



The mathematical work of Evarist Giné

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1. Introduction

Evarist Giné, or Evarist Giné-Masdéu, 1944–2015, was an influential, brilliant and prolific contributor to modern probability theory and mathematical statistics, with a focus on problems arising in infinite-dimensional settings. His work has had a profound impact on modern probability theory, mathematical statistics and recently also machine learning. This article constitutes an attempt to describe Evarist's major mathematical achievements, which we divide into separate subsections each of which are focused on a major area of work, including *probability in Banach spaces, empirical processes, the bootstrap, U-statistics and processes, and mathematical statistics*. We refer to the editorial of this memorial volume for a biographical summary of Evarist's life. A list of all of Evarist's publications, including his three books, can be found at the end of this article.

A unifying aspect of most of Evarist's work was the masterly combination of techniques from real and functional analysis with fundamental ideas from probability theory: Evarist had a deep knowledge of analytic techniques that he often applied with ingenious simplicity in probabilistic problems. At the same time Evarist was a versatile and classically trained probabilist who mastered several core areas ranging from limit theorems and inequalities for sums of independent random variables to Gaussian processes and martingale arguments.

2. The main building blocks of Evarist's work

2.1. The Ph.D. thesis

Evarist wrote his Ph.D. thesis under the supervision of Richard M. Dudley at the Massachusetts Institute of Technology (MIT).

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Five remarkable papers came out of this thesis: Two of them [1,2] were published in the *Annals of Probability* and were side-products of Evarist's time as a graduate student. Paper [1] led Evarist into the area described in the next subsection, and paper [2] with R. Klein established limiting properties of the quadratic variation of processes with Gaussian increments generalizing results by Dudley [9] for Brownian motion.

But the main topic of Evarist's thesis was the construction of computable statistical tests for observations that take values in a general compact Riemannian manifold, published in the *Annals of Statistics* in 1975, see [4]. The handling editor for this substantial paper was Lucien Le Cam, who found the mathematical level so high that the only referee he could think of was Richard Dudley! This paper basically initiated the whole area of 'Sobolev tests' that has been important ever since in the area of directional statistics, and required the development of some important mathematical results of independent interest, such as a proof of the fact that a Sobolev ball defined on any compact Riemannian manifold satisfies the empirical central limit theorem (i.e., is a P -Donsker class—see Section 2.3 for more on this). At that time the general empirical process machinery was not yet available but Evarist succeeded with a clever reduction of the problem to the central limit theorem in $C(S)$ (the space of continuous functions on a compact metric space S) using duality theory for Sobolev spaces. The thesis also required the proof of some exact identities in geometric analysis that were published in a separate paper [3], and led to a (and also Evarist's only) genuinely applied paper [5], published in the *Journal of Geology*. Overall it can be said that this thesis was absolutely outstanding in its breadth and depth.

2.2. Probability in Banach spaces

Departing from the paper [1] that was part of the Ph.D. thesis, Evarist embarked on one of the hot topics in probability theory in the 1970s: the problem of deriving the classical limit theorems for sums $\sum_{i=1}^n X_i$ of centered i.i.d. random variables X_i that take values in an infinite-dimensional Banach space B : For instance the *central limit theorem* (CLT): for a suitable Gaussian random variable G taking values in B one wants to show the distributional limit theorem

$$\frac{1}{\sqrt{n}} \sum_{i=1}^n X_i \rightarrow^d G \quad \text{as } n \rightarrow \infty.$$

While in finite-dimensional spaces a necessary and sufficient condition for the CLT to hold is finiteness of $E\|X_1\|^2$, in infinite dimensions this is not the case, and geometric properties of the Banach space start to play a key role. The mathematical techniques developed in this area during that time were of fundamental importance to many areas in modern mathematics, including geometric functional analysis, concentration of measure, statistical learning and mathematical statistics.

