

Ozone data and Model output combined using Kriging with External Drift .

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IMPACT Project

<http://www.mai.liu.se/impact>

Normalization of Model output with Station data

In many environmental applications:

- **measurements** at points of a region,
- **gridded data** covering the region.

Example: **ground samples** and **remote sensing data**.

This talk:

- combining **station data** and **model output**
→ multivariate geostatistics
- change of support problem
between station data and model output

Multivariate Kriging

Kriging is a special type of **optimal linear prediction** applied to random functions in space or time with the particular requirement that their covariance structure should be known. → **Cokriging**

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Covariance structure: **variograms** (or covariance functions) for a set of variables.

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Covariance structure: **variograms** (or covariance functions) for a set of variables.

Drift: translation-invariant **polynomial drift**, or **external drift**.

Data: Configuration & Neighborhood

Data configuration:

sites of different types of measurements
in a spatial/temporal domain.

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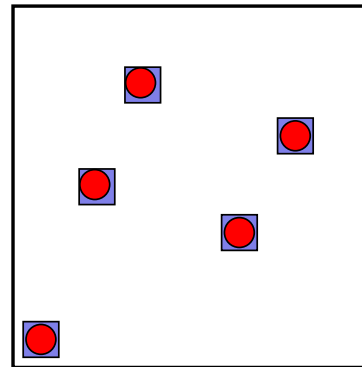
Neighborhood:

a subset of data used in cokriging.

Configurations: Iso- and Heterotopic Data

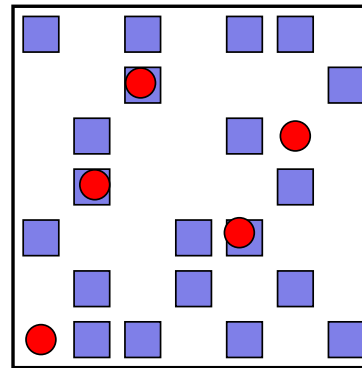
● primary data ■ secondary data

Isotopic data



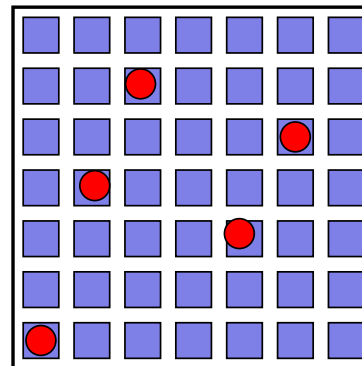
Sample sites
are shared

Heterotopic data



Sample sites
may be different

Dense auxiliary data



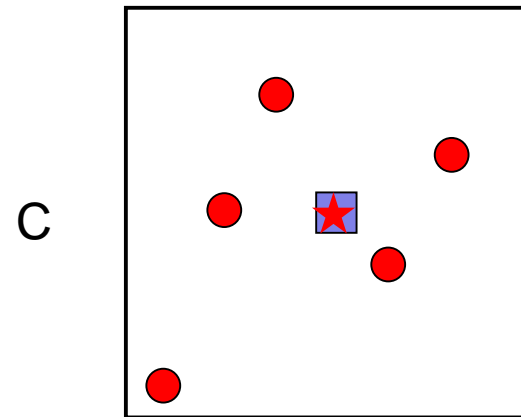
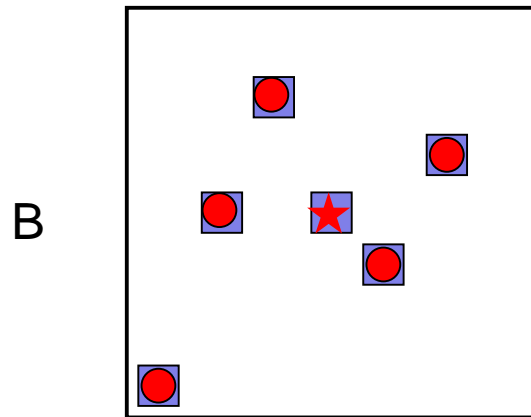
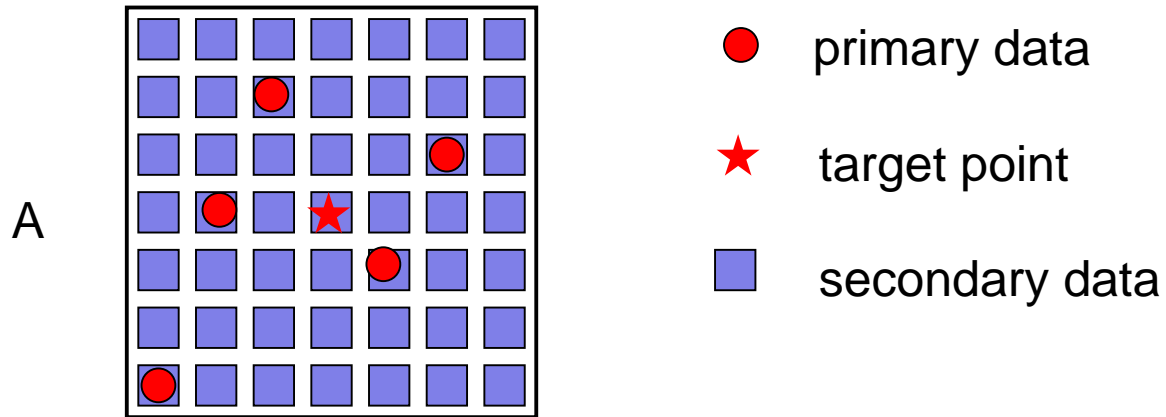
Secondary data
covers whole domain

Kriging with external drift

Estimator:
$$Z^*(\mathbf{x}_0) = \sum_{\alpha=1}^n w_{\alpha} Z(\mathbf{x}_{\alpha})$$

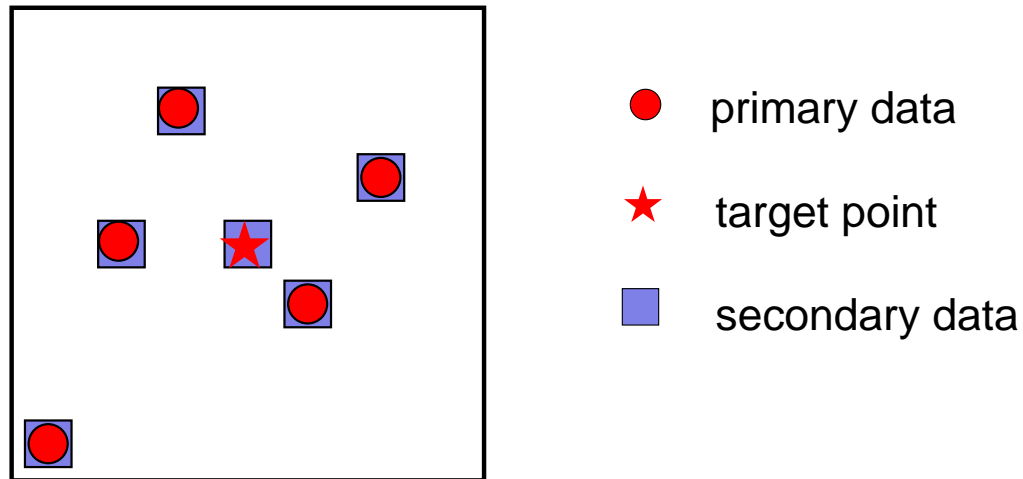
- constrained weights:
$$\sum w_{\alpha} = 1$$
- auxiliary variable $s(\mathbf{x})$ put as a constraint:
$$\sum_{\alpha} w_{\alpha} s(\mathbf{x}_{\alpha}) = s(\mathbf{x}_0)$$
- dense auxiliary data,
- linear relation with primary variable.

Configuration: dense auxiliary data



- A: neighborhood using **all data**
B: **multi-collocated** neighborhood
C: **collocated** neighborhood

Neighborhood: multi-collocated



Chilès & Delfiner (1999), Rivoirard (2000)

- all forms of cokriging: simple, ordinary, universal
- **multi-collocated** cokriging equivalent to **full** cokriging when **proportionality** in the cross-covariance model.

External drift: uses a multi-collocated neighborhood.

Neighborhood: multi-located

Cokriging **with all data** is equivalent to cokriging **with multi-located neighborhood** when (Rivoirard 2000):

$$C_Z(\mathbf{h}) = p^2 C(\mathbf{h}) + C_R(\mathbf{h})$$

$$C_S(\mathbf{h}) = C(\mathbf{h})$$

$$C_{ZS}(\mathbf{h}) = p C(\mathbf{h})$$

where p is a proportionality coefficient,
 $C_R(\mathbf{h})$ is the covariance of the residual,
i.e. the difference between $Z(\mathbf{x})$ and $S(\mathbf{x})$.

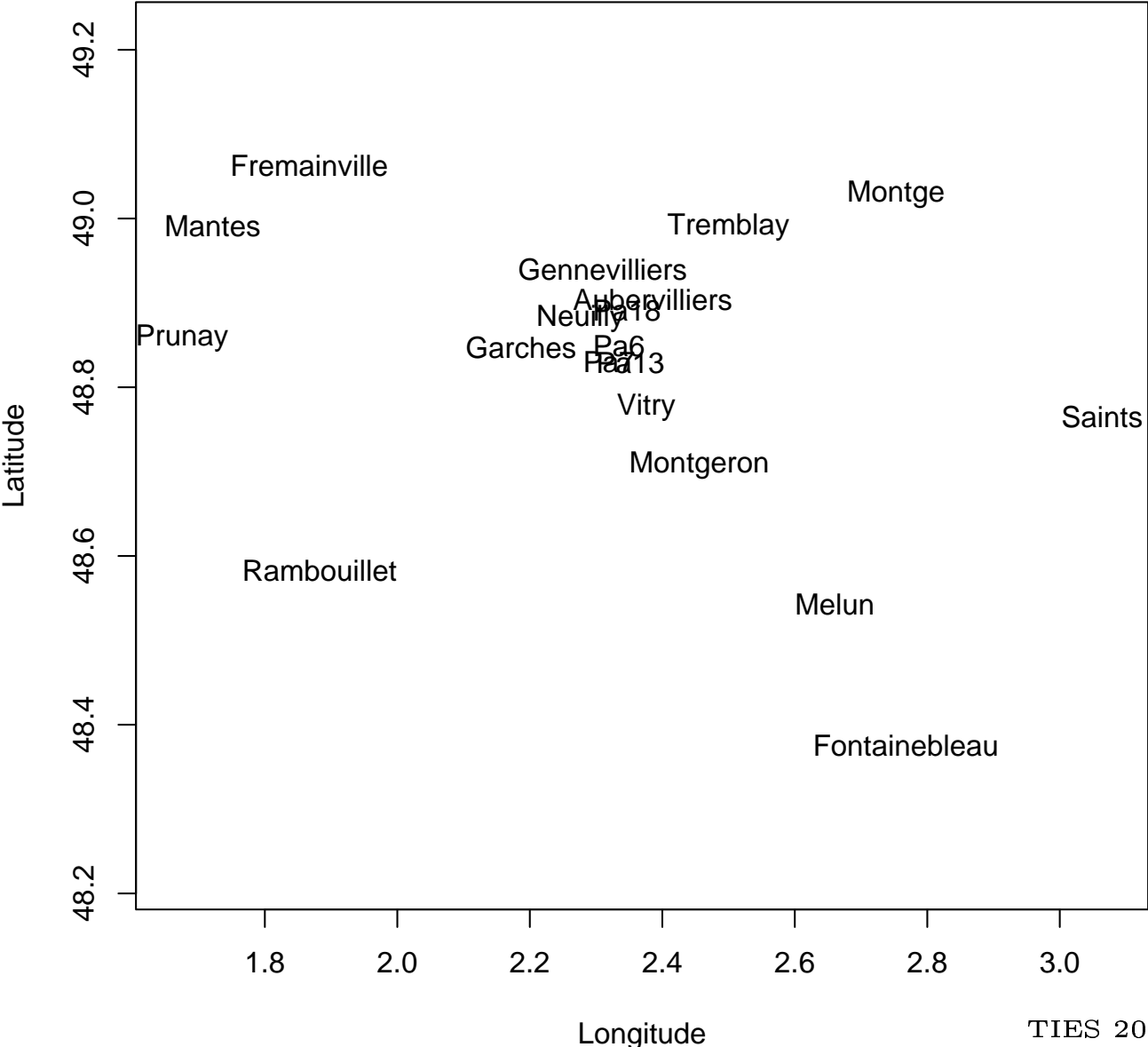
- $C_S(\mathbf{h})$ more regular than $C_Z(\mathbf{h})$,
if $C_R(\mathbf{h})$ less regular than $C(\mathbf{h})$,
- S is smoother than Z .

