

**NON PARAMETRIC TESTS AND CONFIDENCE REGIONS
TO EVALUATE INTRINSIC DIVERSITY PROFILES
OF ECOLOGICAL POPULATIONS**

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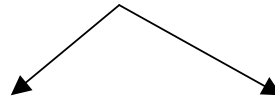
THE AIM

TO EVALUATE THE DIVERSITY OF BIOLOGICAL COMMUNITIES



HOW DO SPECIES EVOLVE AND INTERACT EACH OTHER?

- DIVERSITY INDEX: Shannon, Simpson, Brillouin, etc.



Species richness

the degree of evenness

AVAILABLE
APPROACH



Can be expressed as a function $g(\mathbf{N})$

$$\mathbf{N} = (N_1, N_2, \dots, N_s)$$



number of units belonging to the i -th species

SOME

D

R

A

W

B

A

C

K

S

Using an index
ALTERNATIVE

- ◆ IT IS NOT POSSIBLE TO RECOMMEND A SINGLE INDEX AS SUPERIOR TO ALL OTHERS
- ◆ DIFFERENT INDEXES MAY INCONSISTENTLY RANK A GIVEN PAIR OF COMMUNITIES
- ◆ DESCRIPTION OF WHOLE COMMUNITIES BY ONE STATISTIC RUN THE RISK OF LOOSING MUCH VALUABLE INFORMATION

A DIVERSITY PROFILE APPROACH (Patil and Taillie, 1982)

$$T_j = \frac{1}{N} \sum_{i=j+1}^s N_{(i)}, \quad j = 1, \dots, s-1$$

where $N_{(1)}, N_{(2)}, \dots, N_{(s)}$ are the ranked abundance,

$$N = \mathbf{1}^T \mathbf{N}, \quad \mathbf{T}_s = 0 \text{ and } \mathbf{T}_0 = 1$$

The convex curves obtained by plotting the (j, T_j) pairs

Are termed intrinsic diversity profiles

Let \mathbf{T}_{C_1} and \mathbf{T}_{C_2} be the intrinsic diversity profiles of communities C_1 and C_2

$$\mathbf{T}_{C_1} \leq \mathbf{T}_{C_2}$$



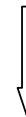
C_2 is more intrinsically
diverse than C_1
(dominance)

\mathbf{T}_{C_1} crosses \mathbf{T}_{C_2}



communities are not
intrinsically comparable

$$\mathbf{T}_{C_1} = \mathbf{T}_{C_2}$$



no differences in diversity
between C_1 and C_2
(equivalence)

However \mathbf{T}_{C_1} and \mathbf{T}_{C_2} are descriptive statistics

THE GOAL
OF THE WORK

AN INFERENTIAL APPROACH TO COMPARE TWO OR MORE
COMMUNITIES ACCORDING TO THEIR DIVERSITY
IS PROPOSED

CONFIDENCE SETS

TESTING HYPOTHESES

LITERATURE

◆ Fattorini and Marcheselli (1999)

HOWEVER, IN THIS SETTING, PROBLEMS ARISE BECAUSE OF

difficulty in assuming any specific
distribution both for the abundance
vector and for the diversity profile

components of diversity
profiles are dependent
and heteroschedastic

PROPOSAL

A NON PARAMETRIC METHODOLOGY BASED ON
BOOTSTRAP IS PRESENTED

F
R
A
M
E
W
O
R
K

LET $\mathbf{n} = (\hat{\pi}_1, \hat{\pi}_2, \dots, \hat{\pi}_n)$ BE *n*.i.d. UNBIASED ESTIMATORS OF THE
RELATIVE ABUNDANCE VECTOR π