Evarist's contributions to the area were substantial, including about 20 papers and culminating in Evarist's first book *The central limit theorem for real and Banach-valued random variables* published in 1980. His work in the area, which was carried out jointly with several co-authors including Alejandro de Acosta, Aloisio Araujo and Joel Zinn, included the derivation of the Lévy–Khintchine formula for infinitely divisible laws in Banach spaces and related characterizations of the domain of attraction of a normal law in Banach spaces [12,14], convergence of moments in the CLT in Banach spaces [15], and CLTs for some specific spaces of functions [7,10,16,28,29].

An elegant application of these techniques to the theory of *random sets* (see [17,25]) is given in the paper [33] jointly with Marjorie Hahn and Joel Zinn: If B is a separable Banach space, we

can define the set $K(B)$ of all non-empty compact subsets of B , which is a complete metric space when endowed with the Hausdorff distance δ . Moreover (Minkowski-) addition is a well-defined operation on $K(B)$ and for $A \in K(B)$ we can even define a norm $\|A\| = \sup\{\|a\|_B : a \in A\}$. A random compact set is any Borel random variable X taking values in $K(B)$, and its expectation EX can be defined in a suitable (‘Bochner’-) sense. Using a clever reduction to the CLT in $C(S)$ with an appropriate choice of S , the paper [33] proved the central limit theorem for random sets: For instance, if $B = \mathbb{R}^d$, $E\|X\| < \infty$ and if the X_i ’s are i.i.d. random compact sets, then

$$\sqrt{n}\delta\left(\frac{1}{n}\sum_{i=1}^n X_i, EX\right)$$

converges in distribution to a suitable norm of a Gaussian process. A result for the case where B is infinite-dimensional is also proved.

2.3. Empirical processes

Techniques for probability in Banach spaces were mostly developed for separable spaces, which is quite reasonable as probability distributions on complete metric spaces necessarily concentrate most of their mass on compact sets (i.e., they are Radon measures). At the same time, a key open problem in the late 1970s was the central limit theorem for empirical processes indexed by abstract classes \mathcal{F} of functions $f : S \rightarrow \mathbb{R}$, and where S is an arbitrary sample space where i.i.d. random variables X_1, \dots, X_n of law P take their values.

The empirical measure $P_n := n^{-1} \sum_{j=1}^n \delta_{X_j}$ is a natural statistical estimator of the unknown law P and it is of importance to understand how close the sample means $P_n f = n^{-1} \sum_{j=1}^n f(X_j)$ are to the true means $Pf = \mathbb{E}f(X)$, uniformly over a large class \mathcal{F} of functions f . In the late 60s—early 70s, Vapnik and Chervonenkis, motivated by applications in statistical theory of pattern recognition (a part of what is nowadays called statistical learning theory) obtained striking necessary and sufficient conditions for the uniform law of large numbers for empirical measures (the Glivenko–Cantelli problem)

$$\sup_{f \in \mathcal{F}} |P_n f - Pf| \rightarrow 0 \quad \text{as } n \rightarrow \infty \text{ a.s.}$$

in the case when $\mathcal{F} = \{I_C : C \in \mathcal{C}\}$, where \mathcal{C} is a class of measurable subsets of S . They later extended these results to the classes of uniformly bounded functions [36]. However, the extension of the celebrated ‘Donsker theorem’, that is, the central limit theorem for classical empirical processes on the real line, to the same general framework remained open. In today’s notation the general empirical processes are written as

$$f \mapsto v_n(f) = n^{1/2}(P_n f - Pf) = \frac{1}{\sqrt{n}} \sum_{i=1}^n (f(X_i) - Ef(X)), \quad f \in \mathcal{F},$$

and the question is whether v_n converges to some Gaussian process $(G_P(f) : f \in \mathcal{F})$ uniformly in $f \in \mathcal{F}$. Even if this looks at first like a very abstract problem, the techniques required for its solution have had the most profound impact on modern theoretical statistics and learning theory. In a seminal paper Dudley [10] studied such limit theorems, and showcased that even in the simplest cases where \mathcal{F} consists of indicators of a class \mathcal{C} of subsets of Euclidean space, techniques very different from those used in probability in Banach spaces are required. This is partly due