CASE STUDY: Ozone in Paris area

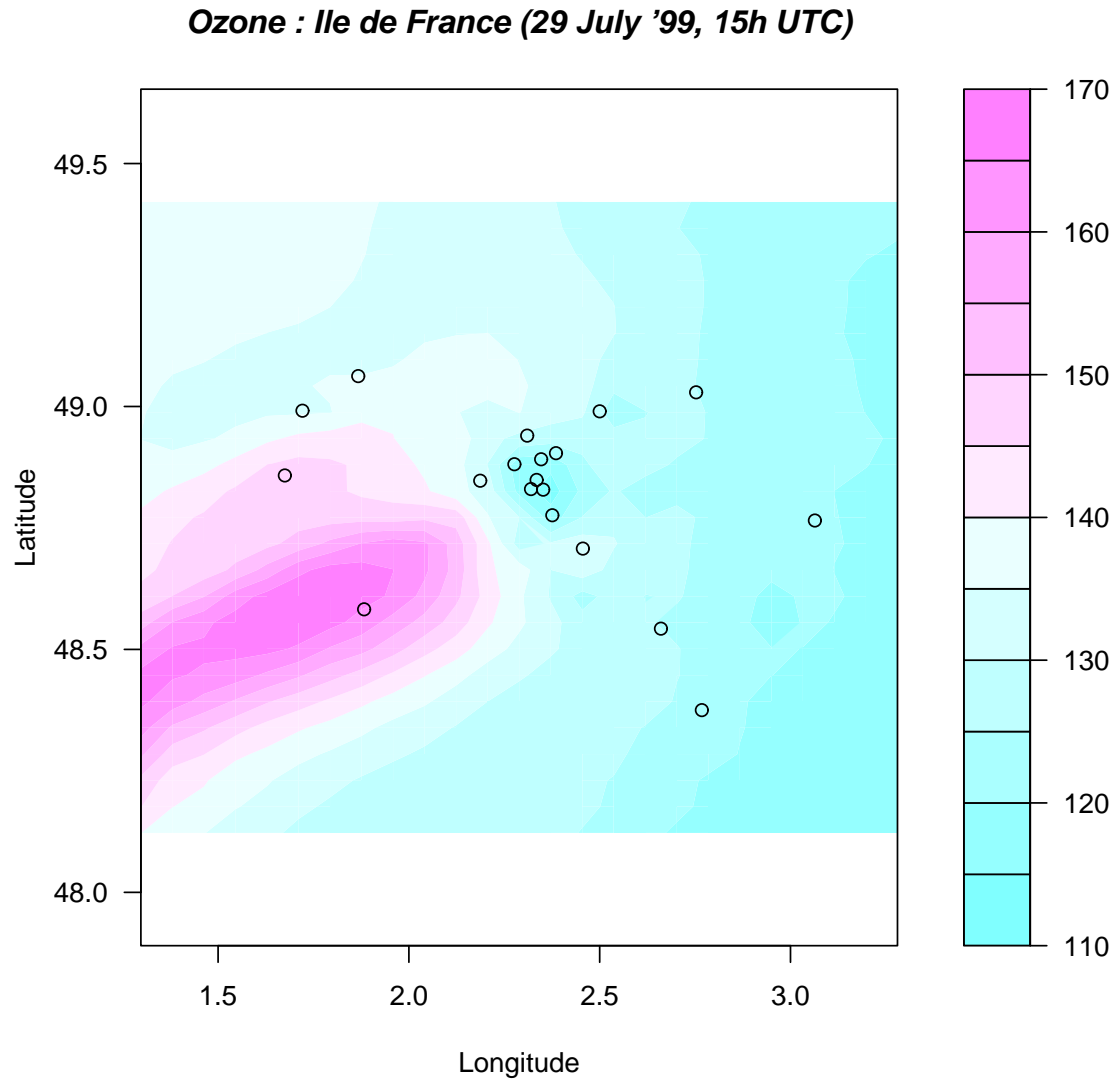
- Ozone hourly values from 19 AirParif stations
- CHIMERE model output (LMD/IPSL):
 - simplified chemistry transport, forced by ECMWF weather forecasts (80 gaseous substances, 200 processes)
 - $50 \times 50 \text{ Km}^2$ large scale resolution with $6 \times 6 \text{ Km}^2$ subgrid in $150 \times 150 \text{ km}^2$ square (Ile-de-France)

We compare two pollution events on 29 and 17 July 1999.

Ozone values at 19 AirParif Stations

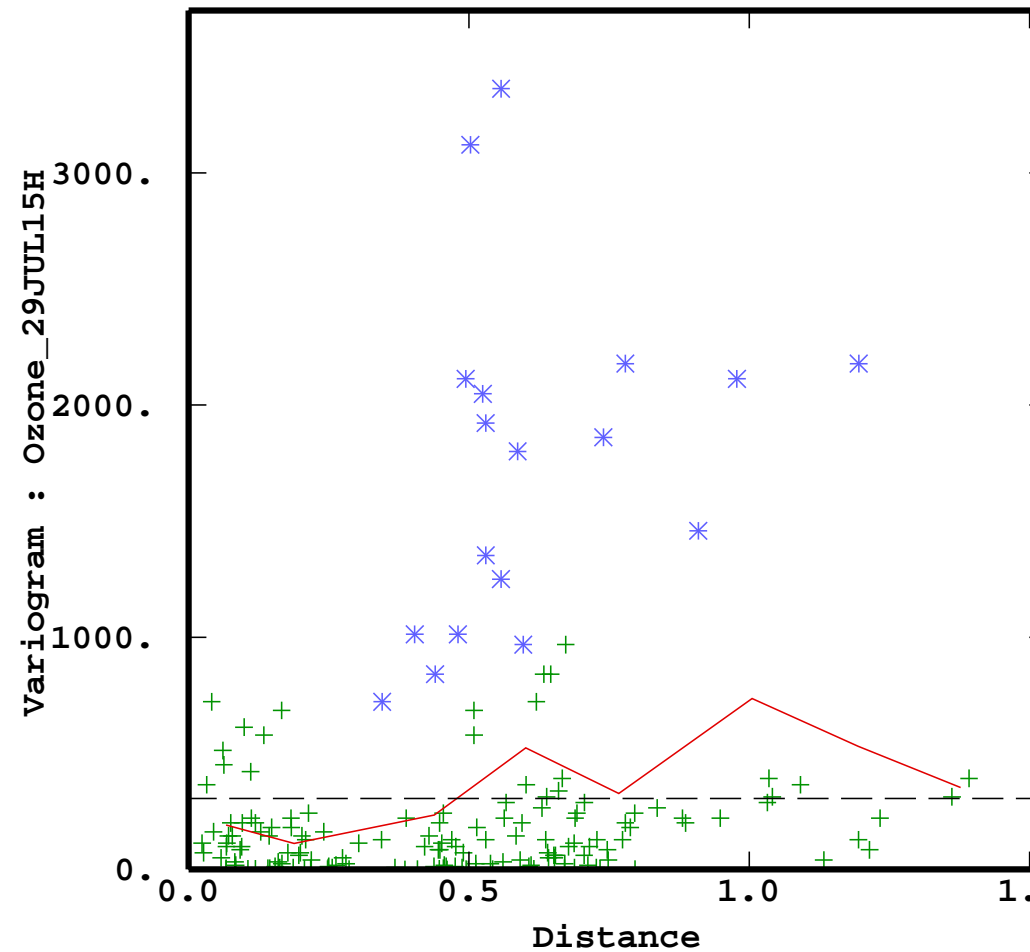
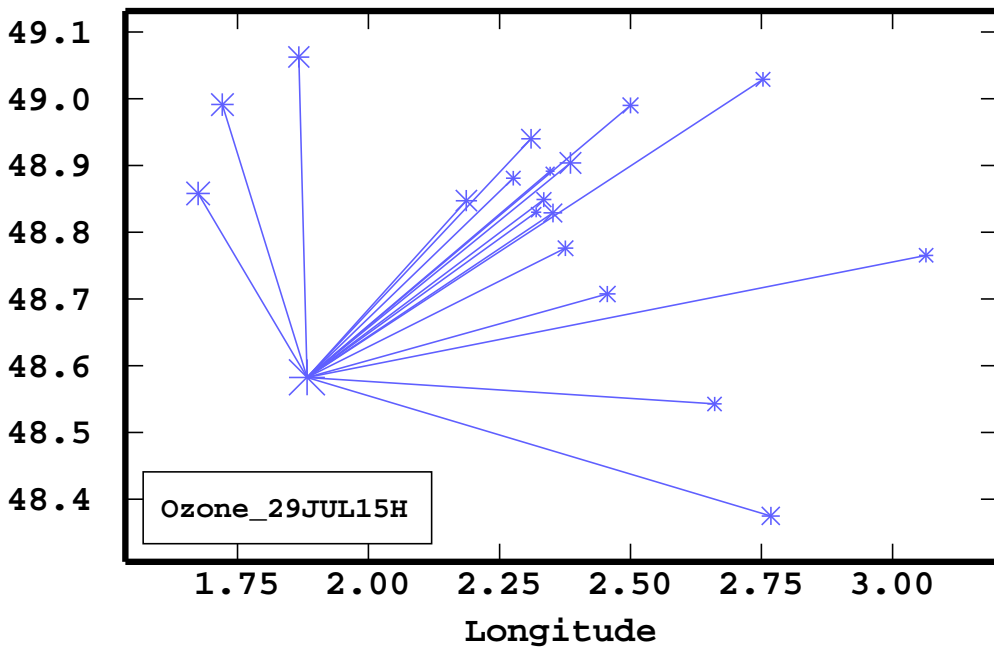


CHIMERE model: 29 July '99, 15h UT



Scale of pollution: **continental** plume

Variogram of Ozone on 29 July '99 at 15h UT

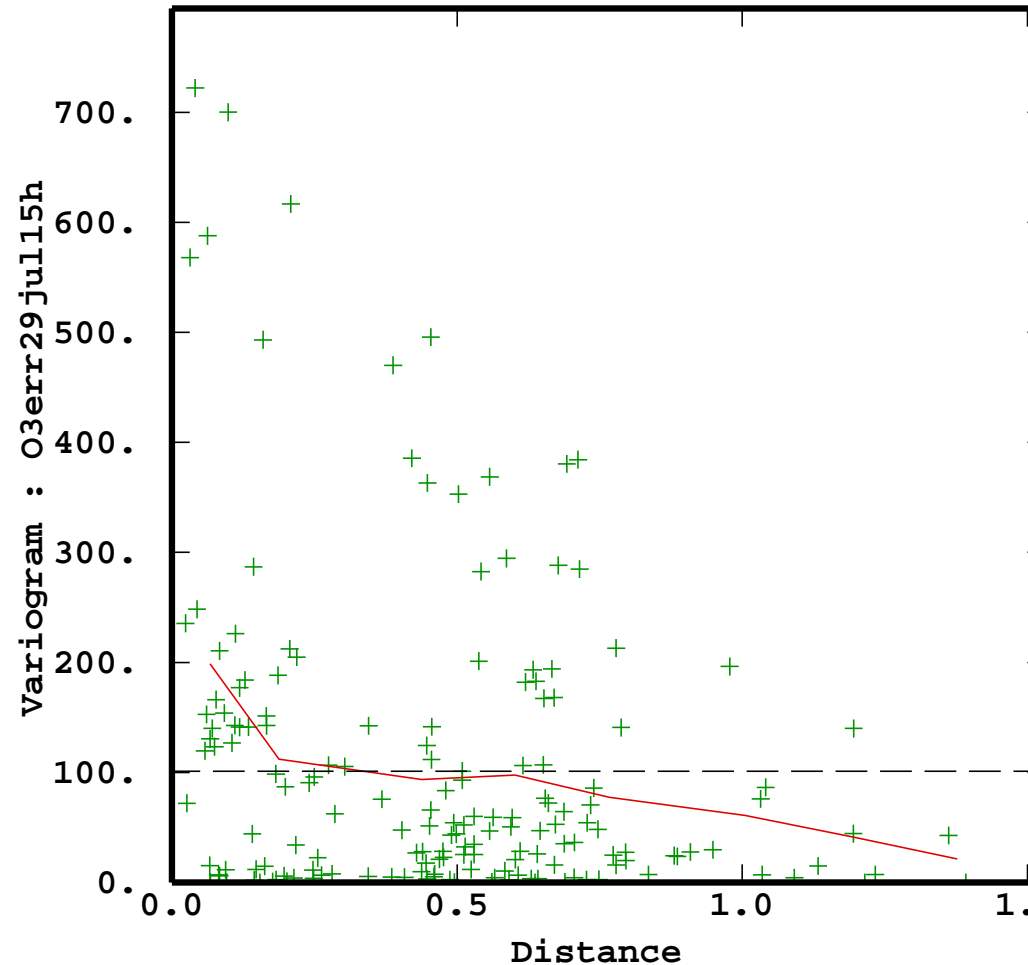
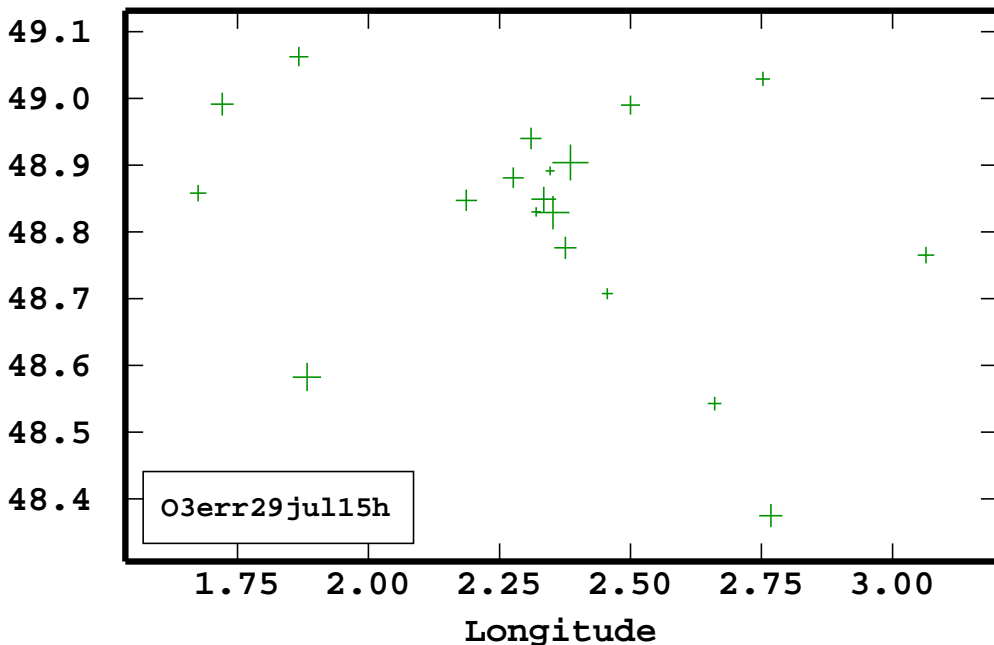