↳ Replicated sampling - Barabesi and Fattorini (1998)

then, an improved unbiased estimator for π is given by

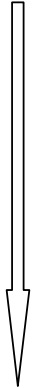
$$\bar{\pi}_n = \frac{1}{n} \sum_{i=1}^n \hat{\pi}_i \quad \text{with variance-covariance } n^{-1} \mathbf{V}$$

and an unbiased and consistent estimator for \mathbf{V} is

$$\hat{\mathbf{V}}_n = \frac{1}{n-1} \sum_{i=1}^n (\hat{\pi}_i - \bar{\pi}_n)(\hat{\pi}_i - \bar{\pi}_n)^T$$

IN THIS SETTING, THE NOW-FAMILIAR NON PARAMETRIC BOOTSTRAP
METHOD (EFRON, 1982) MAY BE SUITABLY ADOPTED ON THE *i.i.d.* SAMPLE
 $\mathbf{n} = (\hat{\pi}_1, \hat{\pi}_2, \dots, \hat{\pi}_n)$

PROBLEM



A SOLUTION

THE INTRINSIC DIVERSITY PROFILE $T(\boldsymbol{\pi})$ IS A FAMILY OF $S-1$ PARAMETRIC FUNCTIONS



ASSERTING CONFIDENCE SET FOR $T(\boldsymbol{\pi})$ INVOLVES DIFFICULT QUESTIONS OF SIMULTANEOUS INFERENCE

OF THIS PROBLEM IS GIVEN BY BERAN'S PROCEDURE (JASA, 1988)




BALANCED AND SIMULTANEOUS CONFIDENCE SETS FOR PARAMETERS EXPRESSED AS A LINEAR COMBINATIONS OF MEANS



IN OUR FRAMEWORK, THE PROCEDURE OF BERAN (1988) MAY BE IMPLEMENTED IN ORDER TO OBTAIN SIMULTANEOUS CONFIDENCE SETS FOR THE INTRINSIC DIVERSITY PROFILE $\mathbf{T}(\boldsymbol{\pi})$

IN FACT, WE CAN EXPRESS THE INTRINSIC DIVERSITY PROFILE AS A LINEAR FUNCTION OF $\boldsymbol{\pi}$


$$\mathbf{T}(\boldsymbol{\pi}) = \{T_{\mathbf{b}}(\boldsymbol{\pi}) = \mathbf{b}'\boldsymbol{\pi} : \mathbf{b} \in B\}$$

WHERE

$$\mathbf{b}_l = \sum_{j=1}^{s+1-l} \mathbf{a}_j \quad l = 1, \dots, s-1 \quad \text{and } B = \{\mathbf{b}_l; l = 1, \dots, s-1\}$$

AND a_1, a_2, \dots, a_{s-1} ARE THE SET OF STANDARD BASIS OF \mathfrak{R}^{s-1}

The statistic proposed for $T_b(\boldsymbol{\pi})$, called by Beran "studentized root", is:

$$R_{n,\mathbf{b}}(\mathbf{n}, T_b(\boldsymbol{\pi})) = n^{1/2} \frac{\mathbf{b}(\hat{\boldsymbol{\pi}}_n - \boldsymbol{\pi})}{[\mathbf{b}'\mathbf{V}_n\mathbf{b}]^{1/2}}$$

◆ A RESULTING CONFIDENCE SET FOR $T(\boldsymbol{\pi})$

$$C_n = \{t \in \mathbf{T} : L_{n,\mathbf{b}} \leq R_{n,\mathbf{b}}(t_{\mathbf{b}}) \leq U_{n,\mathbf{b}}, \forall \mathbf{b} \in \mathbf{B}\}$$

◆ THE PROPOSED BOOTSTRAP VERSION OF SIMULTANEOUS CONFIDENCE SET IS OBTAINED BY TAKING THE CRITICAL VALUES

$$L_{n,\mathbf{b}} = H_{n,\mathbf{b}}^{*-1} \left[H_{\text{inf}}^{*-1} \left(\frac{\alpha}{2} \right) \right]$$

$$U_{n,\mathbf{b}} = H_{n,\mathbf{b}}^{*-1} \left[H_{\text{sup}}^{*-1} \left(1 - \frac{\alpha}{2} \right) \right]$$

corresponding bootstrap estimates
by taking bootstrap samples $\mathbf{n}^* = (\hat{\boldsymbol{\pi}}_1^*, \hat{\boldsymbol{\pi}}_2^*, \dots, \hat{\boldsymbol{\pi}}_n^*)$ from the $\hat{\boldsymbol{\pi}}_i$'s

where, $H_{n,\mathbf{b}}^*$, H_{inf}^* , H_{sup}^* are the corresponding bootstrap estimates of:

