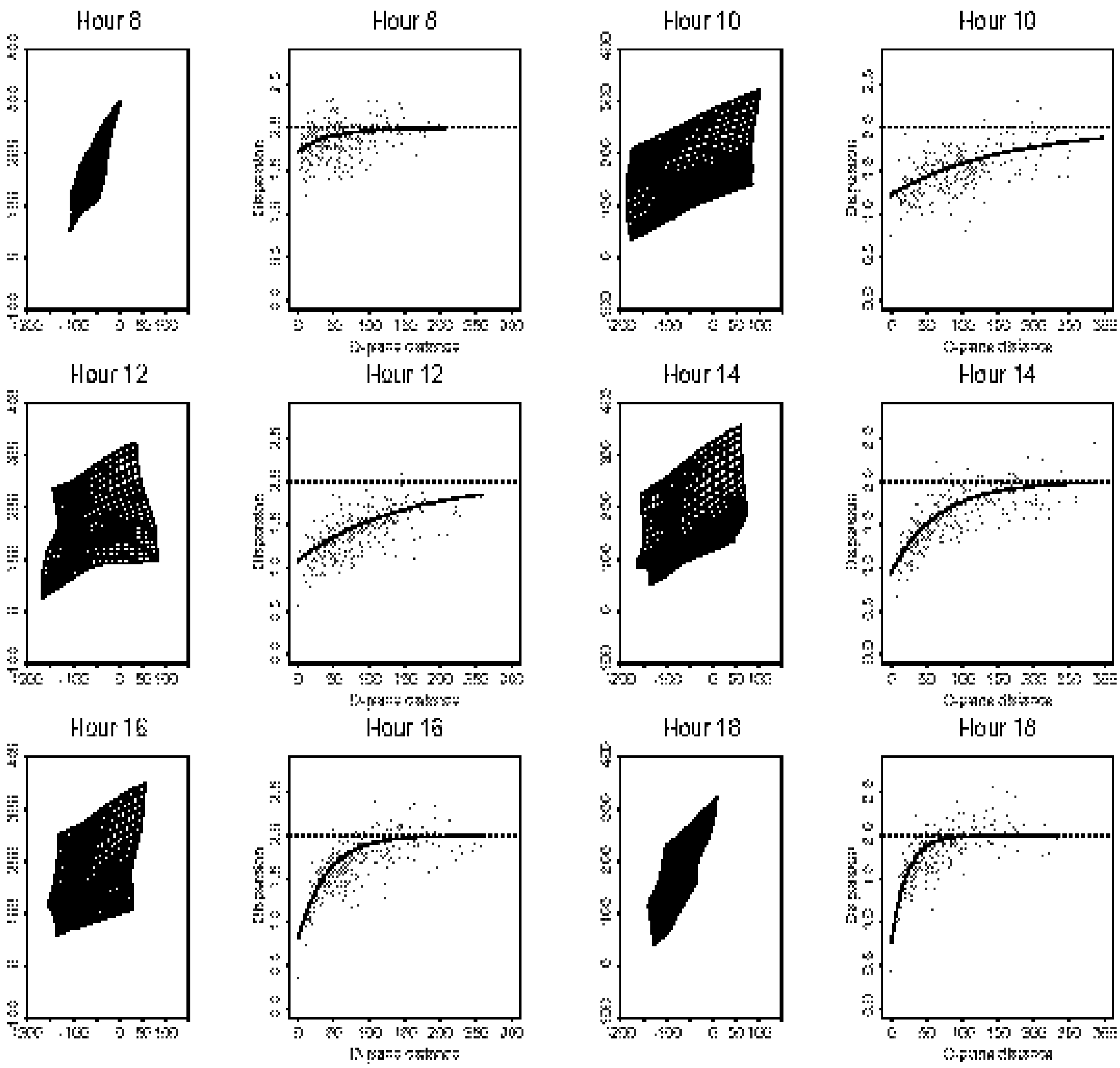
Diurnal cycle



Temporal dependence



Spatial dependence



Estimating a grid square average

$$V_t(s) = \sqrt{Z_t(s)}$$

$$V_t(s) = \mu_t(s) + W_t(s)$$

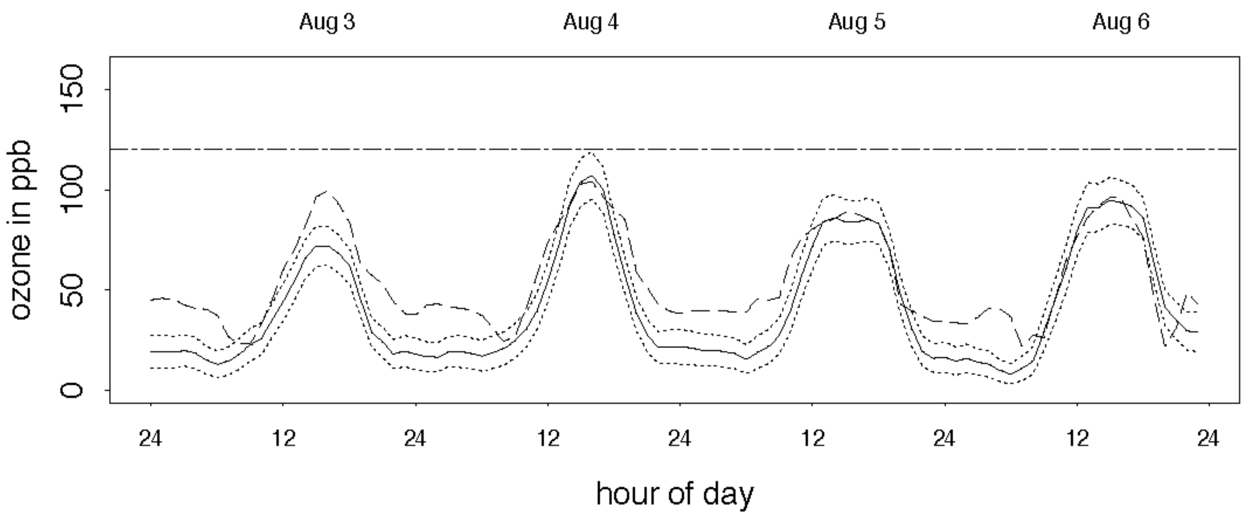
$$W_t(s) = \alpha_1(s) W_{t-1}(s) + \alpha_2(s) W_{t-2}(s) + Y_t(s)$$

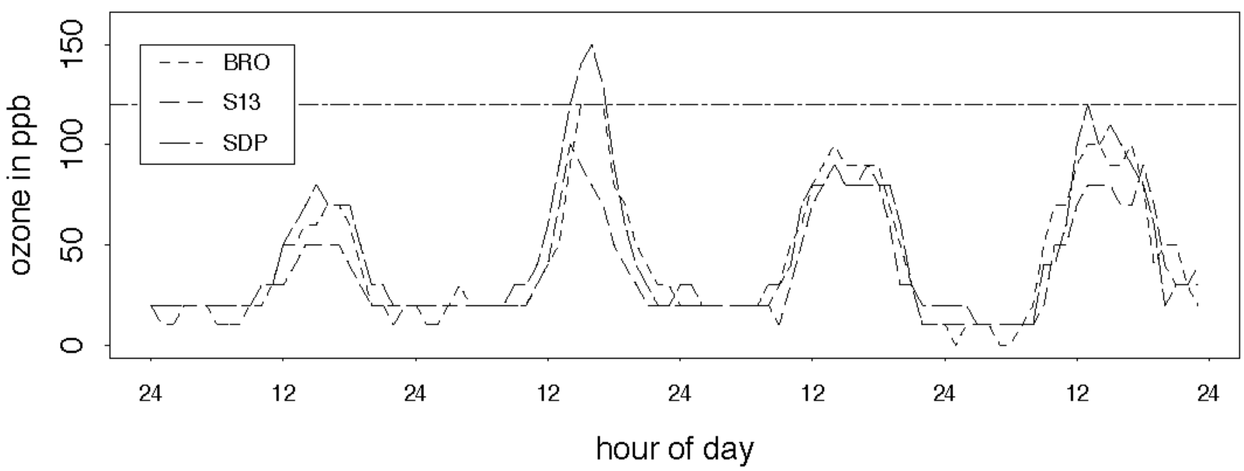
Estimate $\frac{1}{|A|}\int_A V_t^2(s)ds$ **using**

$$\frac{1}{M}\sum E\left\{V_t(s_j)^2 \,\middle|\, \text{data from } 1,\ldots,t\right\}$$