LEFT: Map of **highlighted** pairs of stations

RIGHT: **Variogram cloud**

Variogram of RESIDUALS

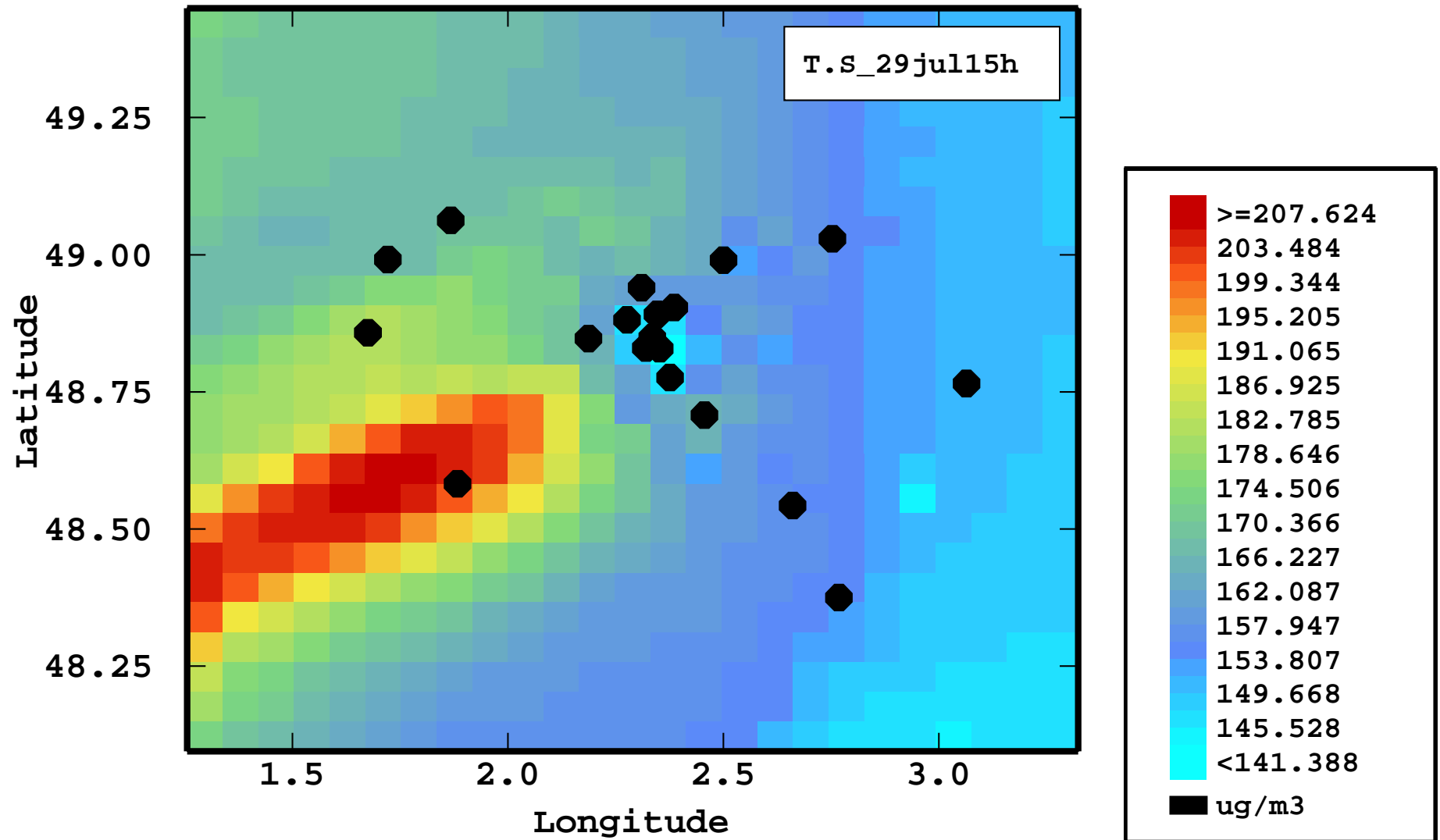
RESIDUALS: AirParif Stations minus CHIMERE Model



Variogram cloud (+) and experimental variogram (—)

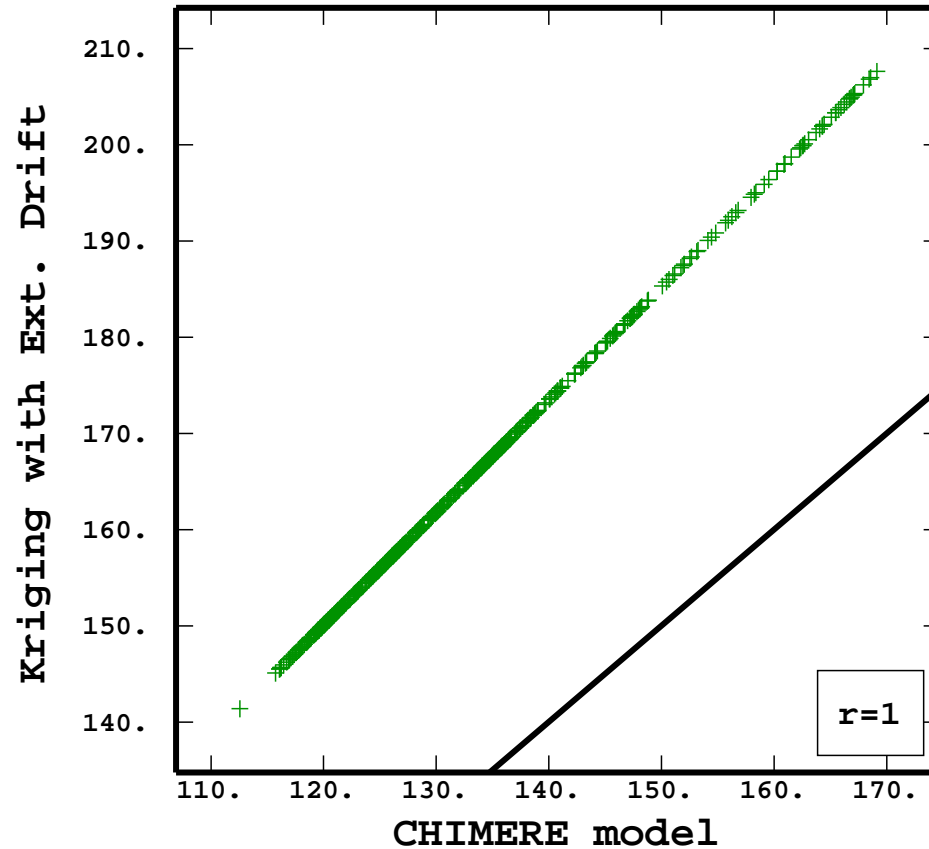
Interpretation: white noise, i.e. **nugget effect** model

Kriging with external drift: 29 July '99 at 15h



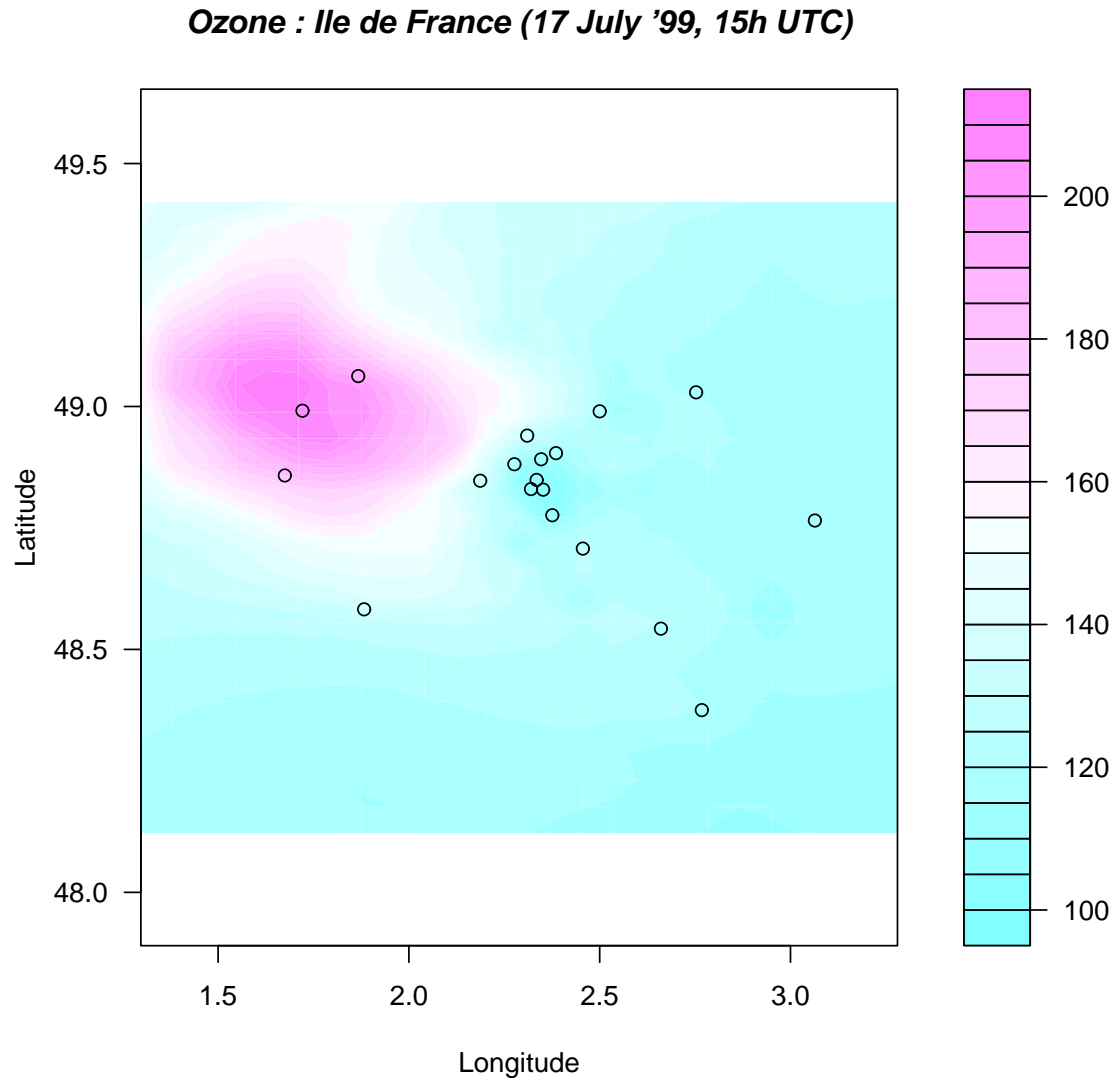
KED model: **no autocorrelation** between residuals

KED vs CHIMERE



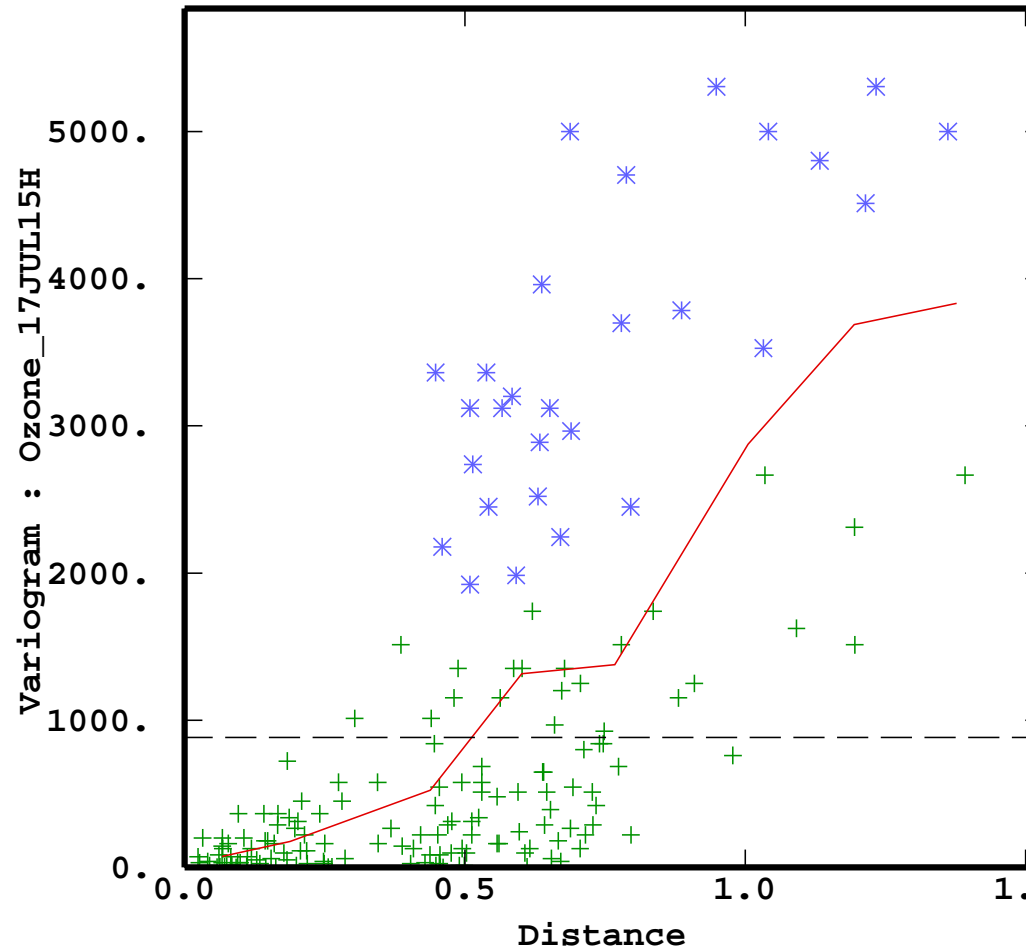
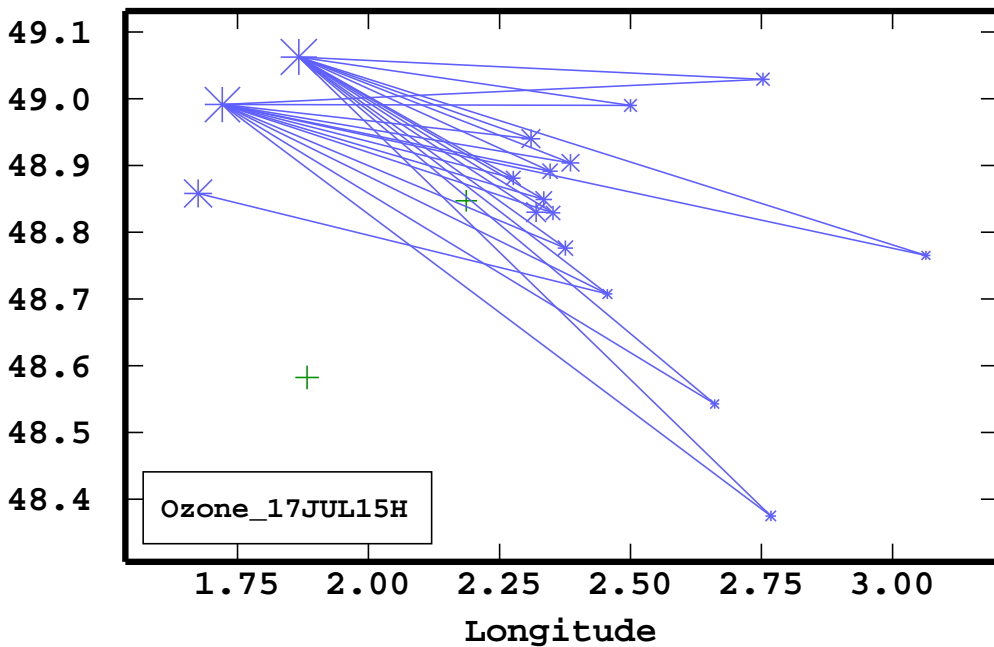
The CHIMERE model is linearly transformed by KED:
amounts mainly to **adding a constant!**