◆ $H_{n,\mathbf{b}}(\boldsymbol{\pi})$ is the cdf of the root $R_{n,\mathbf{b}}$

◆ $\begin{cases} H_{\text{sup}}(\boldsymbol{\pi}) \\ H_{\text{inf}}(\boldsymbol{\pi}) \end{cases}$ cdf of a suitable transformed root $\begin{cases} \sup\{H_{n,\mathbf{b}}(R_{n,\mathbf{b}}, \boldsymbol{\pi}) : \boldsymbol{\pi} \in \mathbf{B}\} \\ \inf\{H_{n,\mathbf{b}}(R_{n,\mathbf{b}}, \boldsymbol{\pi}) : \boldsymbol{\pi} \in \mathbf{B}\} \end{cases}$

respectively.

ADVANTAGES OF THE METHOD

- ◆ C_n is set to have overall coverage probability $1 - \alpha$
- ◆ C_n is balanced, that is

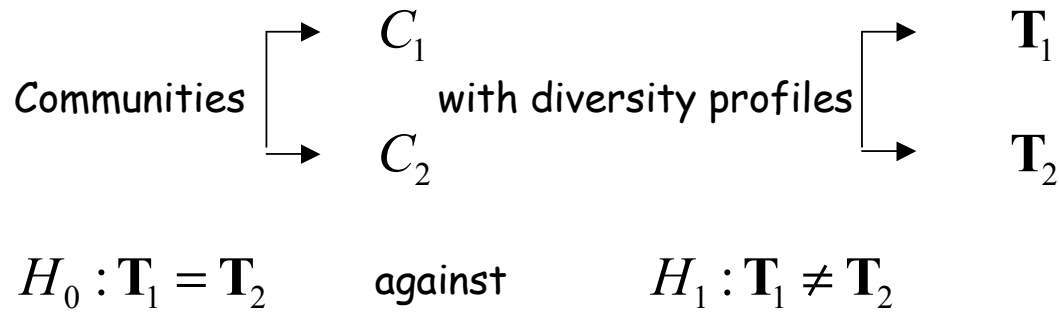


the coverage probability for the confidence interval of each right tail-sum $T_b(\pi)$ remains unchanged as b varies

- ◆ the method is non parametric where other methods such Tukey (1953), Scheffè (1953) and Richmond (1982) require components of π independent and normally distributed

HYPOTHESIS TESTING

IN ORDER TO EVALUATE AND MONITOR BIODIVERSITY
AT DIFFERENT SPATIAL (OR TEMPORAL) SCALES



◆ Gove et al (1994)
jackknifing approach, but

↓
unresolved questions of
simultaneous inference

◆ Fattorini and Marcheselli (1999) proposed an
asymptotically conservative procedure

↓
deriving asymptotic multinormality
distribution for \hat{T}_1 and \hat{T}_2

$\boldsymbol{\theta} = \mathbf{T}_1 - \mathbf{T}_2$ is the $s - 1$ vector of differences in the right tail-sums of the two communities

↳ the equivalence hypothesis $H_0 : \boldsymbol{\theta} = \boldsymbol{\theta}_0 = \mathbf{0}$

