











Natural Thin Plate Splines

$$f(x) = \beta_0 + \beta^T x + \sum_{j=1}^n b_j h_j(x)$$

Coefficients are found via standard penalized LS

$$\min_{\beta,b}(y - X\beta - Eb)^T(y - X\beta - Eb) + \lambda b^T Eb$$

s.t.
$$\sum_{i} b_i = \sum_{i} b_i x_{i1} = \sum_{i} b_i x_{i2} = 0$$

Interpretation: We take an elastic flat plate that interpolates points (x_i, y_i) and penalize its "bending energy"





