CHIMERE model: 17 July '99, 15h UT



Scale of pollution: **local** plume (i.e. within domain)

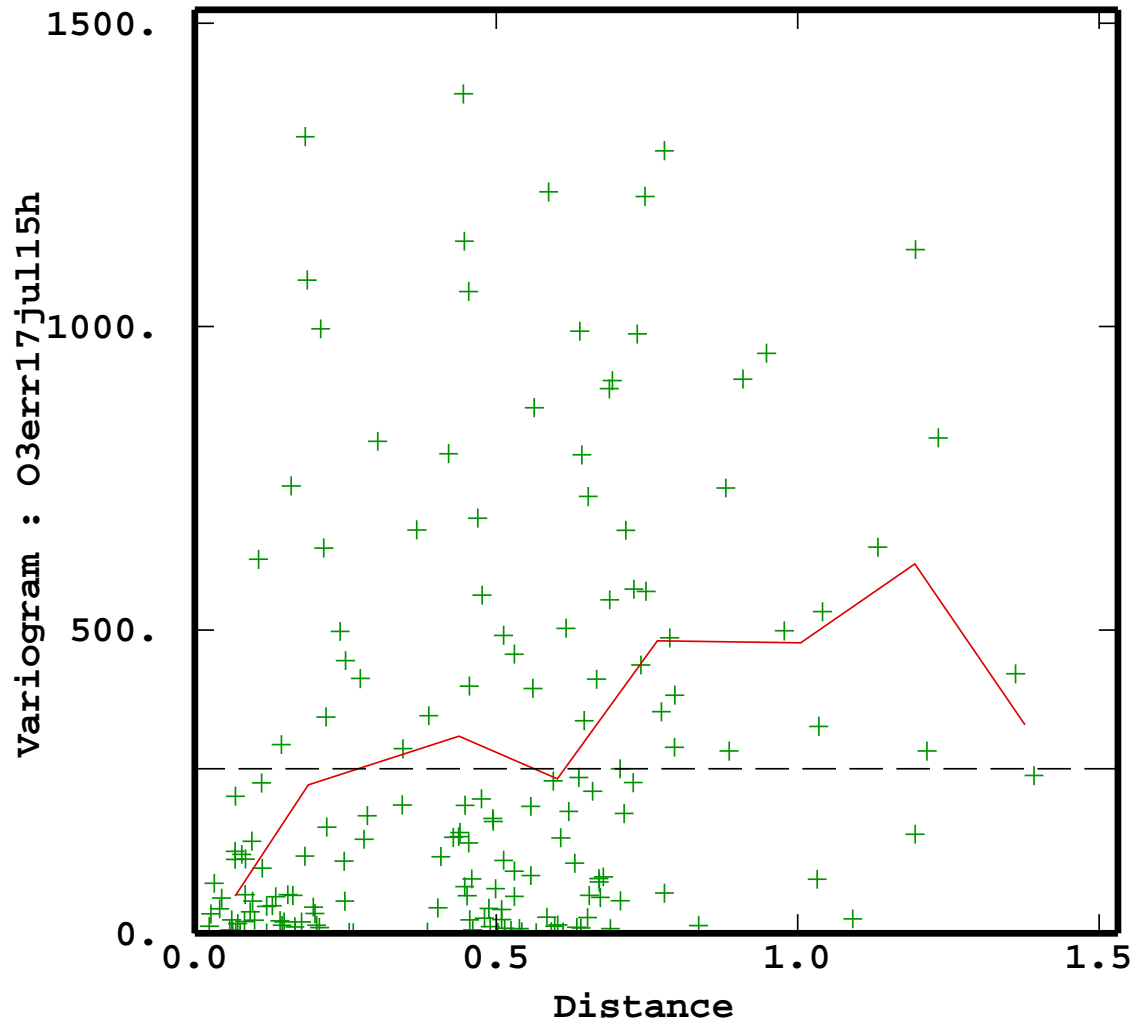
Variogram of Ozone on 17 July '99 at 15h UT



LEFT: Map of **highlighted** pairs of stations

RIGHT: **Variogram cloud**

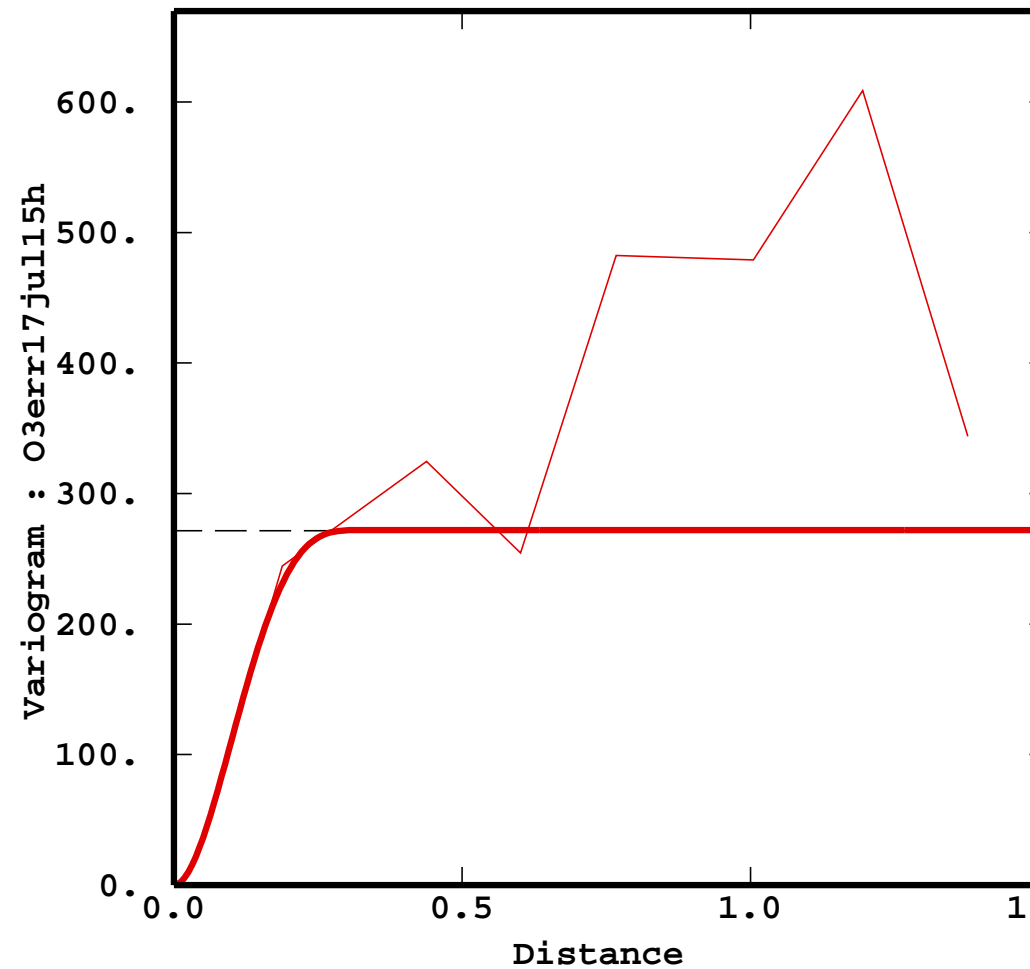
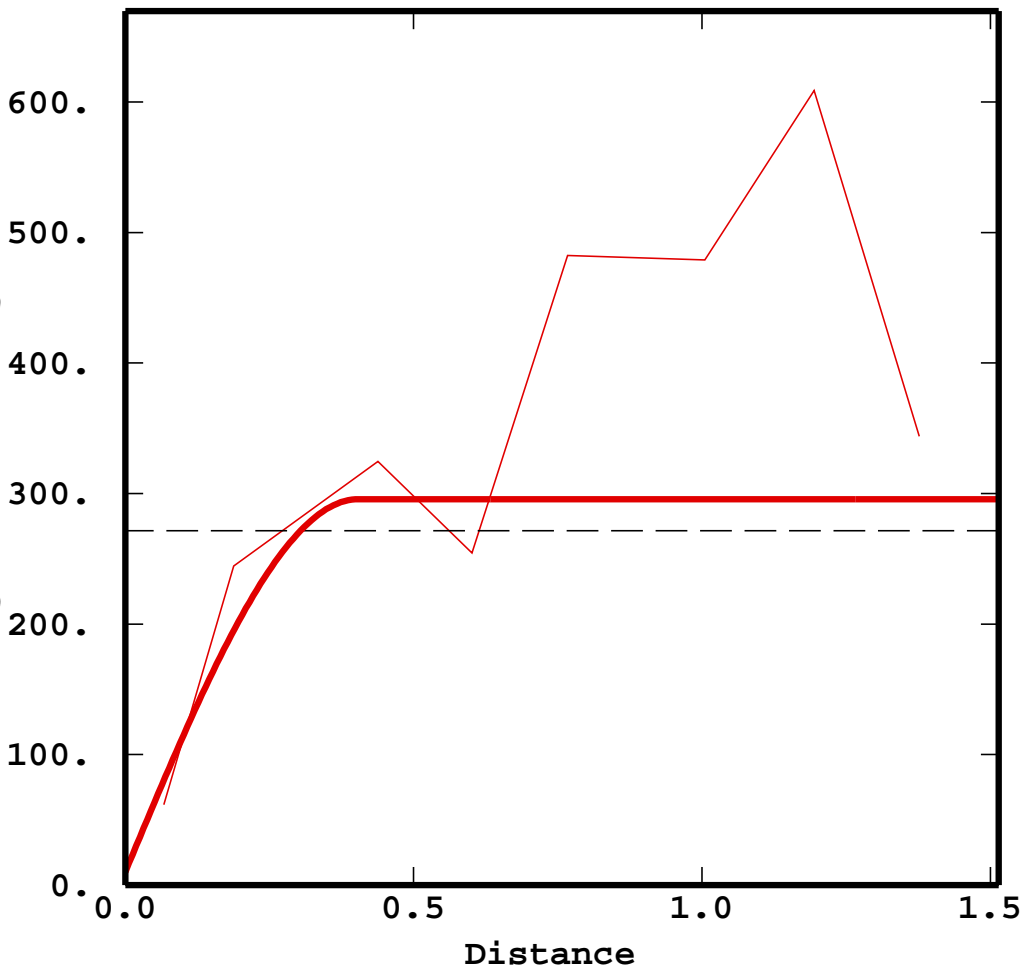
Variogram of RESIDUALS



Variogram cloud (+) and experimental variogram (—)

Interpretation: autocorrelated residuals

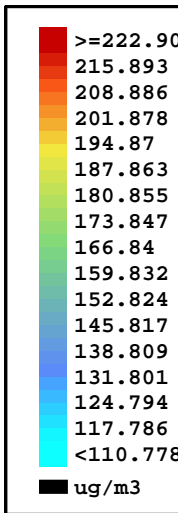
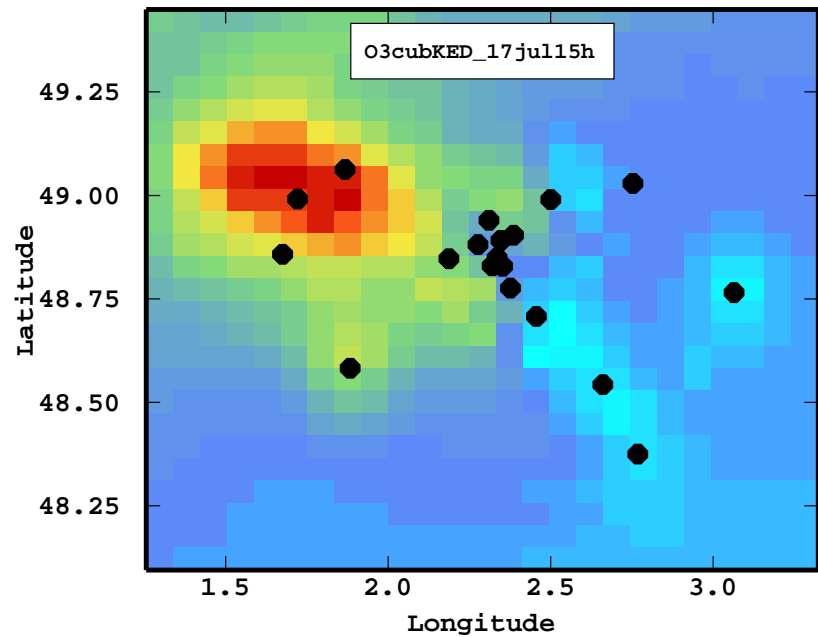
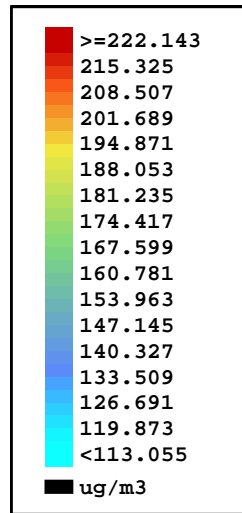
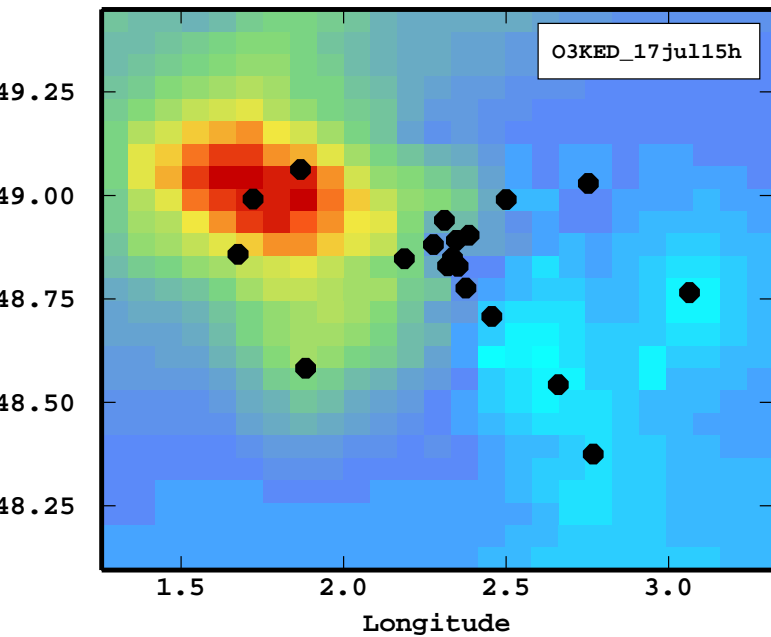
Spherical or Cubic variogram model?