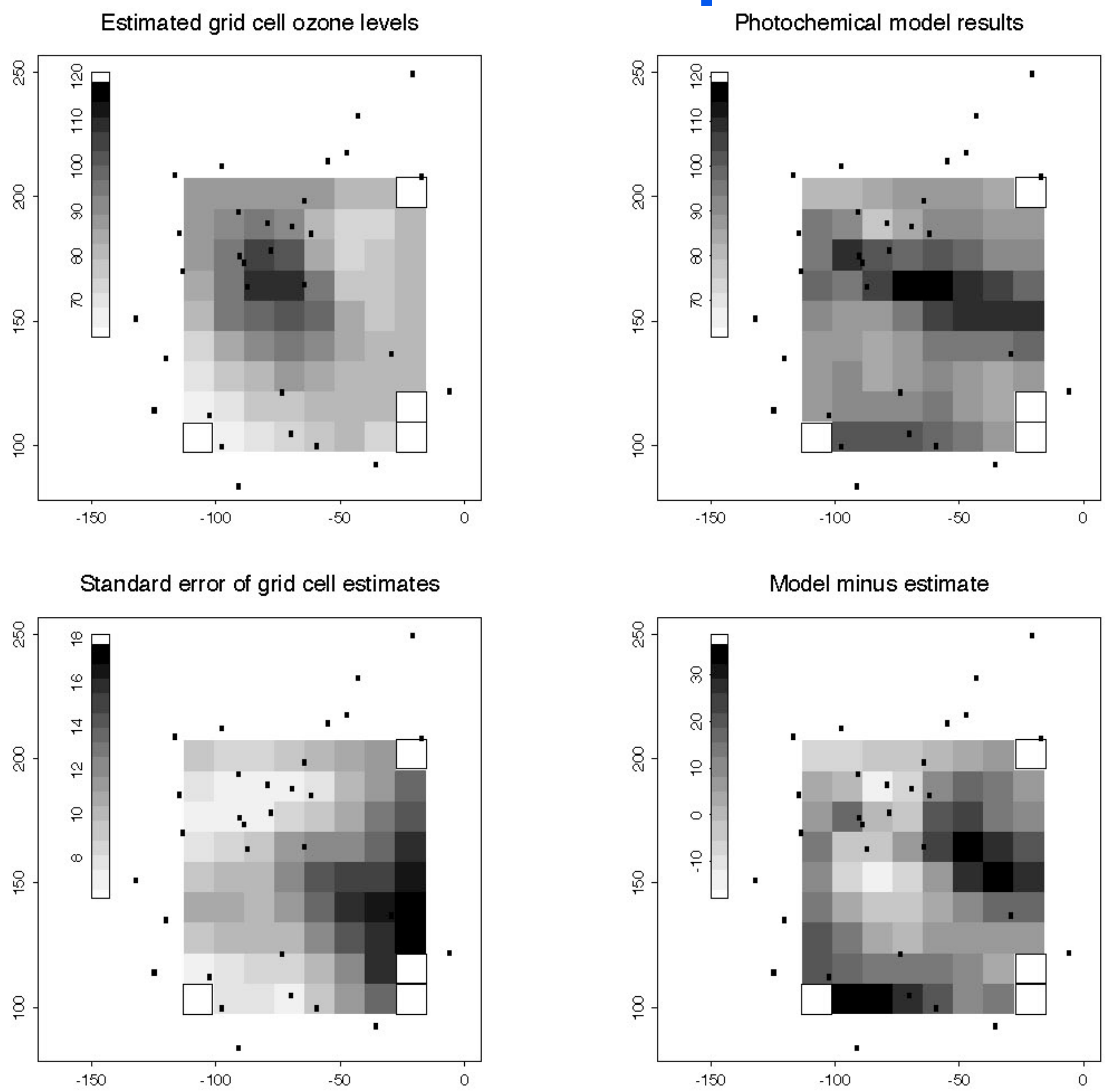
(*not* averages of squares of kriging estimates on the square root scale)

