Statistics 491, Problem Set 9

Wellner; 11/20/13

Reading: Durrett; Chapter 2, pages 92 - 118 .
Ross; Chapter 5, sections 5.1-5.3, pages 291 - 339.

Due: Wednesday, November 27, 2013.

1. Durrett, chapter 2, problem 2.1, page 111.
Suppose that the time to repair a machine is an exponentially distributed random variable with mean 2 hours. (a) What is the probability that the repair take more than 2 hours? (b) What is the probability that the repair takes more than 5 hours given that it takes more than 3 hours?

2. Durrett, chapter 2, problem 2.7, page 111.
Let S and T be exponentially distributed with rates λ and μ. Let $U = \min\{S, T\}$ and $V = \max\{S, T\}$. Find: (a) $E(U)$; (b) $E(V - U)$; (c) $E(V)$. (d) Use the identity $V = S + T - U$ to get a different looking formula for $E(V)$ and verify that the two are equal.

3. Durrett, chapter 2, problem 2.8, page 111.
Let S and T be exponentially distributed with rates λ and μ. Let $U = \min\{S, T\}$, $V = \max\{S, T\}$, and $W = V - U$. Find the variances of U, V, and W.

4. Durrett, chapter 2, problems 2.18 and 2.19, page 113.
(a) Compare the Poisson approximation with the exact binomial probabilities of 1 success when $n = 20$, $p = 0.1$.
(b) Compare the Poisson approximation with the exact binomial probabilities of 1 success when: (i) $n = 10$, $p = 0.1$; (ii) $n = 50$, $p = 0.02$ (changed from $p = 0.2$ to agree with Durrett).

5. Durrett, chapter 2, problem 2.22, page 113.
Suppose that $N(t)$ is a Poisson process with rate 3. Let T_n denote the time of the n arrival. Find: (a) $E(T_{12})$; (b) $E(T_{12}|N(2) = 5)$; (c) $E(N(5)|N(2) = 5$.

6. **Optional bonus problem 1:** Durrett, chapter 2, problem 2.17, page 113.
Let T_i, $i = 1, 2, 3$ be independent exponentials with rate λ_i. (a) Show that for any numbers t_1, t_2, t_3

$$\max\{t_1, t_2, t_3\} = t_1 + t_2 + t_3 - \min\{t_1, t_2\} - \min\{t_1, t_3\} - \min\{t_2, t_3\} + \min\{t_1, t_2, t_3\}.$$

(b) Use (a) to find $E \max\{T_1, T_2, T_3\}$.

7. **Optional bonus problem 2:** Durrett, chapter 2, problem 2.16, page 113. Ron, Sue, and Ted arrive at the beginning of a professor's office hours. The amount of time they will stay is exponentially distributed with means 1, 1/2, and 1/3 hours. (a) What is the expected time until only one student remains? (b) For each student, find the probability they are the last student left. (c) What is the expected time until all three students are gone?