LEFT Model: 285 spherical(h , range=.4) + 10 nugget(h)

RIGHT Model: 273 cubic(h , range=.33)

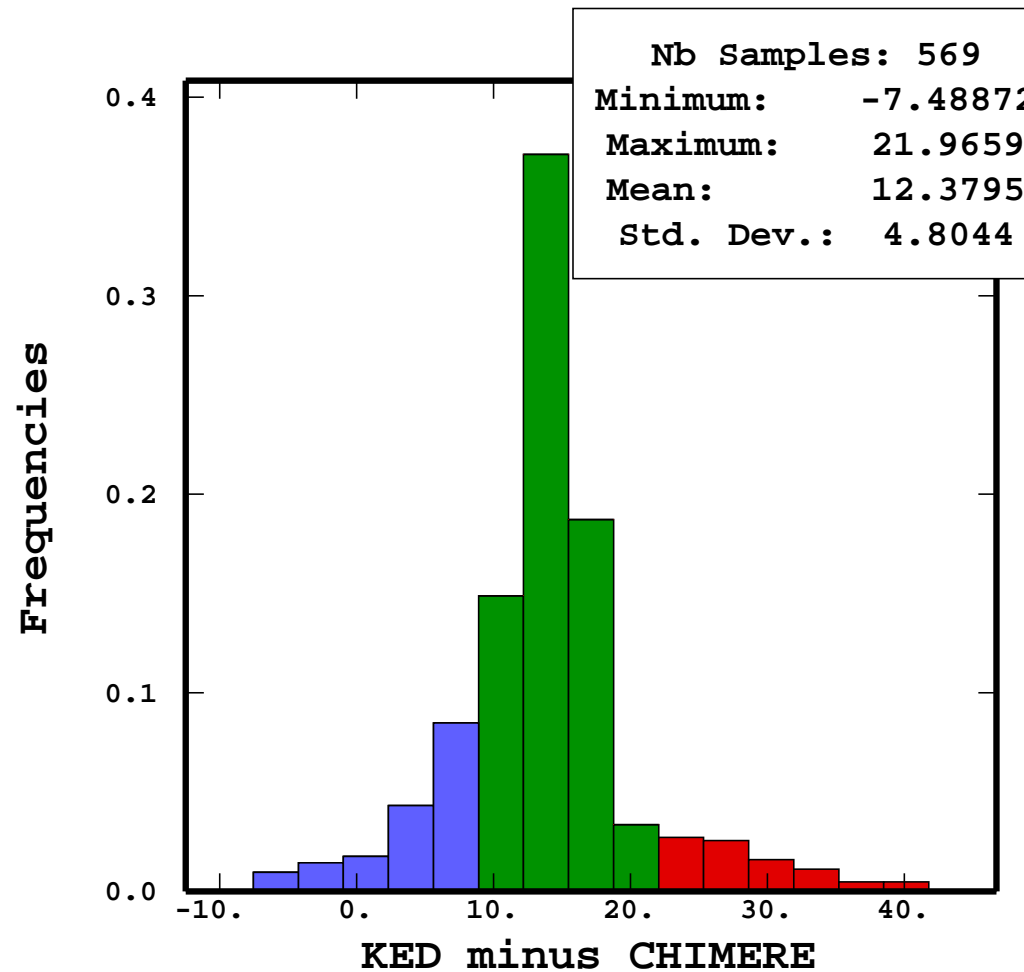
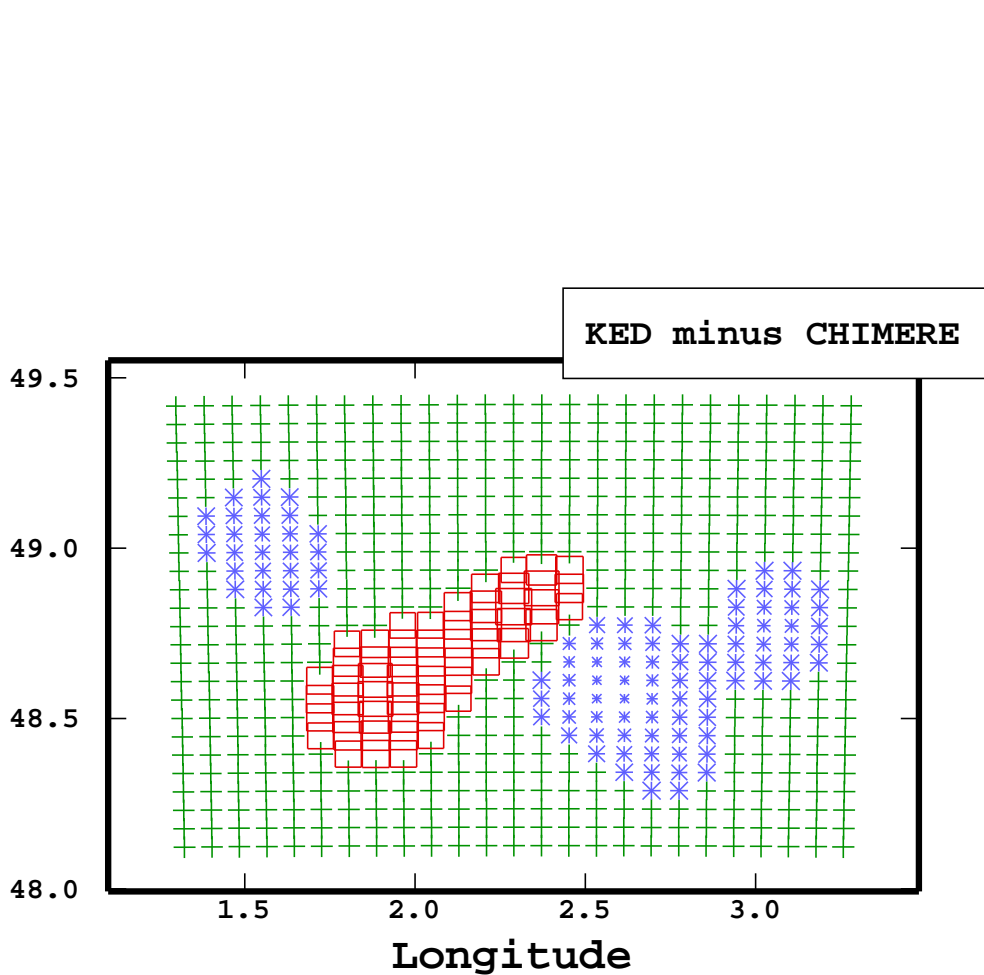
Kriging with external drift: 17 July '99 at 15h



LEFT: Spherical + Nugget variogram model

RIGHT: Cubic variogram model

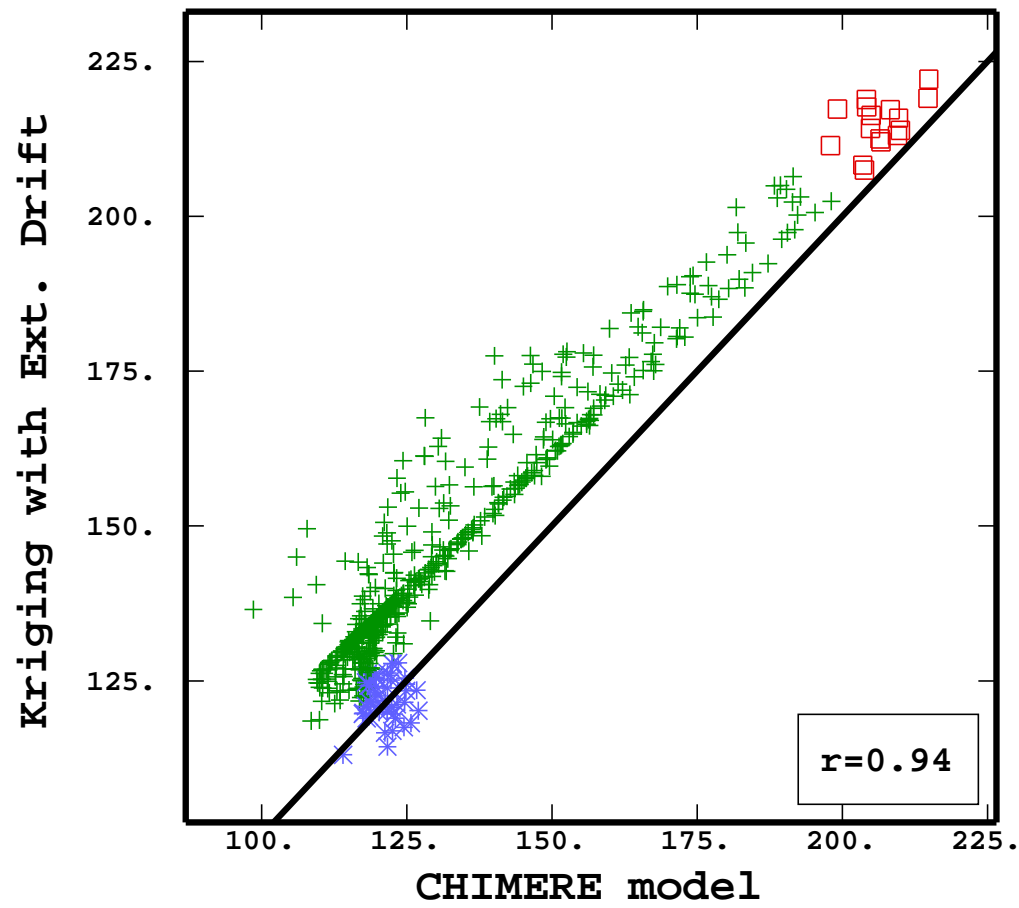
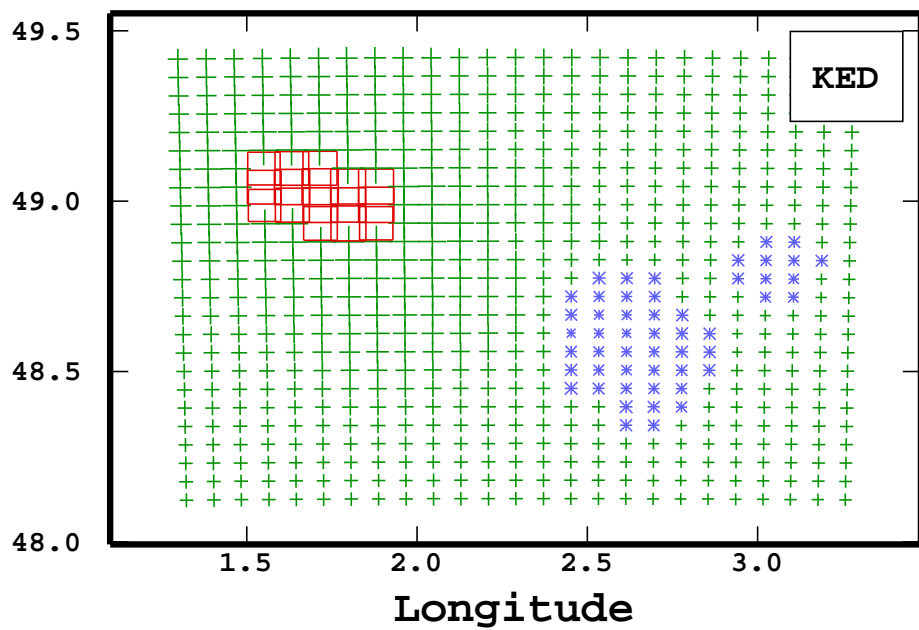
KED minus CHIMERE