Looking at an episode

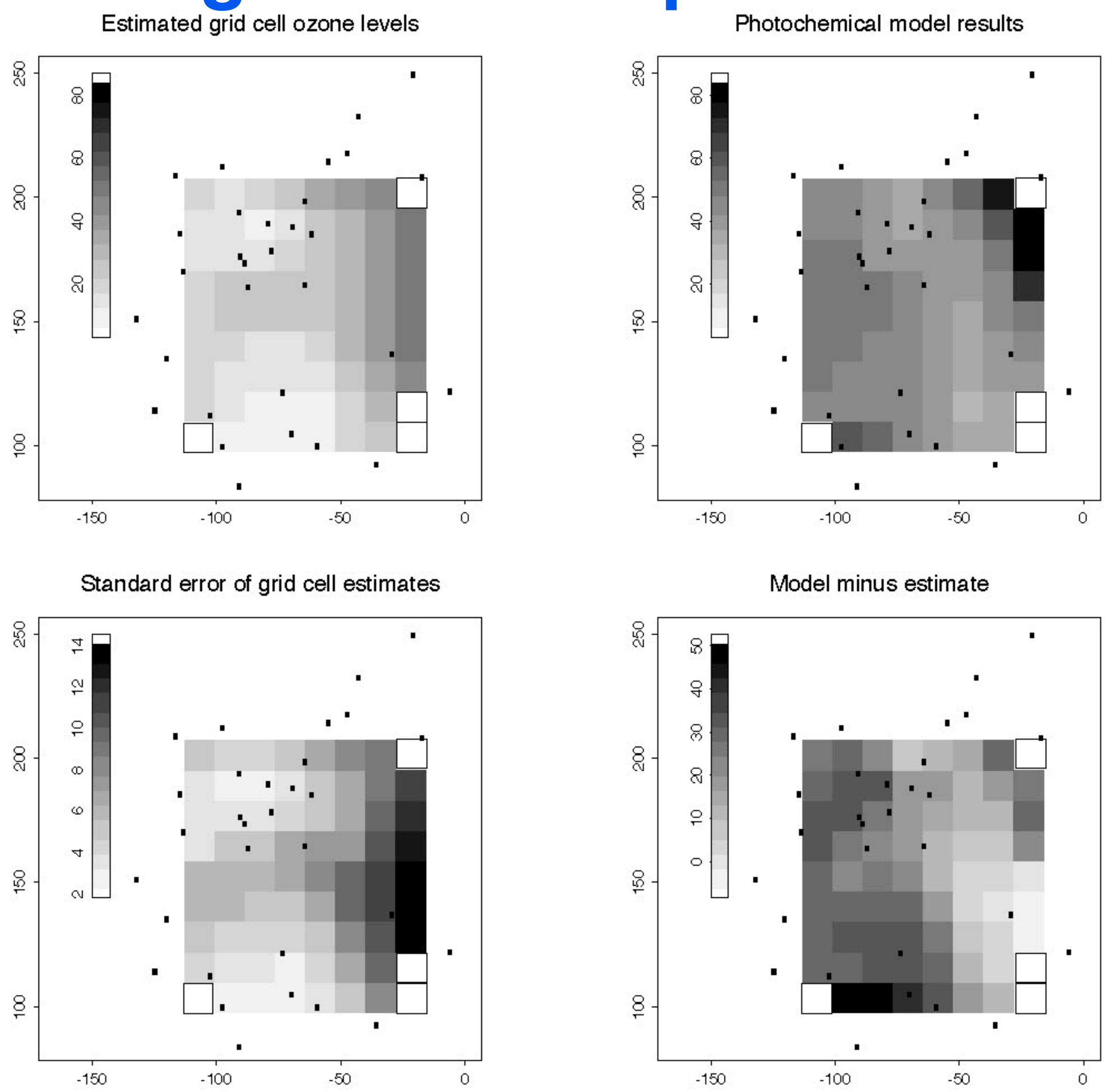




Afternoon comparison



Nighttime comparison



Bayesian melding

Deterministic model: $M(\theta)$

Prior: $f(\theta)$

Induced prior on outputs: $h(m)=M(f(\theta))$

Direct prior on outputs: $g(m)$

Melded prior on outputs:

$$f(m)=h(m)^{\lambda}g(m)^{1-\lambda}$$

Data: x

Likelihood: $f(x|m)$

Posterior: $f(m|x)$

A state space approach

SARMAP study spatially data rich

If spatially sparse data, how estimate grid squares?

$$P = \beta Z + M + A + \delta$$

$$O = (\gamma)Z + B + E$$

P = process model output

O = observations

Z = truth (state variable)

Calculate (Z |P,O) for prediction

Calculate (O |P, β = 1, M = S = A = 0) for model assessment

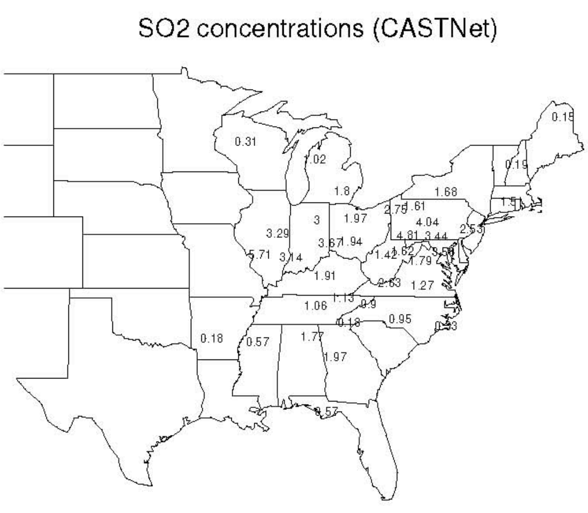
CASTNet and Models-3

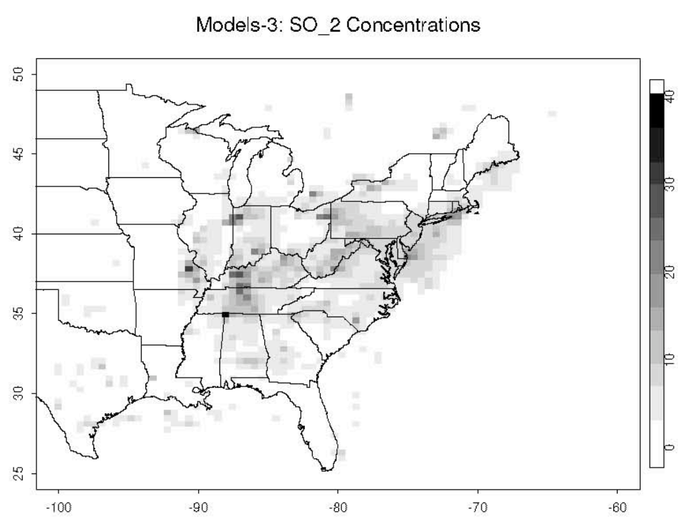
CASTNet is a dry deposition network

Models-3 sophisticated air quality model

Average fluxes on 36x36 km^2 grid

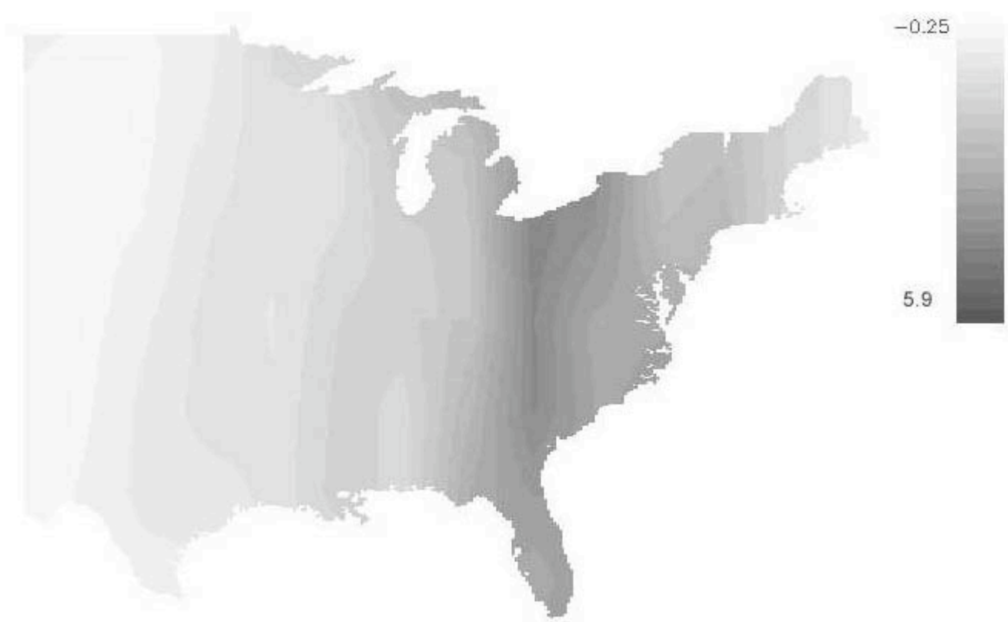
Weekly data and hourly output





Estimated model bias

The multiplicative bias β is taken spatially constant (= 0.5). The additive bias E(M+A+δ) is spatially distributed.



Assessing model fit

Predict CASTNet observation O_i from posterior mean of prediction using Models-3 output P_i and remaining observations O_{-I}.

Average length of 90% credible intervals is 7 ppb

Average length using only Models-3 is 3.5 ppb

The Bayesian hierarchical approach

Three levels of modelling:

Data model:

f(data | process, parameters)

Process model:

f(process | parameters)

Parameter model:

f(parameters)

Use Bayes' theorem to compute posterior

f(process, parameters | data)

Application to Models-3

$$(Z(s_0)\,|\,P,O) \propto \int f(Z(s_0)\,|\,P,O,\theta) f(\theta\,|\,P,O)\,d\theta$$