to the fact that empirical processes typically do *not* concentrate on some Banach space of *continuous* (for some metric) functions on \mathcal{F} – think of the empirical distribution function where $\mathcal{F} = \{1_{(-\infty, t]} : t \in \mathbb{R}^d\}$ – and the resulting lack of separability of the Banach space $\ell^\infty(\mathcal{F})$ of bounded functions on \mathcal{F} in which v_n takes values. Dudley [10] studied the central limit theorem for empirical processes indexed by classes of sets calling such a class \mathcal{C} Donsker for the law P of X_1, \dots, X_n if the CLT holds on the class. He obtained sufficient conditions for Donsker property in terms of the bracketing metric entropy of \mathcal{C} and proved that Vapnik–Chervonenkis classes of sets (the notion also being introduced by Dudley) are Donsker for any law P subject only to proper measurability assumptions. After Dudley’s key contribution some useful sets of sufficient conditions for the CLT for empirical processes to hold were given by V. Koltchinskii [18] in terms of random entropies of function classes and by D. Pollard [31] in terms of their uniform entropies. Le Cam [19] obtained sufficient conditions for the CLT on classes of sets in terms of random entropies. These conditions were similar in spirit to Vapnik–Chervonenkis conditions for the laws of large numbers. The proofs of these results were based on what is now called symmetrization inequalities for tail probabilities. By a conditioning argument, they allowed to reduce the bounds on the sup-norms of empirical processes to the bounds for a subgaussian (conditionally on X_1, \dots, X_n) process of the following type: $\mathcal{F} \ni f \mapsto n^{-1} \sum_{j=1}^n \varepsilon_j f(X_j)$, where $\{\varepsilon_j\}$ are i.i.d. Rademacher random variables independent of $\{X_j\}$. This subgaussian process (called nowadays the Rademacher process) was controlled in terms of entropies of random sets $\{(f(X_1), \dots, f(X_n)) : f \in \mathcal{F}\} \subset \mathbb{R}^n$. Evarist’s entrance to the stage of empirical processes could not have been more impressive, in a 70 page special invited paper [34] in *The Annals of Probability* in 1984, jointly written with Joel Zinn. This paper not only gave some very final results about the CLT for empirical processes, but also introduced powerful new techniques to the area and developed to perfection the techniques used before. In particular, the symmetrization method used by Koltchinskii [18], Pollard [31] and Le Cam [19] has reached its final form in two beautiful Giné–Zinn symmetrization inequalities used ever since, and the reduction of bounding of the empirical process to bounding the Rademacher process has been studied to its full extent. Moreover, this technique was combined with other tools from probability in Banach spaces, such as the multiplier inequality of Pisier, and the paper established connections of the emerging theory of empirical processes with a large body of literature on probability in Banach spaces. The idea of Gaussian or Rademacher symmetrization via the multiplier inequality of Pisier was to play an important role in Evarist’s later work, also with Zinn, on bootstrapping the empirical process; see Section 2.4. Specific important results obtained in the Giné and Zinn [34] paper include the final versions of random entropy conditions for the central limit theorem for empirical processes, the proof of necessity of such conditions for the classes of sets (the necessity of conditions in terms of Vapnik–Chervonenkis shattering numbers was proved later by Talagrand) and the extension of Vapnik–Chervonenkis necessary and sufficient condition for the law of large numbers (Glivenko–Cantelli theorem) to the case of unbounded classes of functions.