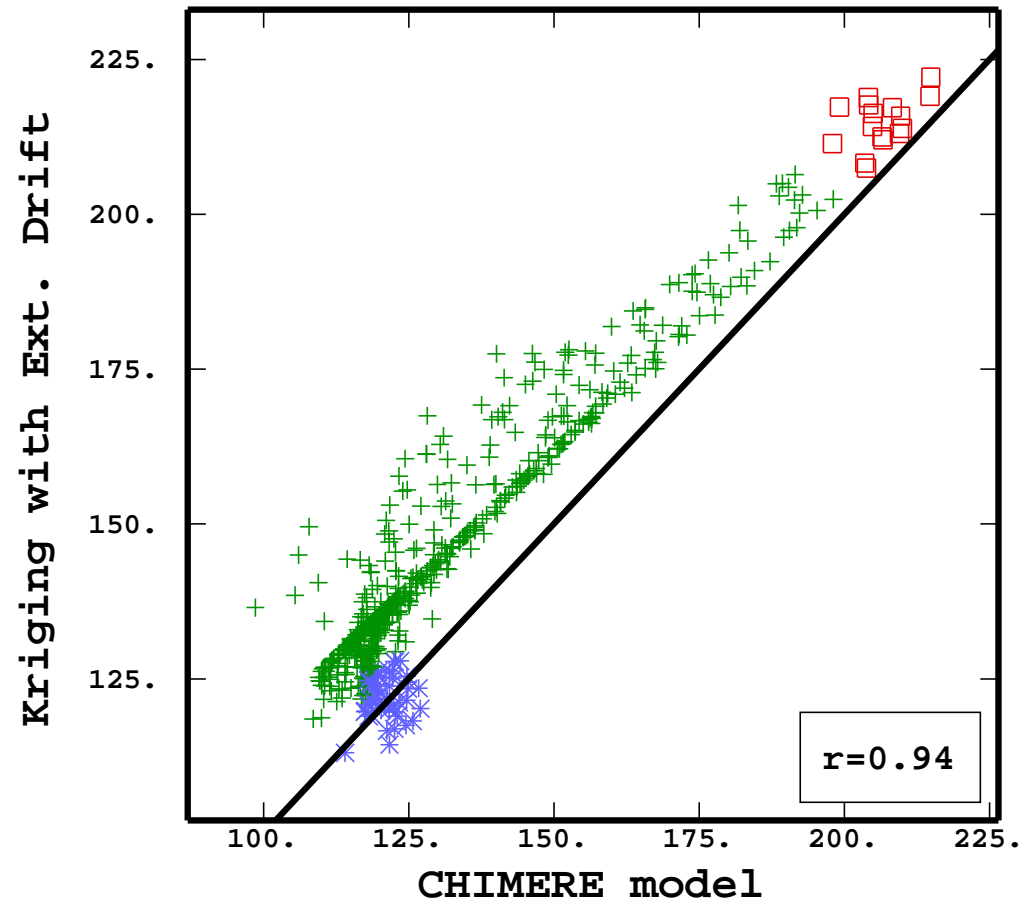
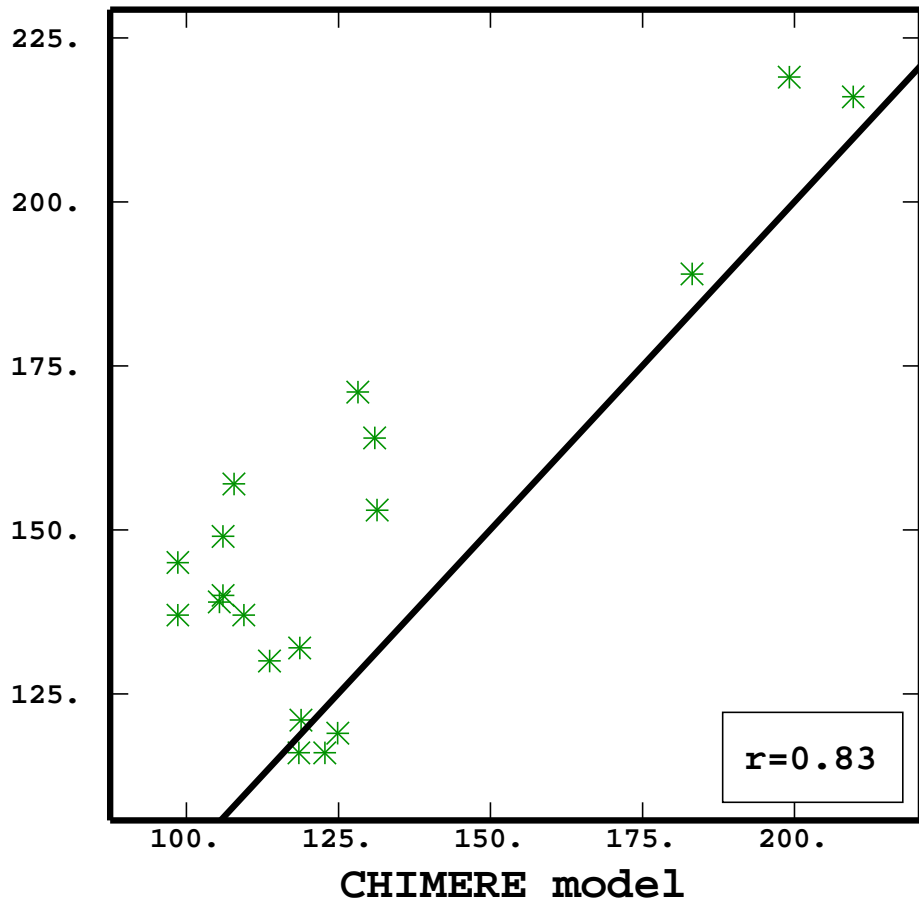
KED model: autocorrelated residuals

→ deformation of CHIMERE surface.

KED vs CHIMERE



AirParif Stations vs CHIMERE



External drift and Change of Support

Basic unsolved question

The measurements from stations and the numerical model output are not on the same support:

- *model output* on $6 \times 6 \text{ km}^2$ cells,
- *measurements* on a smaller support (which size?).

The variability of cell values should be lower than that of point values:

→ **change-of-support** problem?

CASE STUDY: Geostatistical simulation of O₃

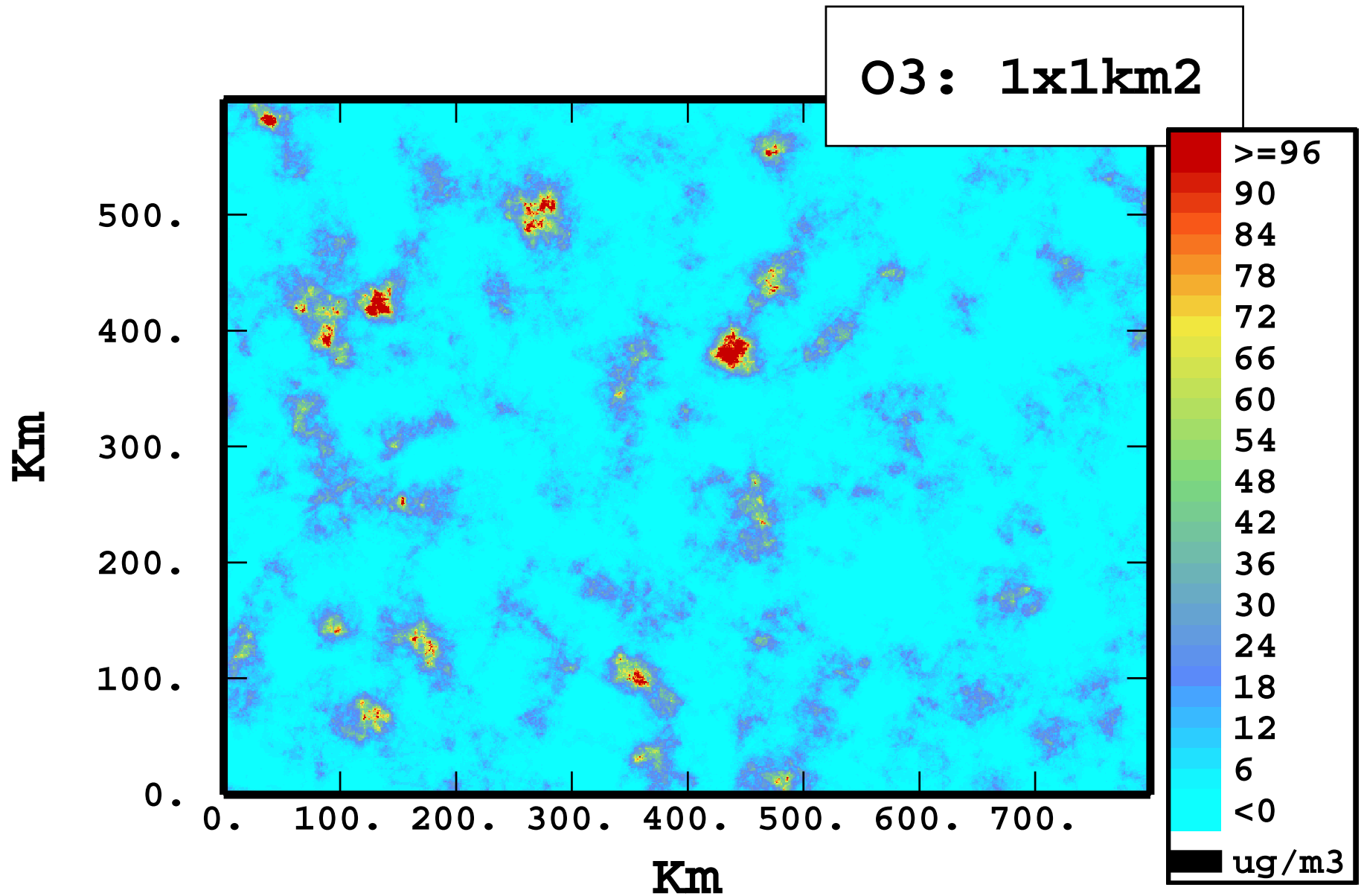
Simulation of the exponential of a Gaussian variable

Region 600 × 800 Km²

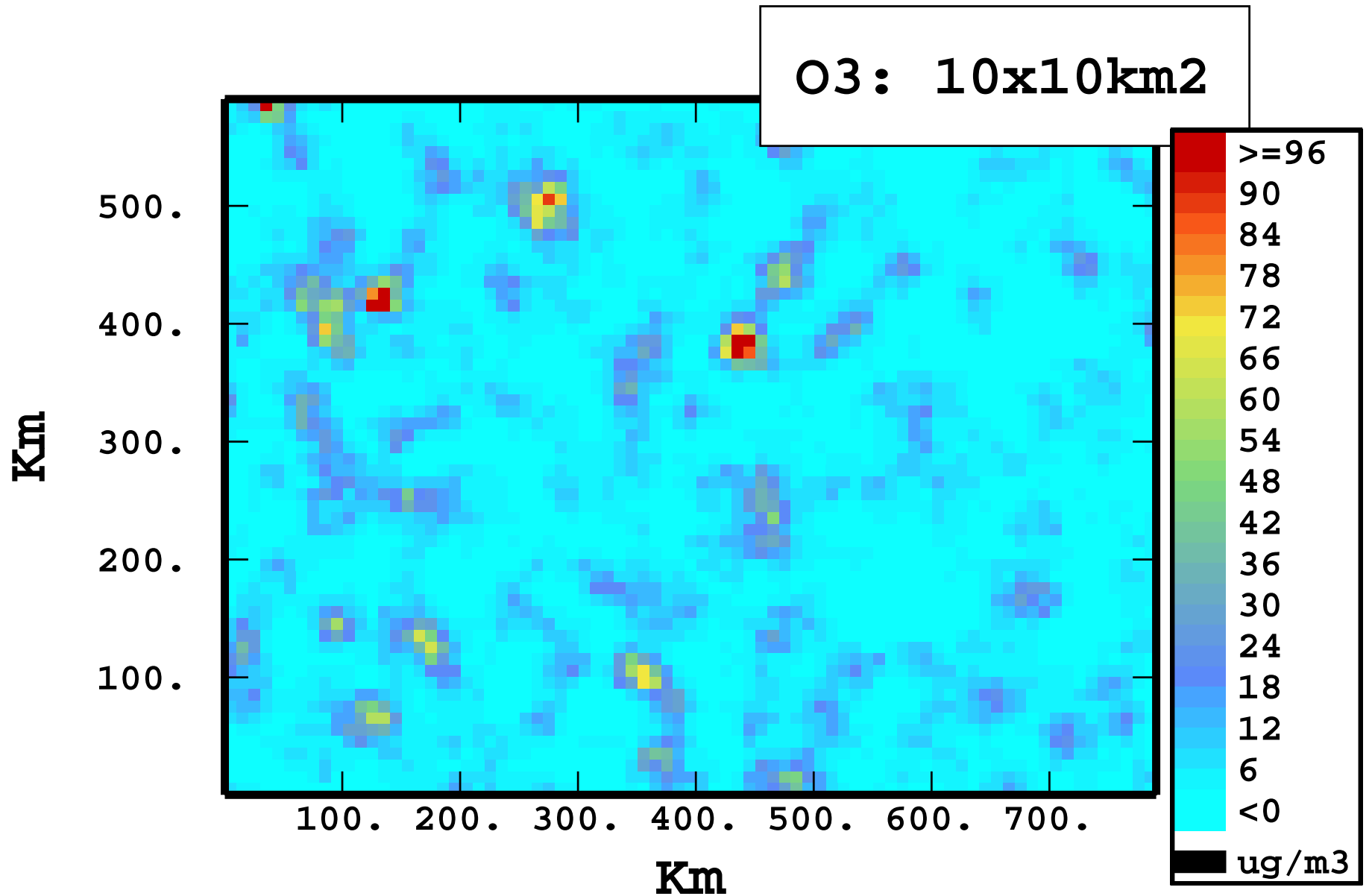
Cells 1 × 1 Km²

Variogram with a range of 50 Km

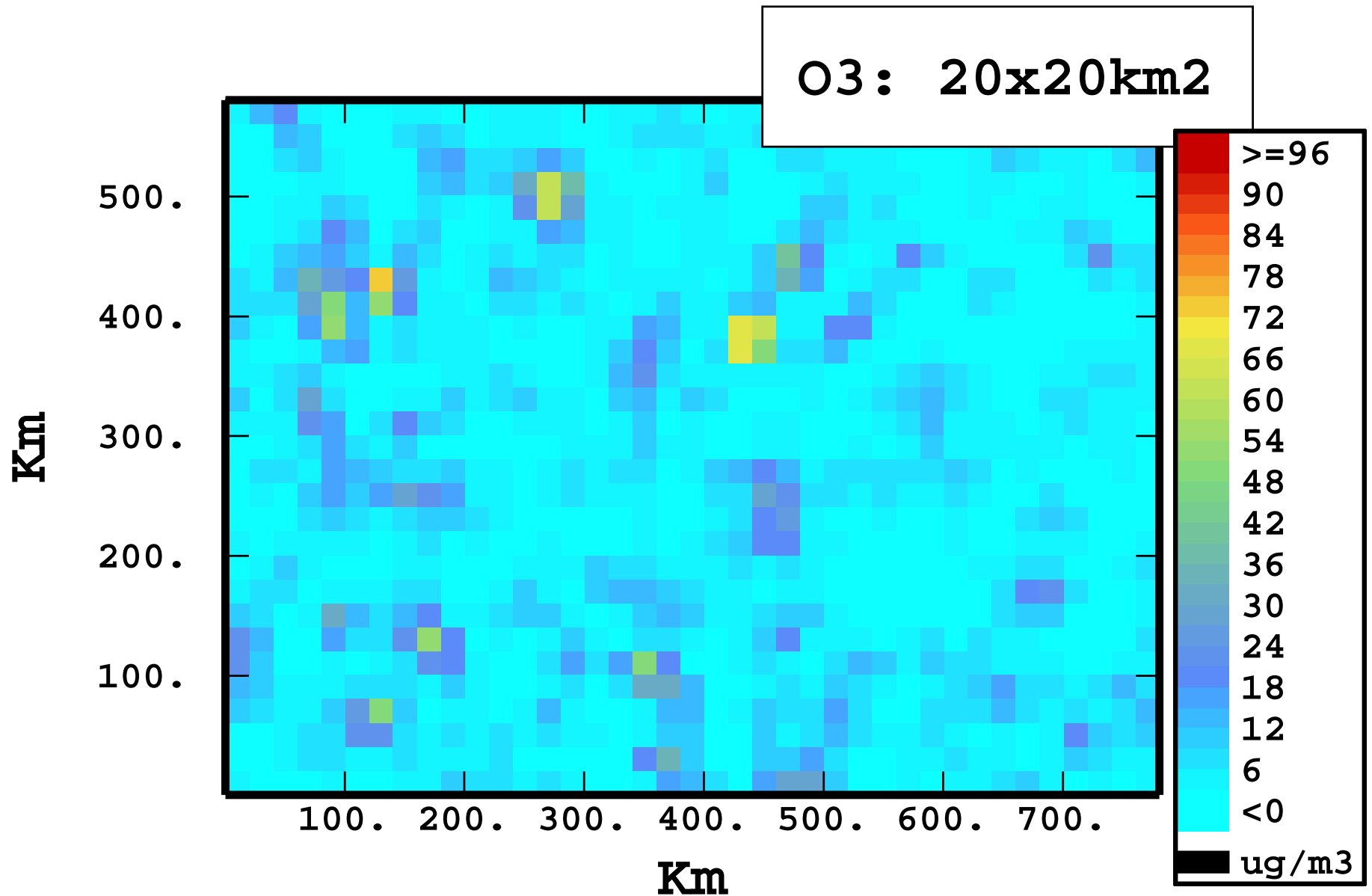
Simulation of Ozone: $1 \times 1 \text{ Km}^2$ support