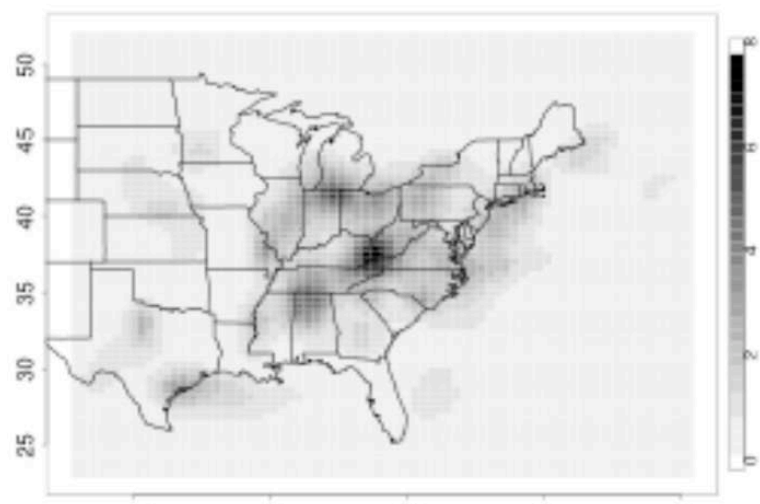
$$\approx \frac{1}{M}\sum_{i=1}^{M} f(Z(s_0\,|\,P,O,\theta^{(i)})$$

where $\theta^{(I)}$ are samples from the posterior distribution of θ

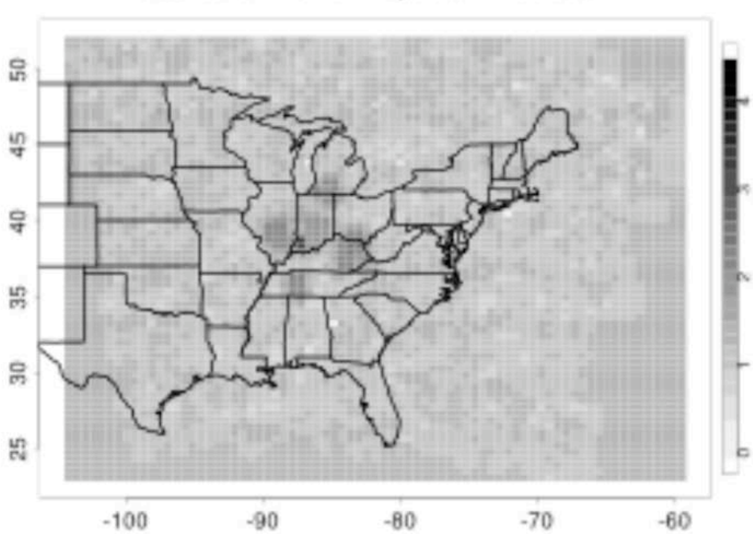
	ME	IL	NC	IN	FL	MI
CASTNet	0.15	3.29	0.90	3.14	0.57	1.02
Models-3	0.33	3.33	5.32	9.59	0.52	1.04
Adjusted Models-3	0.12	2.88	1.09	3.12	0.44	1.01

Predictions

SO2 Concentrations (Bayesian melding)



Standard Error of Bayesian Prediction



Two examples

Precipitation modeling (Tamre Cardoso, PhD UW 2004)

Ocean wind dynamics (Anders Malmberg, PhD Lund U 2005)

Rainfall measurement

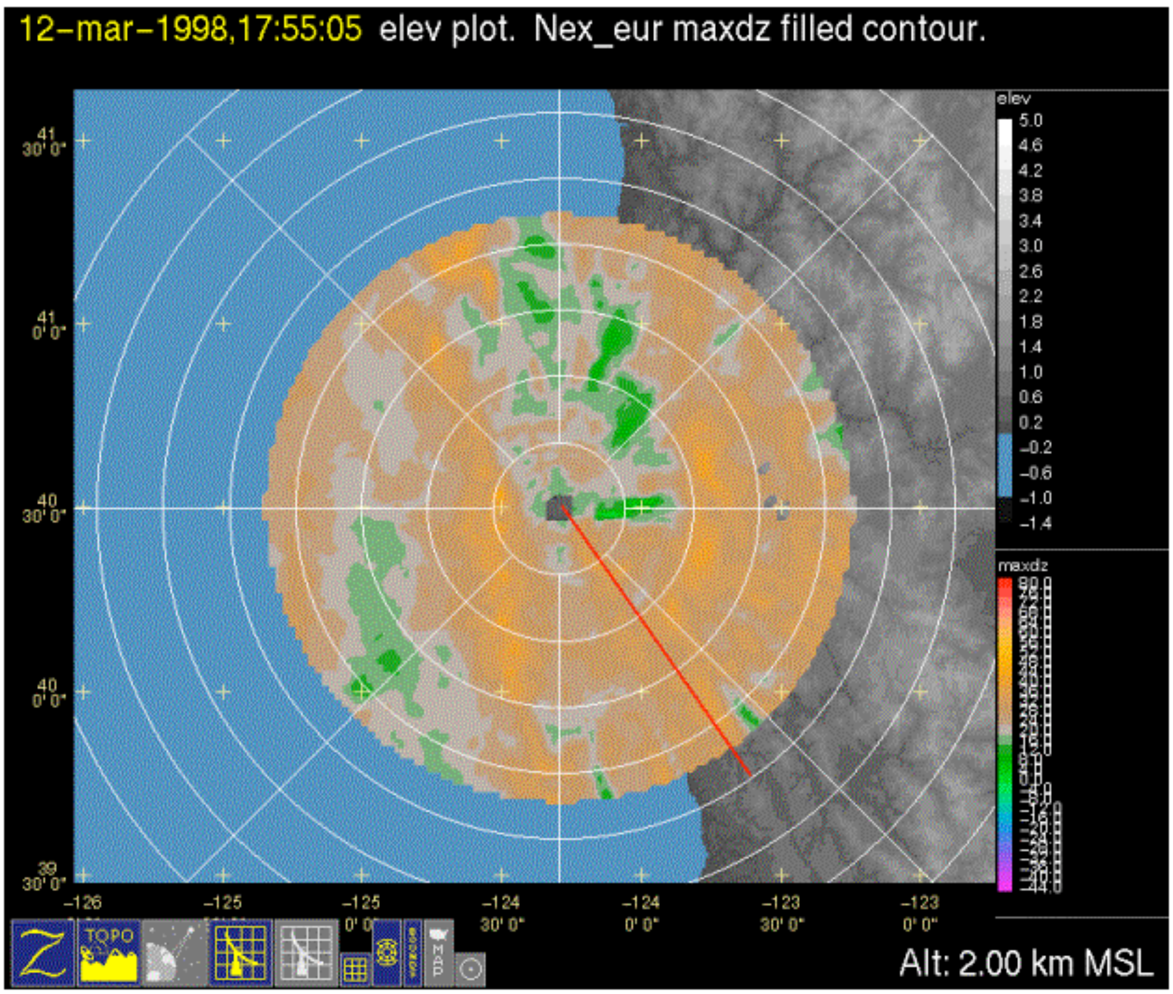
Rain gauge (1 hr)
- High wind, low rain rate (evaporation)
- Spatially localized, temporally moderate

Radar reflectivity (6 min)
- Attenuation, not ground measure
- Spatially integrated, temporally fine

Cloud top temp. (satellite, ca 12 hrs)
- Not directly related to precipitation
- Spatially integrated, temporally sparse

