Statistics 491, Problem Set 1

Wellner; 9/25/13

Reading: Ross; Chapters 1 and 2, pages
Durrett; Appendix A .

Due: Wednesday, October 2, 2013.

1. Ross, problem 20, page 17.

2. Ross, problem 29, page 89.

3. Ross, problem 24, page 88.

4. Ross, problem 68, page 94.

5. Ross, problem 76, page 95.

6. Let A, B, C be independent random variables uniformly distributed on $(0,1)$. What is the probability that $Ax^2 + Bx + C$ has real roots?

7. (a) Suppose that X is distributed according to a Poisson distribution with parameter λ. The parameter λ is itself a random variable whose distribution law is exponential with mean $1/c$: $f_\lambda(t) = c\exp(-ct)1_{[0,\infty)}(t)$. Find the distribution of X.
(b) What if λ follows a Gamma distribution of order α with scale parameter c: i.e. $f_\lambda(t) = c(ct)^{\alpha-1}\exp(-ct)/\Gamma(\alpha)$ for $t > 0$?

8. Let X_1, X_2 be independent random variables with uniform distribution over the interval $[\theta - 1/2, \theta + 1/2]$. Show that $X_1 - X_2$ has a distribution independent of θ and find its density function.

9. Using the central limit theorem for suitable Poisson random variables, prove that

$$\lim_{n\to\infty} e^{-n}\sum_{k=0}^{n}\frac{n^k}{k!} = \frac{1}{2}.$$

10. Suppose we have N chips numbered $1, 2, \ldots, N$. We take a random sample of size n without replacement. Let X be the largest number in the random sample. Show that the probability mass function of X is

$$P(X = k) = \frac{\binom{k-1}{n-1}}{\binom{N}{n}}, \quad \text{for} \quad n, n+1, \ldots, N,$$

and that

$$E(X) = \frac{n}{n+1}(N+1), \qquad Var(X) = \frac{n(N-n)(N+1)}{(n+1)^2(n+2)}.$$