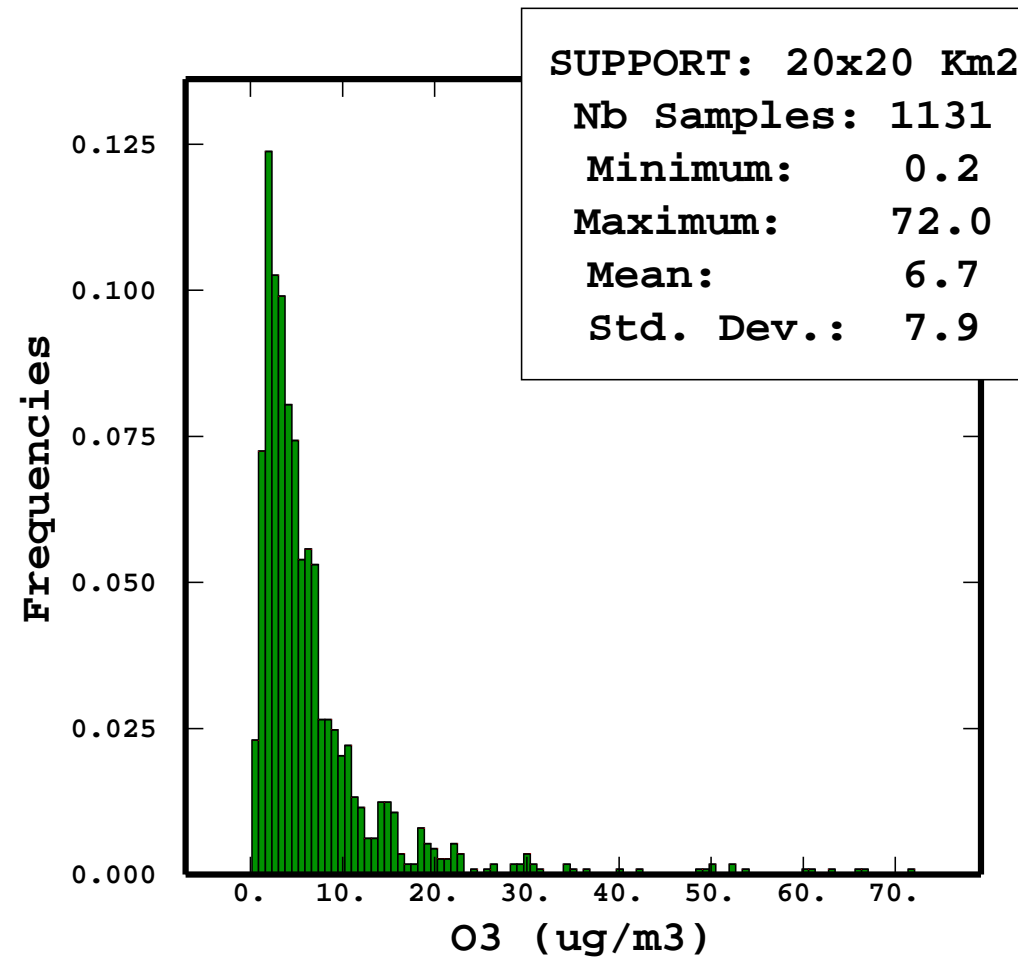
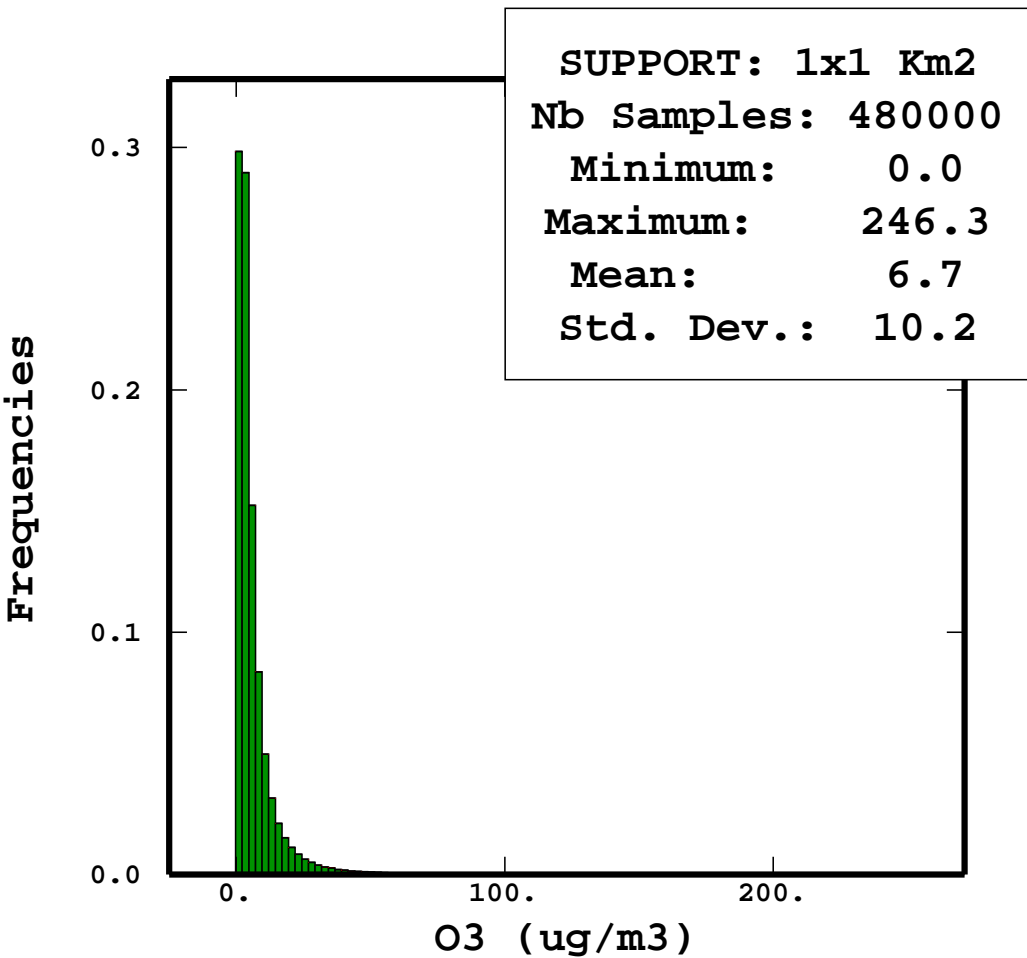
Simulation of Ozone: 10×10 Km² support



Simulation of Ozone: 20×20 Km² support

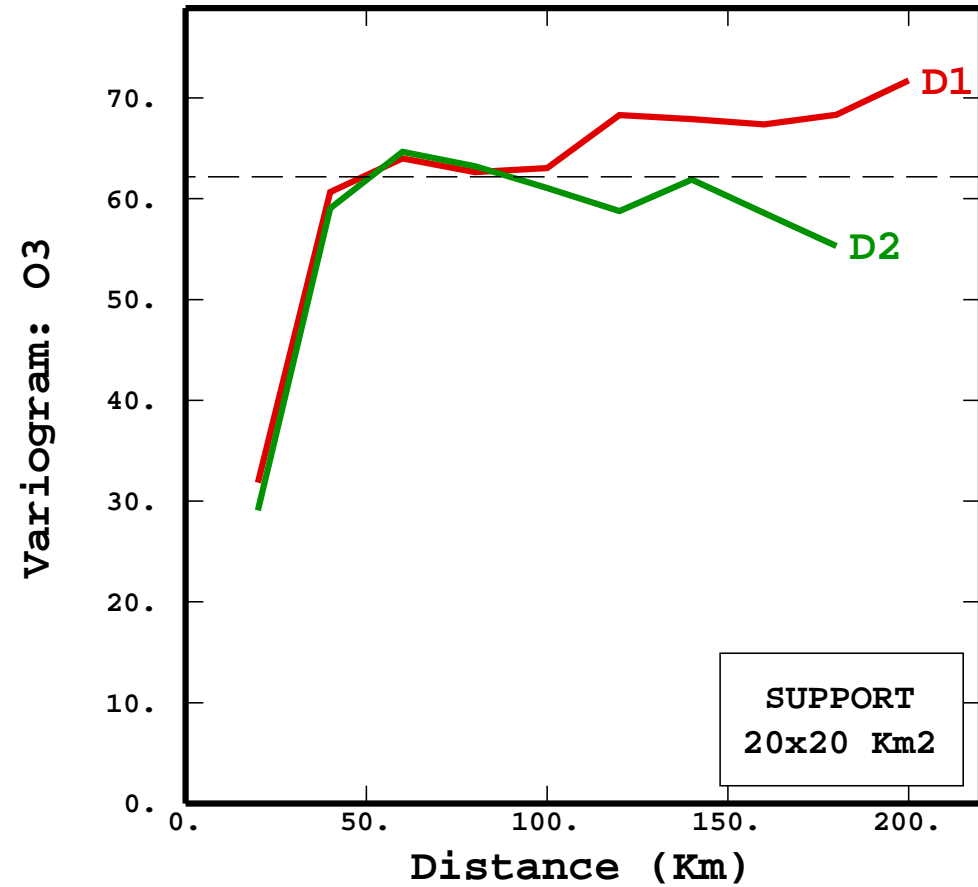
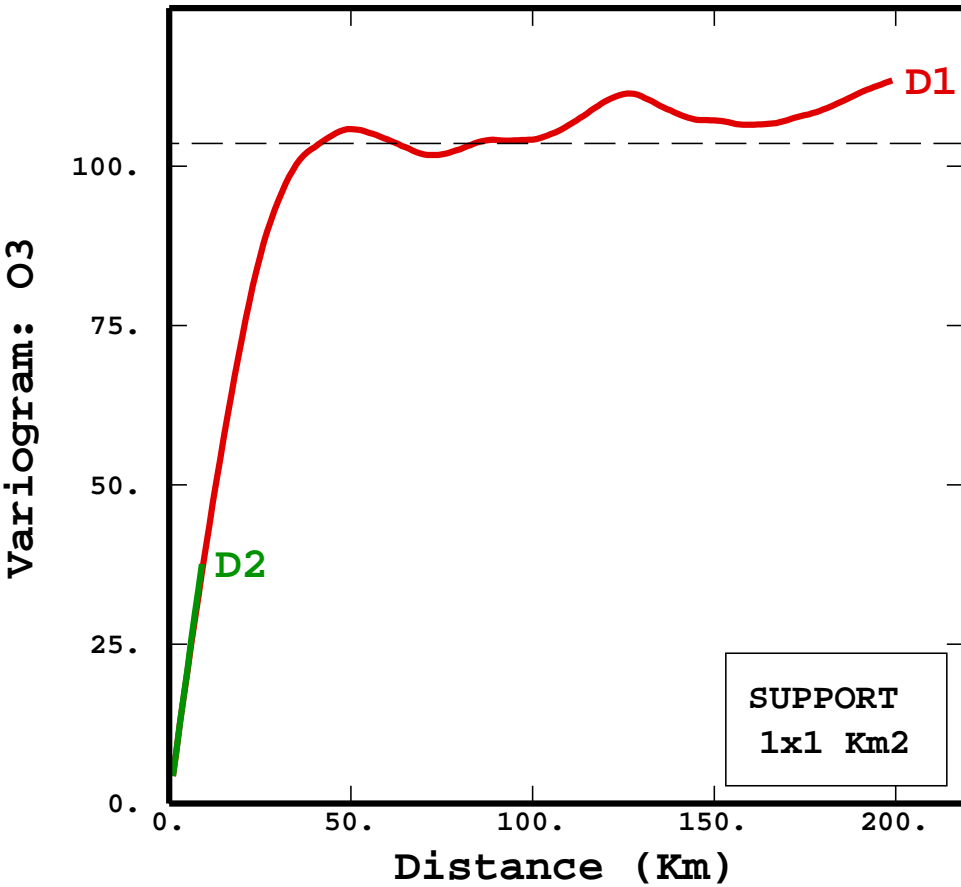


Simulation of Ozone



Increasing the support: the means are equal,
but the extremes and the variance are reduced

