Statistics 492, Problem Set 6

Wellner; 2/25/14

Reading: Klebaner, chapter 6, pages 151-167,
Klebaner, chapter 7, pages 185-212.

Due: Tuesday, 4 March 2014.

1. Klebaner, Exercise 5.3, page 149. Solve the SDE $dX(t) = B(t)X(t)dt + B(t)X(t)dB(t)$, subject to $X(0) = 1$.

2. Klebaner, Exercise 5.4, page 149. Solve the SDE $dX(t) = X(t)dt + B(t)dB(t)$, subject to $X(0) = 1$. Comment on whether it is a diffusion type SDE.

3. Klebaner, Exercise 5.8, page 149. Find the stochastic logarithm of $B^2(t) + 1$.

4. **Optional bonus problem:** Klebaner, Exercise 5.12, page 149. Let $X(t)$ satisfy the following SDE for $0 \le t \le T$: $dX(t) = \sqrt{X(t) + 1}dB(t)$ subject to $X(0) = 0$. Assuming that Itô integrals are martingales, find $EX(t)$ and $E(X^2(t))$. Let $m(u, t)$ be the moment generating function of $X(t)$. Show that it satisfies the PDE

$$\frac{\partial m}{\partial t} = \frac{u^2}{2} \cdot \frac{\partial m}{\partial u} + \frac{u^2}{2} m.$$