Statistics 492, Problem Set 7

Wellner; 3/4/14

Reading: Klebaner, chapter 6, pages 151-167,
Klebaner, chapter 7, pages 185-212.

Due: Tuesday, 11 March 2014.

1. Klebaner, Exercise 4.18, page 122: Let $X(t) = tB(t)$. Find its quadratic variation $[X, X](t)$.

2. Klebaner, Exercise 4.19, page 122: Let $X(t) = \int_0^t (t - s)dB(s)$. Find $dX(t)$ and its quadratic variation $[X, X](t)$. Compare to the quadratic variation of Itô integrals. (This is connected to what is known at *integrated Brownian motion*, $\int_0^t B(s)ds$. Two-sided integrated Brownian motion. This arises in the limit theory of nonparametric estimation of convex functions: let f be a convex function on $\mathbb{R}$, and suppose that we observe $Y(t)$ where $dY(t) = f(t)dt + \sigma dB(t)$. Then $Y(t) = \int_0^t f(s)ds + \sigma B(t) \equiv F(t) + \sigma B(t)$ and
$$Z(t) \equiv \int_0^t Y(s)ds = \int_0^t F(s)ds + \sigma \int_0^t B(s)ds;$$
here the most interesting f for the limit theory is $12t^2$. Then $F(t) = 4t^3$ and $\int_0^t F(s)ds = t^4$. See Groeneboom, Jongbloed, and Wellner, A canonical process for estimation of convex functions: the "invelope" of integrated Brownian motion $+t^4$. *Ann. Statist.* 29 (2001), 1620 - 1652.)

3. Klebaner, Exercise 5.5, page 149.

4. **Optional bonus problem:** Find another application of the Feynman-Kac formula. (Hints in lecture on 3/6/14.)