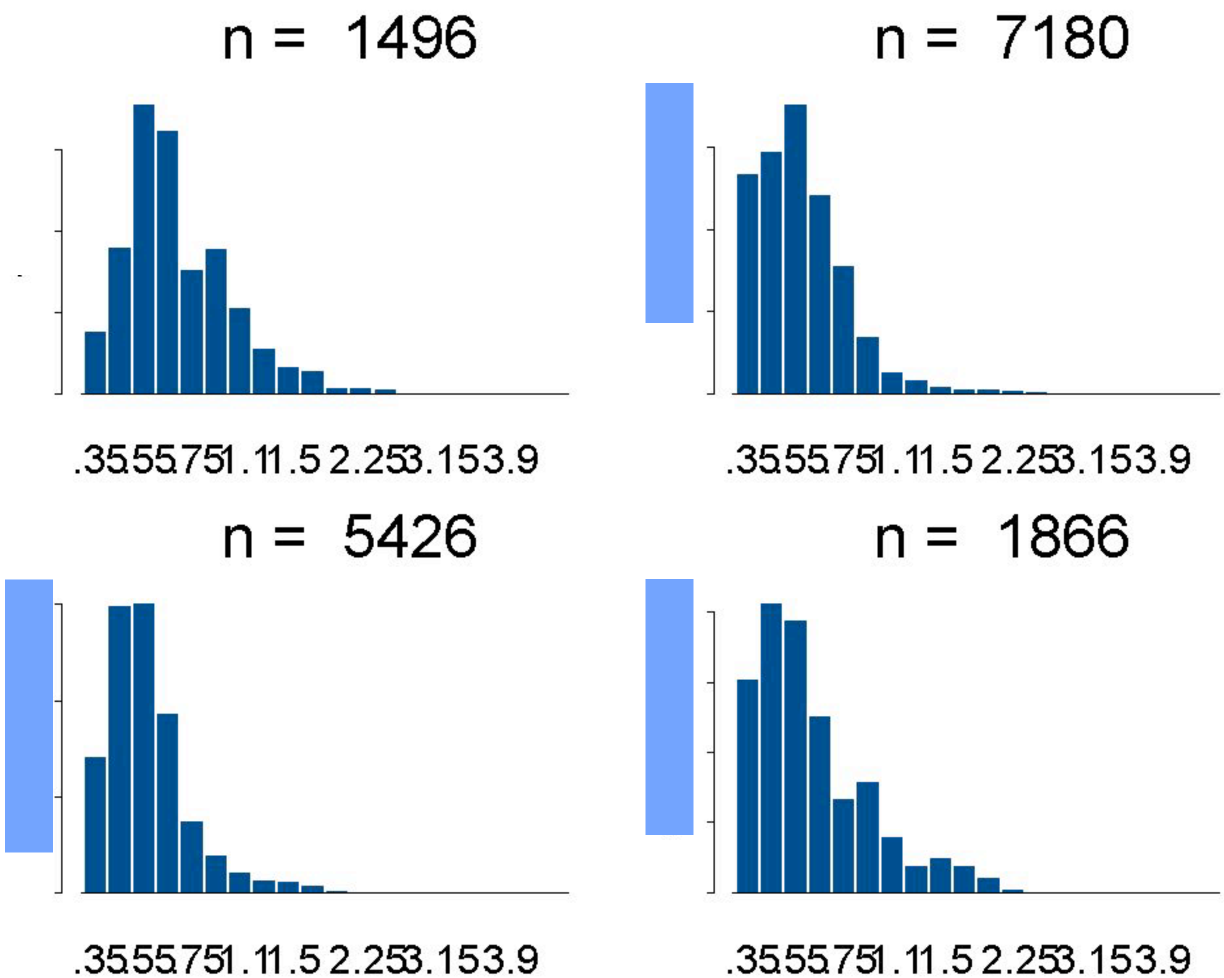
Distrometer (drop sizes, 1 min)
- Expensive measurement
- Spatially localized, temporally fine

Radar image



12–mar–1998,17:55:05 elev plot. Nex_eur maxdz filled contour.
elev
5.0
4.6
4.2
3.8
3.4
3.0
2.6
2.2
1.8
1.4
1.0
0.6
0.2
–0.2
–0.6
–1.0
–1.4
maxdz
41 30' 0"
41 0' 0"
40 30' 0"
40 0' 0"
39 30' 0"
–126
–125
–125
–124
–124
–123
–123
30' 0"
0' 0"
30' 0"
0' 0"
TOPO
Z
MAP
Alt: 2.00 km MSL

Drop size distribution



Basic relations

Rainfall rate:

$$R(t) = c_R \frac{\pi}{6} \int_0^{\infty} D^3 v(D) N(t) f(D) dD$$

v(D) terminal velocity for drop size D

N(t) number of drops at time t

f(D) pdf for drop size distribution

Gauge data:

$$G(t) \sim N\left(g(w(t)) \int_{t-\Delta}^{t} R(s) ds, \sigma_G^2 \right)$$

g(w) gauge type correction factor

w(t) meteorological variables such as wind speed

Basic relations, cont.

Radar reflectivity:

$$Z_D(t) = c_Z\left(\int_0^{\infty} D^6 v(D) N(t) f(D)\, dD\right)$$

Observed radar reflectivity:

$$Z(t) \sim N(Z_D(t), \sigma_Z^2)$$

Structure of model

Data: $[G|N(D),\theta_G]$ $[Z|N(D),\theta_Z]$

Processes: $[N|\mu_N,\theta_N]$ $[D|\xi_t,\theta_D]$

log GARCH LN

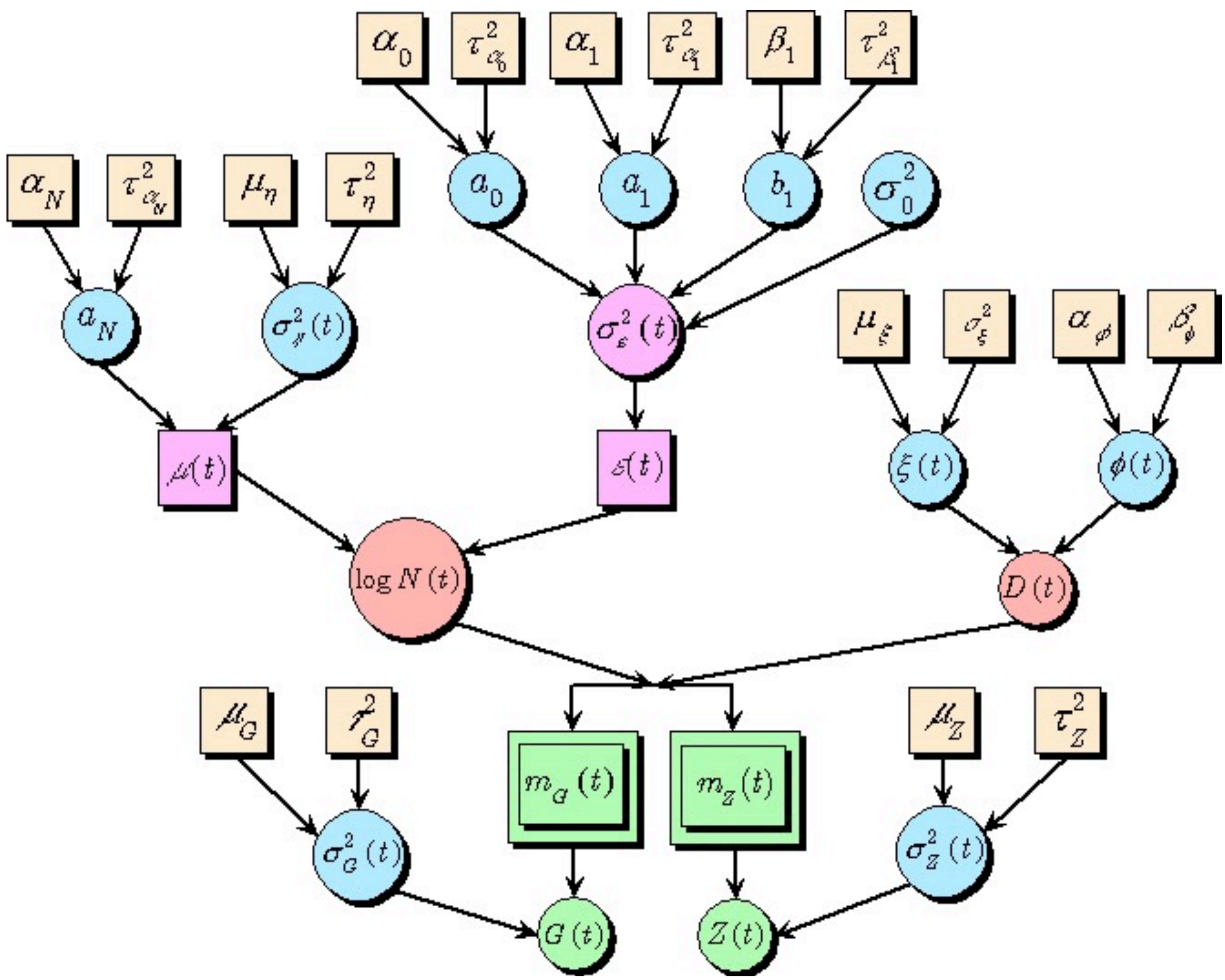
Temporal dynamics: $[\mu_{N(t)}|\theta_\mu]$