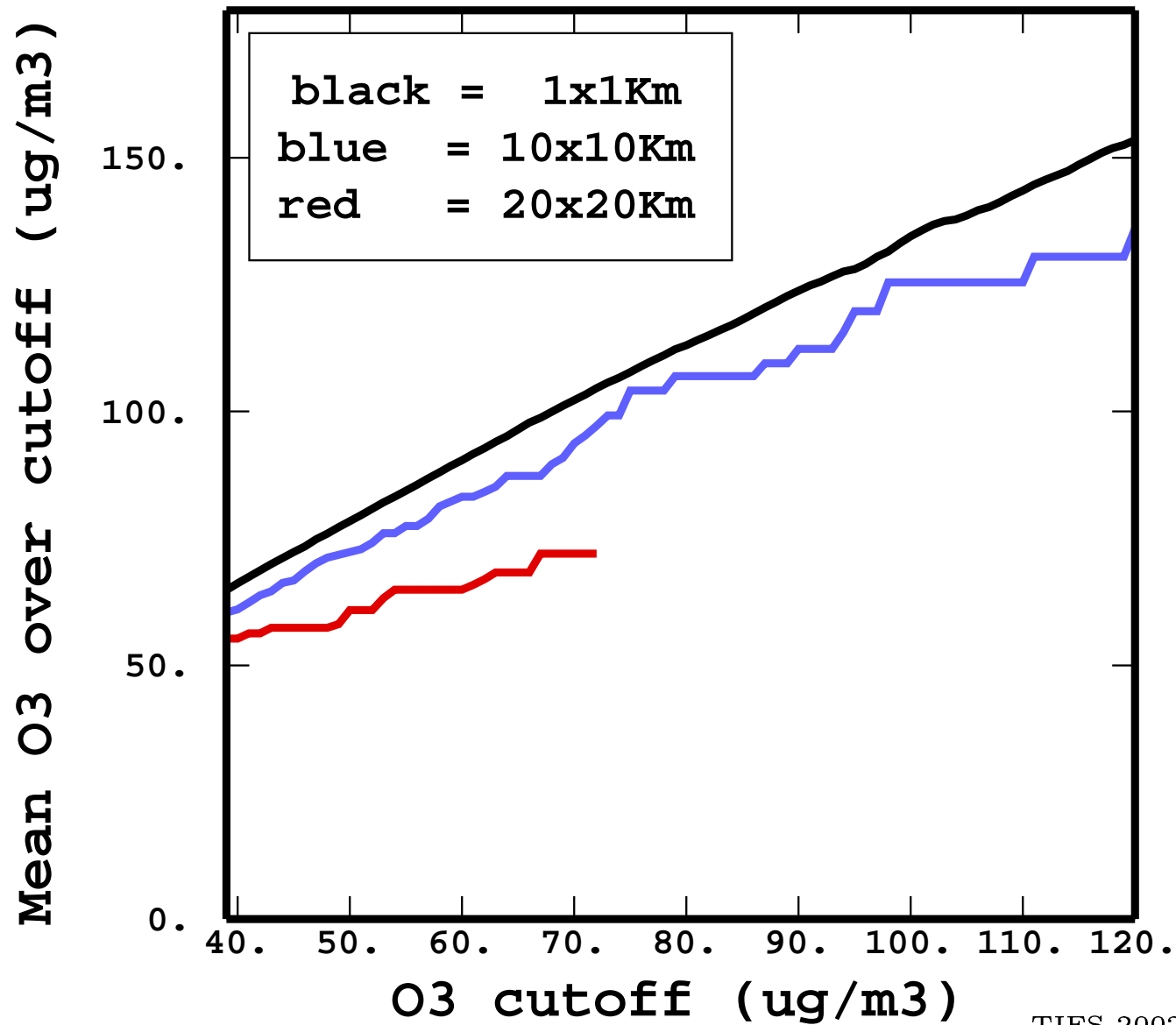
Simulation of Ozone



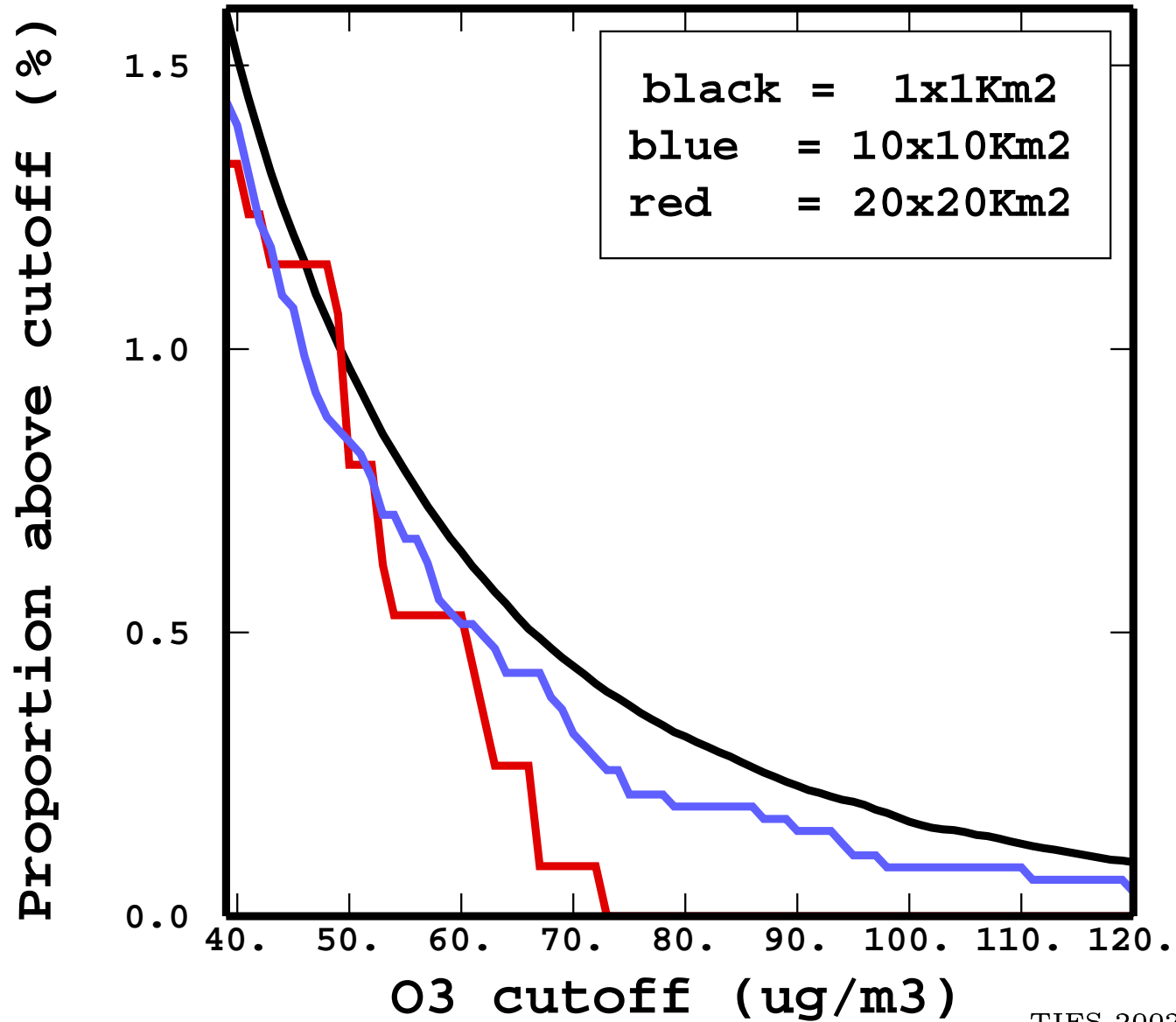
Increasing the support:

the **range** increases

Simulation of Ozone



Simulation of Ozone



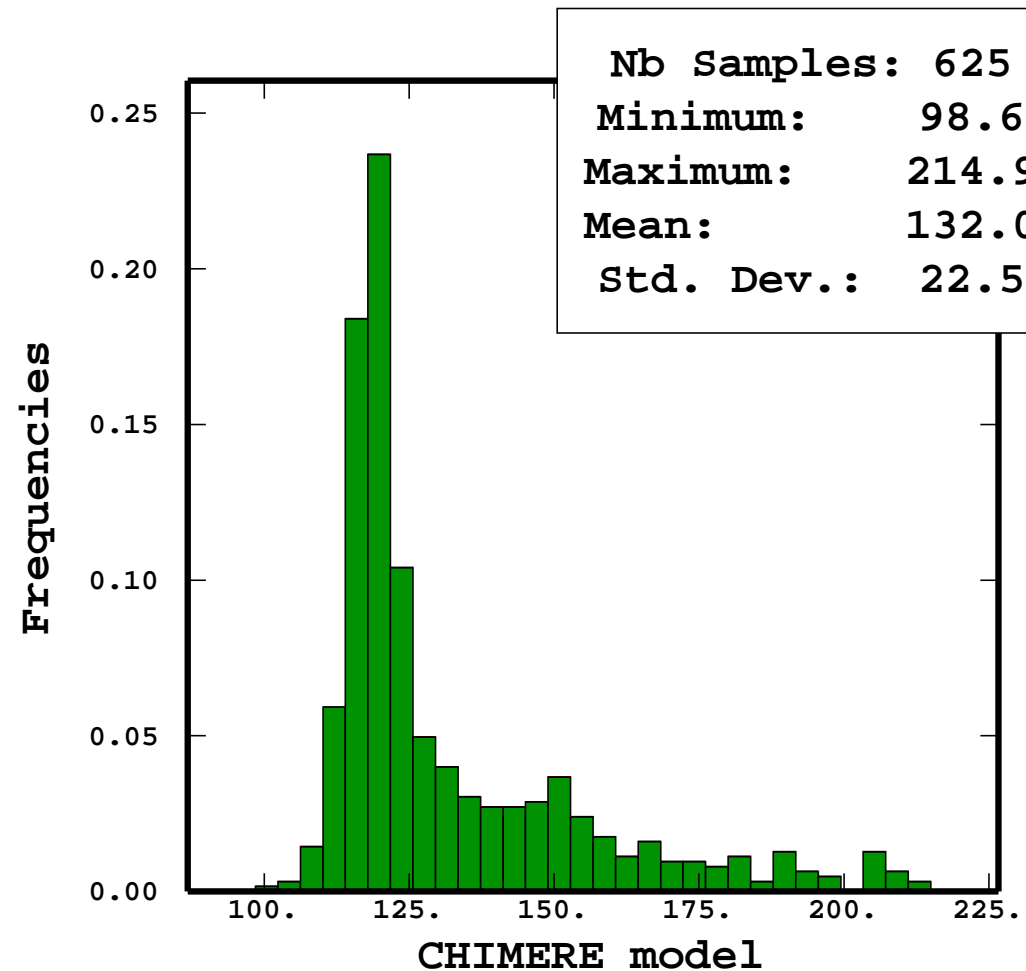
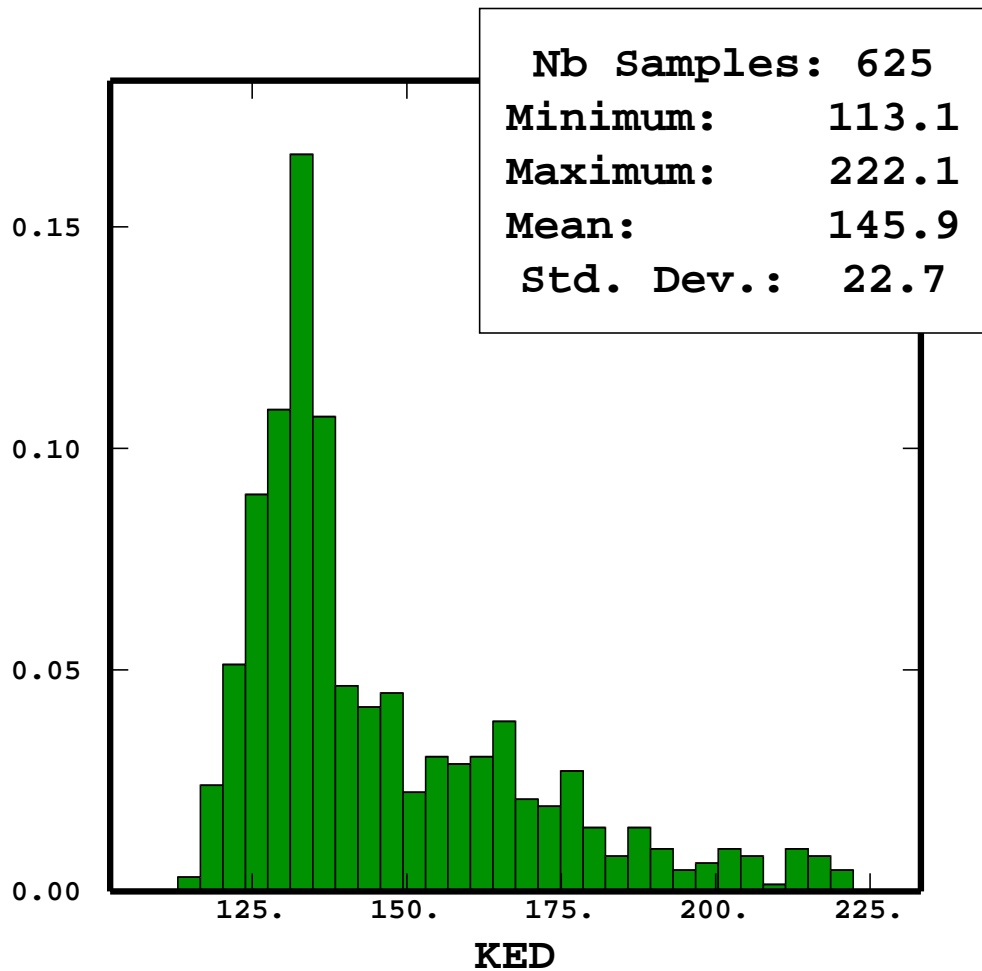
CASE STUDY: Ozone in Paris area

What about

the **support effect**

for **KED** and **CHIMERE** ?

KED vs CHIMERE (17 July '99, 15hUT)



In this example, at least, KED **does not alter** the distribution of the CHIMERE model except for a shift in the mean.

CONCLUSION

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 - KED recalibrates the **mean** of the model output,

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- Kriging with External Drift (KED) is a simple method for **normalizing** model output with station data.
- In our example:
 - KED recalibrates the **mean** of the model output,
 - the **shape of the histogram** is preserved:
no support effect for the normalized model output.

References

Selected references

- Chilès J.P., Delfiner P. (1999) *Geostatistics: Modeling Spatial Uncertainty*. Wiley, New York
- Hudson, G. & Wackernagel, H. (1994) Mapping temperature using kriging with external drift: theory and an example from Scotland. *International Journal of Climatology*, 14, 77–91
- Rivoirard, J. (2000) Which models for collocated cokriging? *Mathematical Geology*, 33 (2).

Air pollution case study

- Lajaunie, C. & Wackernagel, H. (2000) *Geostatistical normalization: Case Studies*. Tech. Rep. N-31/01/G, Centre de Géostatistique, Fontainebleau. Download: <http://cg.ensmp.fr/~hans/projects.html>

Ebro estuary case study

- Wackernagel, H., Bertino, L., Sierra, J.P. & Gonzalez del Rio, J. (2002) Multivariate kriging for interpolating data from different sources. In: Anderson CW, Barnett V, Chatwin P & El Shaarawi AH (eds) *Quantitative Methods for Current Environmental Issues*, 57-75, Springer-Verlag, London.