In this framework and applying Beran's procedure, a suitable statistical test is

$$G_{n,\mathbf{b}}(\boldsymbol{\theta}_0) = \frac{\bar{\boldsymbol{\theta}} - \boldsymbol{\theta}_0}{\left[\frac{\mathbf{b}' \bar{\mathbf{V}}_1 \mathbf{b}}{n_1} \right]^{\frac{1}{2}} \left[\frac{\mathbf{b}' \bar{\mathbf{V}}_2 \mathbf{b}}{n_2} \right]^{\frac{1}{2}}}, \quad \forall \mathbf{b} \in \mathbf{B}$$

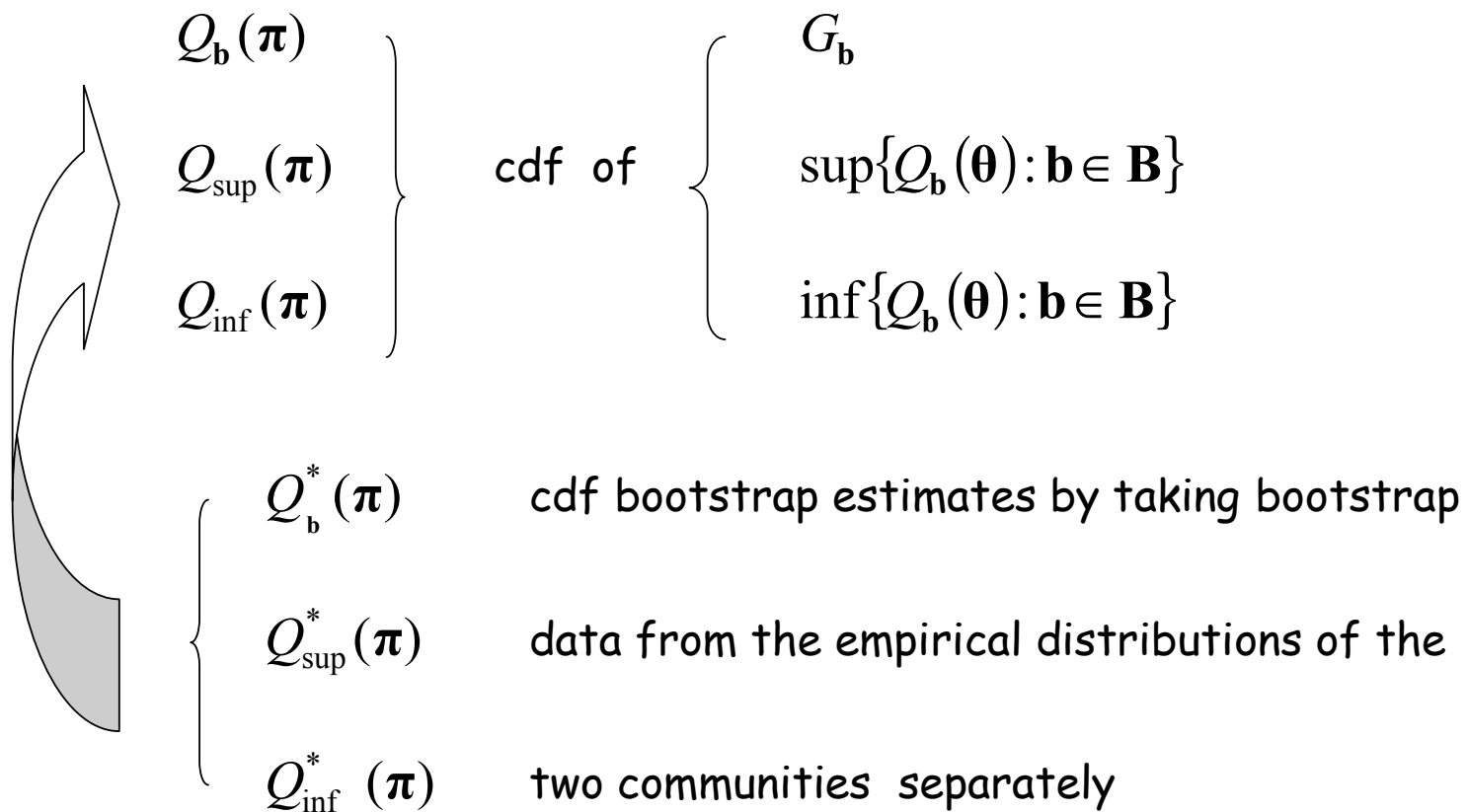
where,

$$\begin{array}{ccc} \bar{\boldsymbol{\theta}} = \{\mathbf{b}'(\bar{\boldsymbol{\pi}}_1 - \bar{\boldsymbol{\pi}}_2) : \mathbf{b} \in \mathbf{B}\} & \bar{\boldsymbol{\pi}}_1, \bar{\boldsymbol{\pi}}_2 & \bar{\mathbf{V}}_1, \bar{\mathbf{V}}_2 \\ \downarrow & \downarrow & \downarrow \\ \boldsymbol{\theta} & \boldsymbol{\pi}_1, \boldsymbol{\pi}_2 & \mathbf{V}_1, \mathbf{V}_2 \end{array}$$

are straightforward estimators of the population counterparts.

THE KEY IDEA:

to construct a simultaneous confidence set for parameter θ in order to define a critical area of the test under $H_0 : \theta = \theta_0 = \mathbf{0}$



finally, the $1-\alpha$ simultaneous confidence set for θ is

$$D_{\theta,1-\alpha} = \{\theta : l_b \leq G_b \leq u_b, b \in \mathbf{B}\}$$

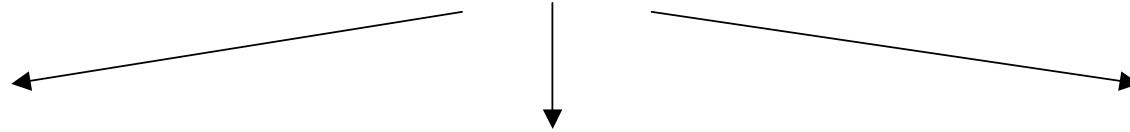
where

$$l_b = Q_b^{*-1} \left[Q_{\text{inf}}^{*-1} \left(\frac{\alpha}{2} \right) \right]$$

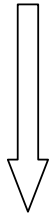
$$u_b = Q_b^{*-1} \left[Q_{\text{sup}}^{*-1} \left(1 - \frac{\alpha}{2} \right) \right]$$