AR(1)

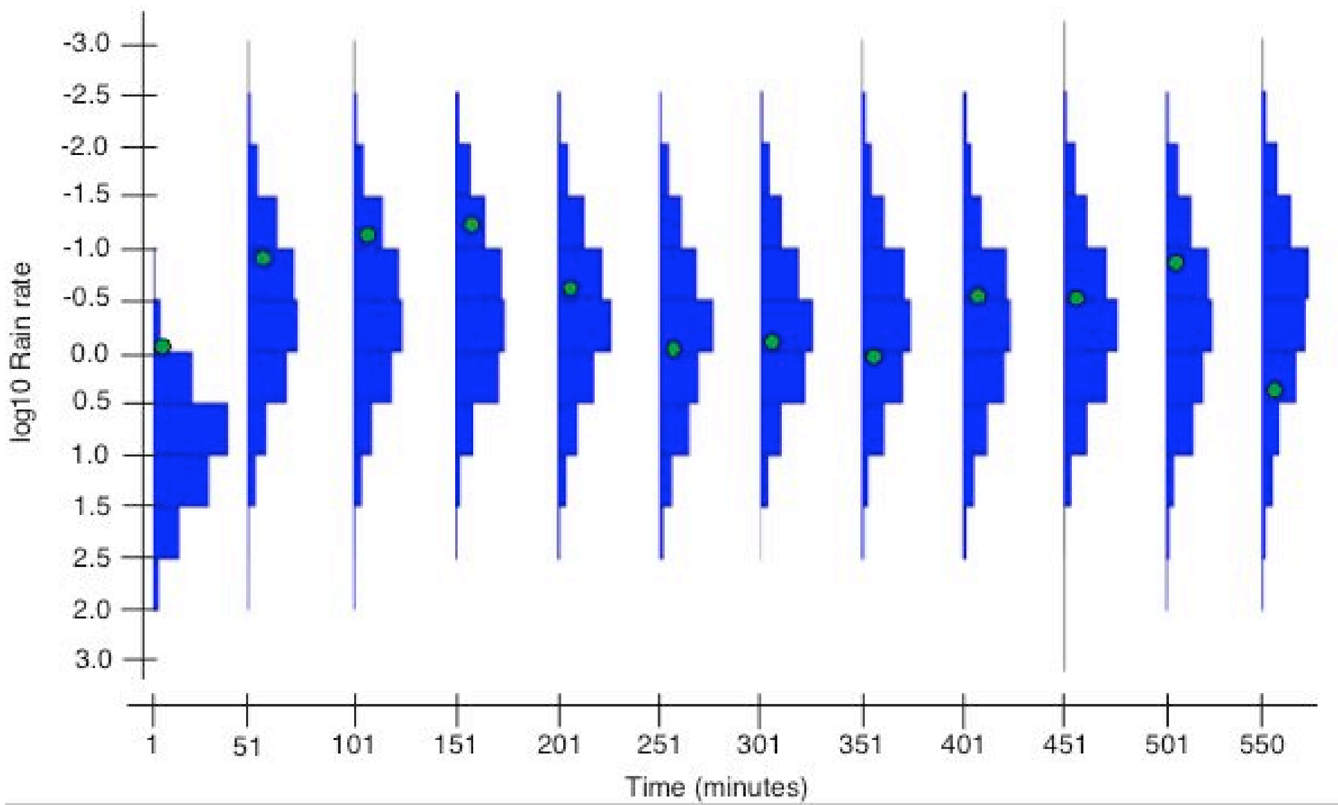
Model parameters: $[\theta_G,\theta_Z,\theta_N,\theta_\mu,\theta_D|\theta_H]$

