Statistics 491, Problem Set 5

Wellner; 10/23/13

Reminder:	Midterm exam 1, Monday, October 28 Math/Stat 394 Probability Review notes at http://www.stat.washington.edu/jaw/COURSES/491-2/f13.394.ho.html
Reading:	Ross; Chapter 4, pages 191 - 230 Durrett; Chapter 5, 185-207 ,
Due:	Wednesday, October 30, 2013.

1. Durrett, chapter 1, page 75: problem 1.2.
   Five white balls and five black balls are distributed in two urns in such a way that each urn contains five balls. At each step we draw one ball from each urn and exchange them. Let X_n be the number of white balls in the left urn at time n. Compute the transition matrix for X_n.

2. Durrett, chapter 1, page 75: problem 1.5.
   Consider a gambler's ruin chain (as in Durrett, Example 1.1, pages 1,2, Example 1.11, page 11) with $N = 4$. That is, if $1 \le i \le 3$, $P(i, i+1) = 0.4$ and $P(i, i-1) = 0.6$, but the endpoints are absorbing states $P(0, 0) = 1$ and $P(4, 4) = 1$. Compute $P^3(1, 4)$ and $P^3(1, 0)$.

3. Durrett, chapter 1, page 75: problem 1.6.
   A taxicab driver moves between the airport A and two hotels B and C according to the following rules. If he is at the airport, he will be at one of the two hotels next with equal probability. If at a hotel, then he returns to the airport with probability 3/4 and goes to the other hotel with probability 1/4.
   (a) Find the transition matrix for the chain.
   (b) Suppose the driver begins at the airport at time 0. Find the probability for each of his three possible locations at time 2 and the probability he is at hotel B and time 3.

4. Durrett, chapter 1, page 84: problem 1.45:
   Consider a general chain with state space $S = \{1, 2\}$ and write the transition probability matrix as

$$\mathbf{P} = \begin{pmatrix} 1-a & a \\ b & 1-b \end{pmatrix}$$

where $0 < a < 1$ and $0 < b < 1$. Use the Markov property to show that

$$P(X_{n+1} = 1) - \frac{b}{a+b} = (1 - a - b) \left\{ P(X_n = 1) - \frac{b}{a+b} \right\},$$

and then conclude

$$P(X_n = 1) = \frac{b}{a+b} + (1 - a - b)^n \left\{ P(X_0 = 1) - \frac{b}{a+b} \right\}.$$

This shows that if $0 < a + b < 2$, then $P(X_n = 1)$ converges exponentially fast to its limiting value, $a + b$.