Statistics 491, Problem Set 7

Wellner; 11/6/13

Reading: Ross; Chapter 4, pages 243 - 290
Durrett; Chapter 1, pages 58 - 91 .

Due: Wednesday, November 13, 2013.

1. Durrett, chapter 1, problem 1.7, page 75.
   Suppose that the probability that it rains today is 0.3 if neither of the last 2 days was rainy, but 0.6 if at least one of the last 2 days was rainy. Let the weather on day n, W_n, be R for rain, or S for sun. W_n is not a Markov chain, but the weather for the last two days, $X_n = (W_{n-1}, W_n)$ is a Markov chain with 4 states $\{RR, RS, SR, SS\}$. (a) Compute its transition probability matrix. (b) Compute the two-step transition probability matrix. (c) What is the probability it will rain on Wednesday given that it did not rain on Sunday and Monday?

2. Durrett, chapter 1, problem 1.8, pages 75-76.
   Consider the following transition matrices. Identify the transient and recurrent states, and the irreducible closed sets in the Markov chains. Give reasons for your answers.

(a)	**1**	**2**	**3**	**4**	**5**
1	0.4	0.3	0.3	0	0
2	0	0.5	0	0.5	0
3	0.5	0	0.5	0	0
4	0	0.5	0	0.5	0
5	0	0.3	0	0.3	0.4

(b)	**1**	**2**	**3**	**4**	**5**	**6**
1	0.1	0	0	0.4	0.5	0
2	0.1	0.2	0.2	0	0.5	0
3	0	0.1	0.3	0	0	0.6
4	0.1	0	0	0.9	0	0
5	0	0	0	0.4	0	0.6
6	0	0	0	0	0.5	0.5

(c)	**1**	**2**	**3**	**4**	**5**
1	0	0	0	0	1
2	0	0.2	0	0.8	0
3	0.1	0.2	0.3	0.4	0
4	0	0.6	0	0.4	0
5	0.3	0	0	0	0.7

(d)	**1**	**2**	**3**	**4**	**5**	**6**
1	0.8	0	0	0.2	0	0
2	0	0.5	0	0	0.5	0
3	0	0	0.3	0.4	0.3	0
4	0.1	0	0	0.9	0	0
5	0	0.2	0	0	0.8	0
6	0.7	0	0	0.3	0	0

3. Durrett, chapter 1, problem 1.41, page 83. (Reflecting random walk on the line)
   Consider the points $1, 2, 3, 4$ to be marked on a straight line. Let X_n be a Markov

chain that moves to the right with probability 2/3 and to the left with probability 1/3, but subject this time to the rule that if X_n tries to go to the left from 1 or to the right from 4 it stays put. Find: (a) the transition probability matrix for the chain, and (b) the limiting amount of time the chain spends at each site.

4. Durrett, chapter 1, problem 1.74, page 91.
   Consider the aging chain on $\{0, 1, 2, \ldots\}$ in which for any $n \geq 0$ the individual gets 1 day older from n to $n+1$ with probability p_n (with $\sum_{n=0}^{\infty} p_n = 1$), but dies and returns to age 0 with probability p_n. Find conditions that guarantee that: (a) 0 is recurrent; (b) positive recurrent. (c) Find the stationary distribution.

5. Durrett, chapter 1, problem 1.75, page 91. The opposite of the aging chain is the renewal chain with state space $\{0, 1, 2, \ldots\}$ in which $P(i, i-1) = 1$ when $i > 0$. The only nontrivial part of the transition probability matrix is $P(0, i) = p_i$ (with $\sum_{i=0}^{\infty} p_i = 1$). Show that this chain is always recurrent, but is positive recurrent if and only if $\sum_{n=0}^{\infty} n p_n < \infty$.

6. Durrett, chapter 1, problem 1.77, page 91.
   Consider a branching process as defined in Example 1.8 in which each family has a number of children that follows a shifted geometric distribution $p_k = p(1-p)^k$ for $k \geq 0$ (which counts the number of failures before the first success when success has probability p). Compute the probability that starting from one individual the chain will be absorbed at 0.