Hyperparameters: θ_H

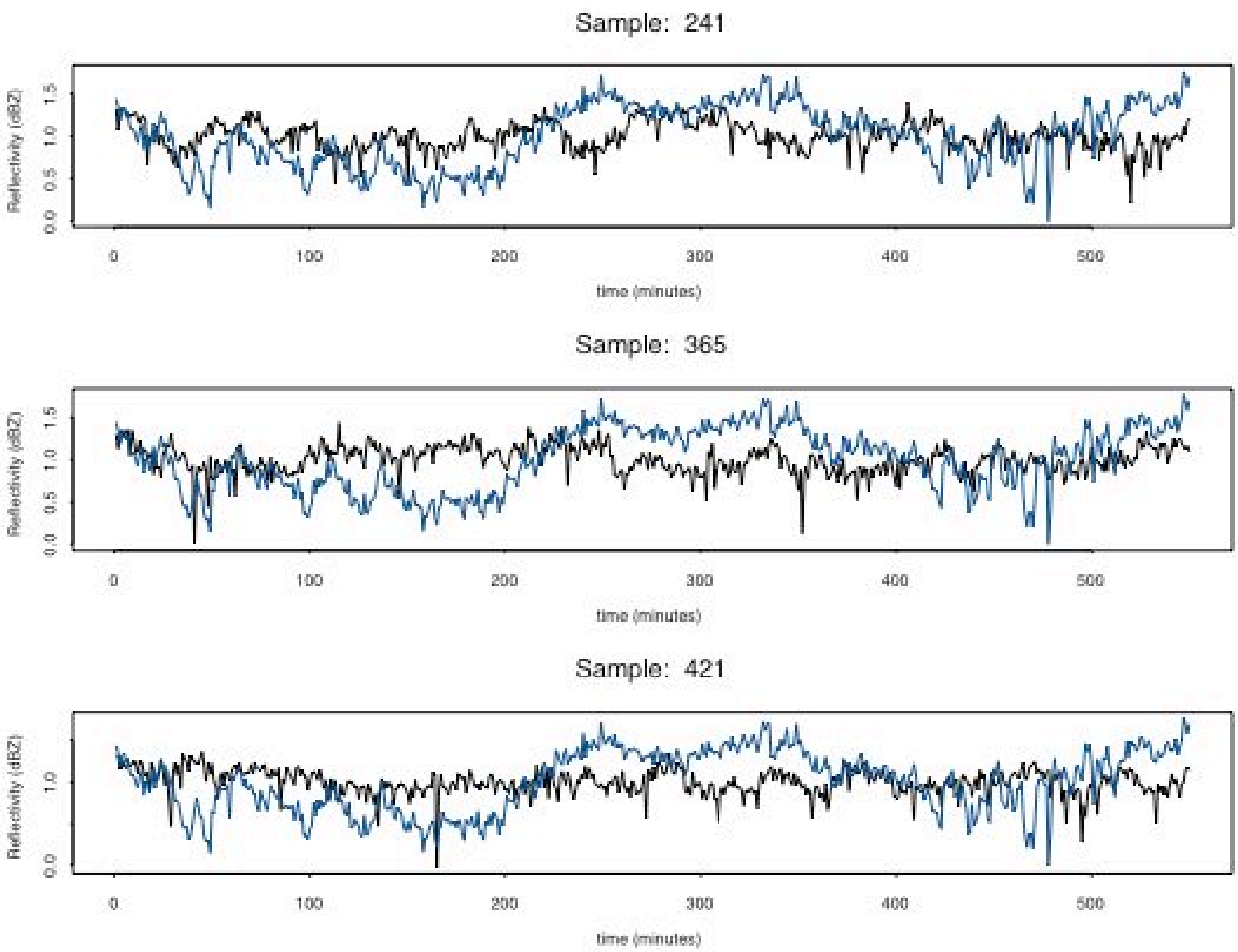
MCMC approach



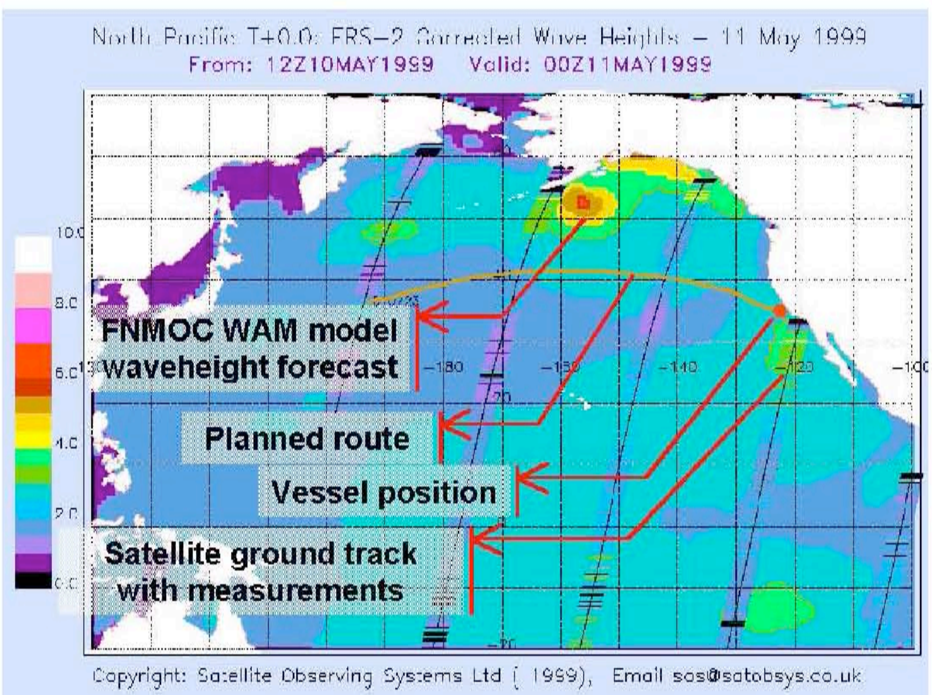
Observed and predicted rain rate



Observed and calculated radar reflectivity



Wave height prediction

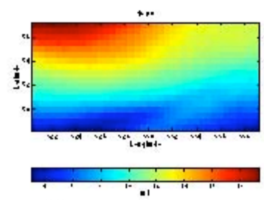


General wave height contour map from the Wave Assimilation Model overlaid with satellite measurements.

Merge forecasts and satellite observations and provide a measure of uncertainty.

Misalignment in time and space

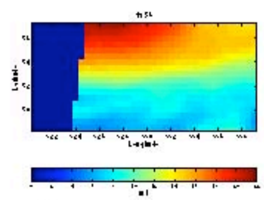
ECMWF data



$\mathbf{Y}^M(t) = (Y^M(\mathbf{s}_1, t), \ldots, Y^M(\mathbf{s}_N, t))$

- Spatial resolution: 0.5 degrees in longitude and latitude. The spatial grid consists of $N = 684$ locations.
- Temporal resolution: Data on a six hour basis.

QuikSCAT data



$\mathbf{Z}^M(\tau) = (Z^M(\mathbf{s}_1, \tau), \ldots, Z^M(\mathbf{s}_{N_\tau}, \tau))$

- Spatial resolution: 25×25 km $\Rightarrow$ smoothed to ECMWF grid. $\mathbf{s} \in \mathbf{S}(\tau)$ where $\mathbf{S}(\tau)$ and N_τ depends on time.
- Temporal resolution: QuikSCAT completes an orbit in 101 minutes with a recurrence period of 4 days.

The Kalman filter

Gauss (1795) least squares

Kolmogorov (1941)-Wiener (1942)

dynamic prediction

Follin (1955) Swerling (1958)

Kalman (1960)

recursive formulation

prediction depends on how far current state is from average

Extensions

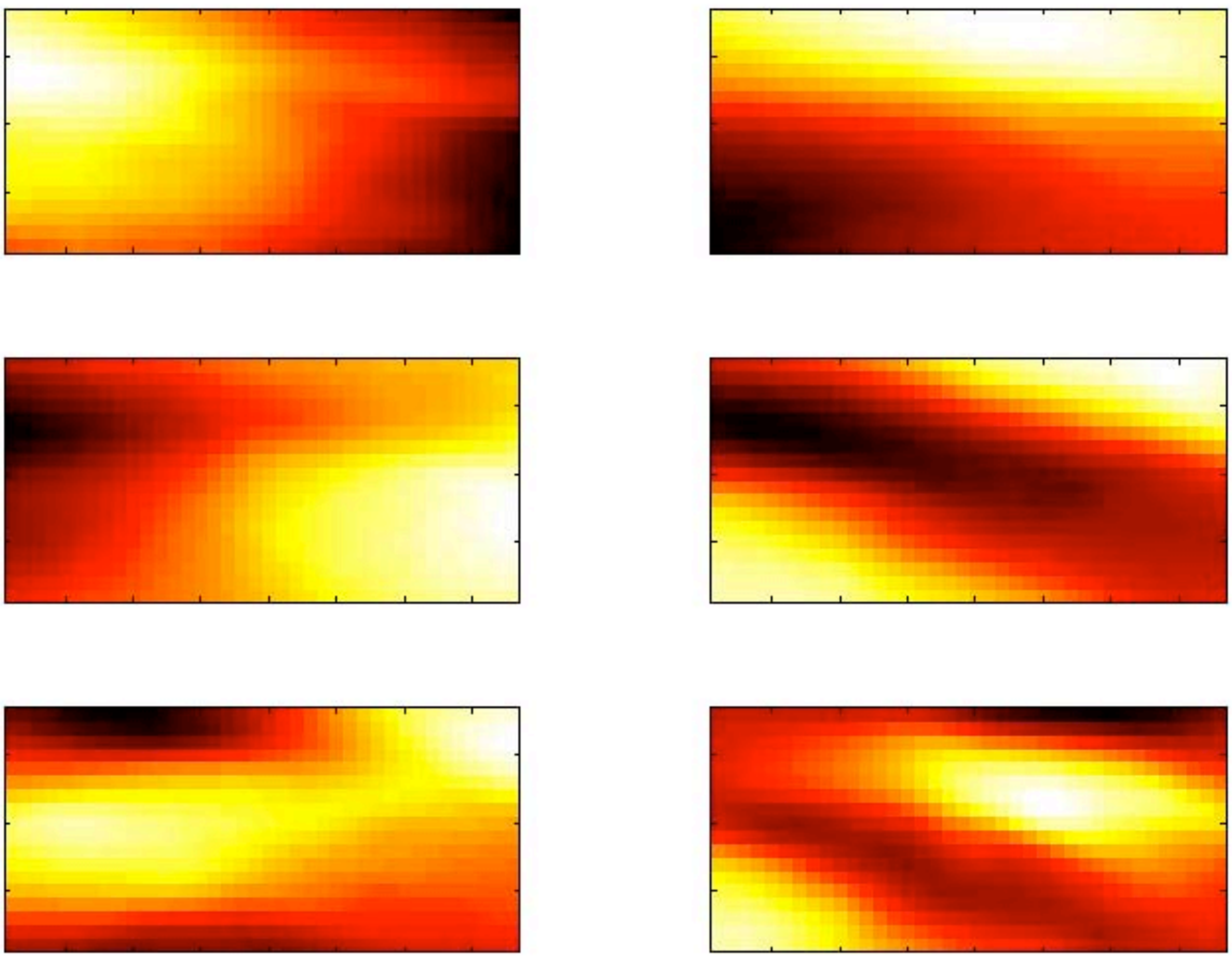
A state-space model

Write the forecast anomalies as a weighted average $Y(s,t) = \sum a_i(t)\phi_i(s)$ of EOFs (computed from the empirical covariance) plus small-scale noise.

