Wavelet Analysis of Climate-Related Flood Variability

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Outline

1. Motivation
2. Wavelet Analysis of Time Series
3. Climate and Floods
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Motivation: Floods and Non-stationarity

- The assumption that floods are IID in time is at odds with the fact that climate naturally varies at all scales.
  - Evidence of regime-like or quadi-periodic behavior and systematic trends in climate variables over the last century.
  - Attributing *cause* for these non-stationarities in a finite record is difficult, given the underlying dynamics.

Motivation: California Flooding and Precipitation

- Catastrophic floods and trends may be understood in terms of large-scale circulation pattern anomalies.
  - Vulnerability of California to extreme flooding events.
  - Tropical Pacific processes (ENSO, MJO) may influence precipitation.
  - Large fraction of winter precipitation related to extratropical processes; such as PNA and TNH.
  - Here we investigate Pacific/North American index and CA precipitation.
- Utilize the non-decimated discrete wavelet transform (MODWT) to determine dominant time scales between processes.
  - Power of discrimination using multiscale quantiles.
  - Wavelet cross-correlation within scales and between scales.

Introduction to Orthonormal Transforms

1. Orthonormal discrete Fourier transform (DFT) of $\mathbf{X}$ (of length N):

$$\mathbf{F} = \mathcal{F}\mathbf{X}$$

   - $\mathcal{F}$ is an $N \times N$ matrix of complex exponentials.
   - Decomposition of $\mathbf{X}$ is on a frequency by frequency basis.
   - FFT: $O(N \log N)$ operations.

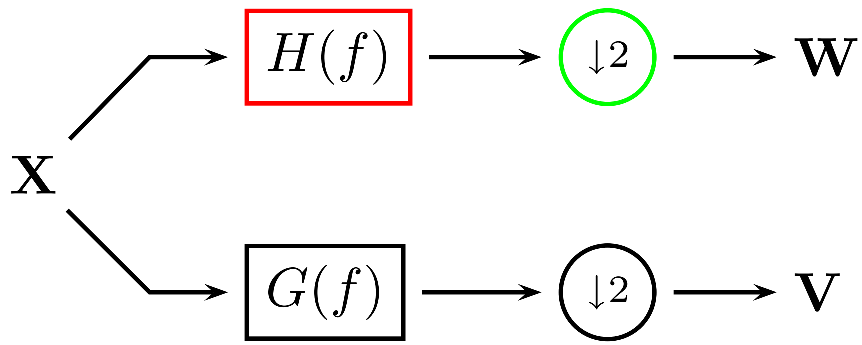
2. Orthonormal discrete wavelet transform (DWT) of $\mathbf{X}$ (of length N):

$$\mathbf{W} = \mathcal{W}\mathbf{X}$$

   - $\mathcal{W}$ is an $N \times N$ matrix of wavelet functions.
   - Decomposition of $\mathbf{X}$ is on a scale by scale basis.
   - Pyramid algorithm: $O(N)$ operations.

DWT: Filtering Interpretation

- Let $\mathbf{h} = (h_0, \ldots, h_{L-1})$ be the vector of wavelet (high-pass) filter coefficients (e.g., Daubechies, 1992).
- Let $\mathbf{g} = (g_0, \ldots, g_{L-1})$ be the vector of scaling (low-pass) filter coefficients.
- Graphical representation of the DWT applied to a dyadic length vector $\mathbf{X}$:



The length N vector $\mathbf{X}$ has been convolved with the filter $\mathbf{h}$, whose discrete Fourier transform is $H(f)$, and downsampled by two in order to produce a new vector $\mathbf{W}$ of length $N/2$ (similarly for $\mathbf{g}$).

DWT: Properties

- Downsampling operation may be omitted to produce a redundant (maximal overlap) DWT.
  - N MODWT coefficients per scale.
- Wavelet coefficients at level j are associated with changes of length $\tau_j = 2^{j-1}$ or oscillations of length $2\tau_j = 2^j$.
- Approximate compensation for phase shifts so that wavelet coefficients may be aligned with the original observations.
- Energy preserving transform: $\mathrm{var}\{X_t\} = \sum_{j=1}^{J} \mathrm{var}\{W_{j,t}\} + \mathrm{var}\{V_{J,t}\}$.
- Length of wavelet filter may be adjusted to balance between frequency localization, boundary affects, smoothness, vanishing moments, etc.

DWT: Wavelet Cross-Correlation

- Let X_t and Y_t be two time series of interest.
- The wavelet correlation of $\{X_t, Y_t\}$ at scale τ_j is defined to be:

$$\rho_{XY}(\tau_j) = \frac{\text{cov}\{\widetilde{W}_{j,t}^{(X)}, \widetilde{W}_{j,t}^{(X)}\}}{\sigma_X(\tau_j) \cdot \sigma_Y(\tau_j)},$$

  where $\sigma_X^2(\tau_j) = \text{var}\{\widetilde{W}_{j,t}^{(X)}\}$ is the wavelet variance for scale τ_j.
- Wavelet cross-correlation: allow time series to be delayed by an integer δ.
- Compare wavelet coefficients across scales (multiscale wavelet cross-correlation, MWCC).

Climate and Floods: Precipitation Data

- Central California has a modified Mediterranean climate.
- Major flooding occurs predominantly in the middle of the wet season.
- Floods are produced by strong onshore atmospheric flow patterns.
- Snowmelt is a contributing factor, but rain is the main ingredient.
- California precipitation data consists of 185 rain gauges.
  - 3-day running totals of precipitation from Jan. 1948 through Dec. 1999.
  - Computed quantiles (5%, 25%, 50%, 75%, 95%) over all stations.
  - Also have fraction of stations with "significant" rainfall.
  - Only looked at meteorological winters defined to be DJF.

Climate and Floods: Pacific/North American (PNA) Index

- Daily PNA index is constructed by projecting the daily 500mb height anomalies over the N. Hemsiphere onto the loading pattern of the PNA.
- The PNA pattern is one of the most prominent modes of low-frequency variability in the N. Hemisphere extratropics.
- The time series of the PNA pattern also indicates substantial interseasonal, interannual and interdecadal variability.
- Daily PNA index was obtained from NOAA/CPC.
  - Observations from Jan. 1948 through Feb. 2002.
  - Only looked at meteorological winters defined to be DJF (1948-1999).

Climate and Floods: Results

- Partition multiscale precipitation by extreme swings in PNA:
  - Positive PNA values associated with multiscale precipitation amounts centered away from zero.
  - Negative PNA values associated with small multiscale precip amounts.
- PNA leading precipitation at (j, δ) in wavelet cross-correlation implies:
  - PNA mode is forcing precipitation at scale τ_j.
  - Both PNA and precipitation are forced by MJO or other remote forcing.
- 1986 and 1997 are major flood years:
  - Scale 2 (1986, 1997) and scale 3 (1986) of PNA leads precipitation anywhere from 3-7 days.
- 1988 is a major MJO year:
  - Several scales of PNA lead CA precipitation (τ_3) mainly from 2-6 days.

Discussion

- Explored multi-scale relationships between precipitation and climate indicator (PNA index).
- Future Directions:
  - Granger causality: lagged values of A should help predict current values of B and lagged values of B should not predict current values of A.
  - Additional variables? streamflow, SOI, MJO, etc.
  - Start looking at time trends across time scales.
  - Want to provide insight for **flood risk prediction**.

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