are the critical values obtained from the empirical bootstrap distribution

DECISION RULE



if both $\bar{\theta} \geq D_{\theta,1-\alpha}$
and $\bar{\theta} \leq D_{\theta,1-\alpha}$ hold



reject H_0
communities not
intrinsically comparable

if $\bar{\theta} \subseteq D_{\theta,1-\alpha}$



accept H_0
equivalence

if $\bar{\theta} \geq D_{\theta,1-\alpha}$
or $\bar{\theta} \leq D_{\theta,1-\alpha}$



reject H_0
dominance

AN APPLICATION

AVIAN COMMUNITIES OF 4 PARKS IN MILAN (ITALY)



for a detailed description of the sample design
Fattorini and Marcheselli (1999)

- ◆ for each park we compute the diversity profile as $\mathbf{T}(\bar{\pi}_n)$ and derive simultaneous confidence sets
- ◆ in order to rank the park according to their diversity we evaluate an estimate of θ for each of the possible couples of parks
- ◆ under the null hypothesis of no difference in diversity we derive a 0.95 simultaneous confidence set for θ

RESULTS

- ◆ Groane and Trenno parks turn out to be the most intrinsically and the least intrinsically diverse, respectively
- ◆ Lambro and Forlanini parks can be located at an intermediate diversity level being these parks equivalent in terms of intrinsic diversity profiles
- ◆ Confidence regions for Groane park clearly show departures from symmetry

Therefore, we have the following ranking of the four parks:

1. Groane (the most diverse)
2. Lambro and Forlanini (intermediate level of diversity)
3. Trenno (the least diverse)