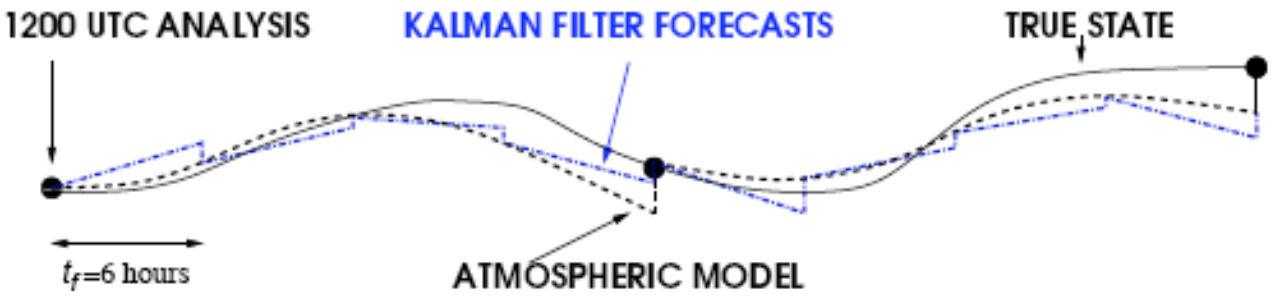
The average develops as a vector autoregressive model:

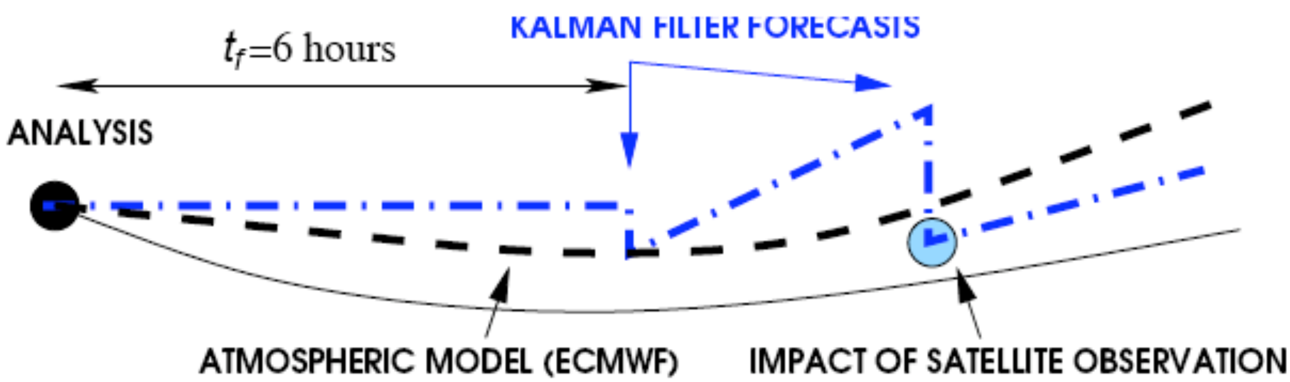
$$Y(s,t+\tau) = \int w_s(u)Y(u,t)du + \eta(s,t+\tau)$$

EOFs of wind forecasts

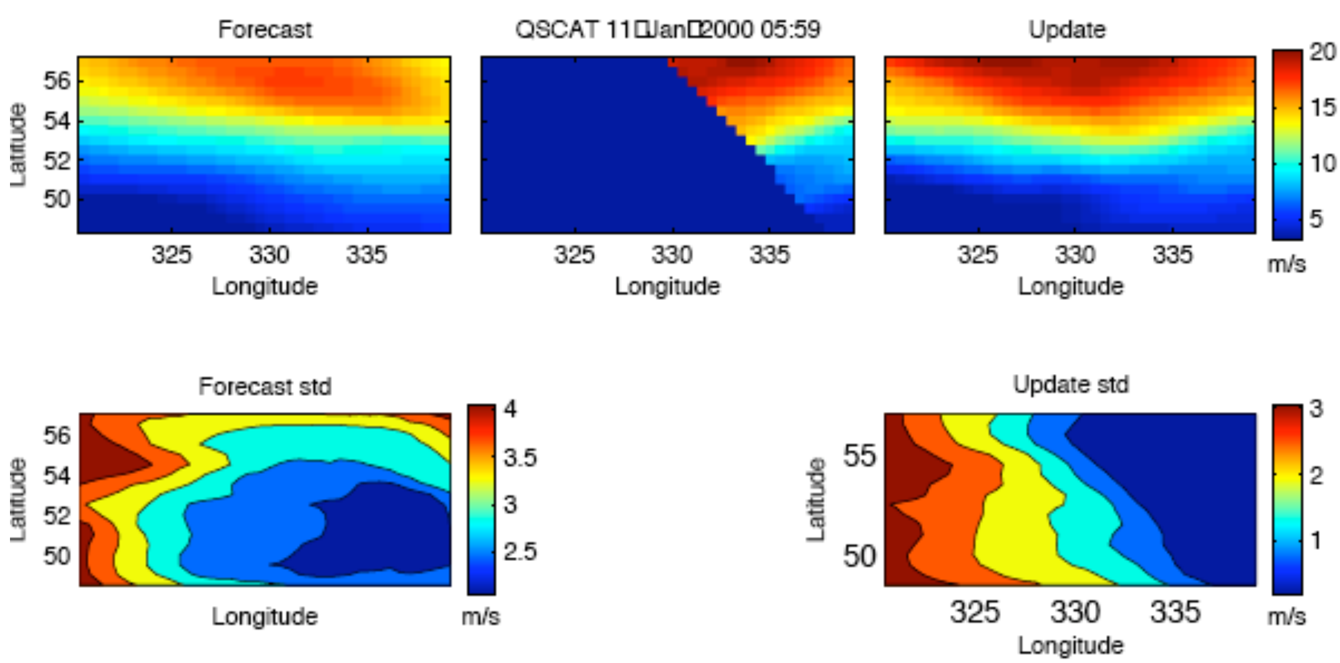


Kalman filter forecast emulates forecast model





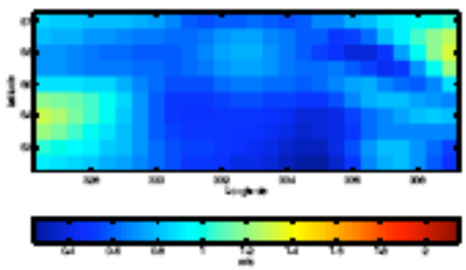
The effect of satellite data



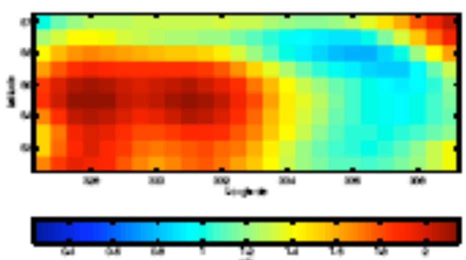
Model assessment

Difference from current forecast of

Previous forecast



Kalman filter



Satellite data assimilated