Another deep and beautiful result that Evarist obtained in empirical process theory is the Gaussian characterization of uniform Donsker classes. Again in joint work with Joel Zinn, published in the *Annals of Probability* [53], Evarist asked the question: when does the CLT for the empirical process holds uniformly in the distribution P of the X_i ’s? More precisely, if β is a metric for weak convergence in $\ell^\infty(\mathcal{F})$ and G the limiting Gaussian process, when is it true that

$$\sup_P \beta(v_n^P, G_P) \rightarrow 0 \quad \text{as } n \rightarrow \infty$$

where the supremum extends over all probability measures in P ? Such a result is of key importance for the statistical interpretation of the CLT, as in its typical applications it is used to infer unknown properties of P and hence no prior knowledge of P should be required for its validity. In essence this is a question about the structure of the class \mathcal{F} , and \mathcal{F} is called uniform Donsker if the last limit holds true. The striking result of [53] is that a necessary and sufficient condition for the uniform Donsker property is that the limit G_P is *pregaussian* uniformly in P (which effectively means that the sample-continuity of the Gaussian process G_P in its argument $f \in \mathcal{F}$ for the intrinsic covariance metric holds uniformly in P). Therefore, to check whether a class \mathcal{F} is a uniform Donsker class is a problem that can be decided entirely in terms of properties of Gaussian processes, which is in remarkable contrast to the otherwise existing gap between the Donsker and the pregaussian property. If much of Evarist’s work in empirical process theory was inspired by the idea of establishing powerful connections between empirical and (sub-)Gaussian processes, this result may be considered as a high-point of that program.

2.4. The Bootstrap

A fundamental idea in statistics, due to Efron [12], is the resampling methodology known as the *bootstrap*. It can be used for inference and confidence sets in situations where limit distributions exist but are not accessible for the statistician (because they are complex or depend on unknown parameters). This can be illustrated in the situation where X_1, \dots, X_n are i.i.d. random variables from law P with mean μ . Then we can draw at random from the sample values to create a bootstrap sample: Let $X_{ni}^b, i = 1, \dots, n$, be i.i.d. draws of the random variable X_n^b with law $\mathbb{P}_n(X_n^b = X_i) = 1/n$ for every $i = 1, \dots, n$. If $\bar{X}_n = \frac{1}{n} \sum_{i=1}^n X_i$ is the sample mean and $\bar{X}_n^b = \frac{1}{n} \sum_{i=1}^n X_{ni}^b$ is the mean of the resampled values, then the idea is that the (known, given the X_i ’s) distribution of $\bar{X}_n^b - \bar{X}_n$ is pivotal for the (unknown) distribution $\bar{X}_n - \mu$. Thus, when computing quantiles for the latter, we can resort to computing (approximate) quantiles of the former. That this works in many situations is fundamentally linked to the central limit theorem, and Evarist (together with Joel Zinn) made several substantial contributions to the field, among them [46] showing necessity of the sufficient conditions found in Bickel and Freeman [3] and providing a general bootstrap CLT for empirical processes, that basically completely settles the question of ‘consistency’ of the (nonparametric) bootstrap. The key result of the 1990 Annals of Probability paper [51] with Joel Zinn is that, for v_n^b the bootstrap empirical process,

$$v_n^b \rightarrow^d G_P \text{ (in probability) if and only if } v_n \rightarrow^d G_P.$$

The 1990 paper remains a tour de force of techniques from empirical process theory and probability theory in Banach spaces in several respects, including: (a) providing another use of the ‘‘multiplier inequality’’ of Pisier which played a role with Gaussian multipliers in their 1984 paper, this time with (symmetrized) Poisson multipliers after applying Poissonization and symmetrization to the multinomial weights of Efron’s bootstrap; (b) connecting the striking results of Ledoux and Talagrand [20,21] concerning multiplier and conditional multiplier CLT’s in Banach spaces to an important set of statistical questions.

Evarist, in collaboration with several co-authors (notably his late student, Miguel Arcones), also addressed a number of other issues in connection with bootstrap resampling methods. Arcones and Giné [47] show that the bootstrap of the sample mean of i.i.d. random variables with finite variance ‘‘works’’ if both the bootstrap sample size m_n and the original sample size go to infinity. In [54] they studied several bootstrap test of symmetry, and in [57] they showed

how to bootstrap U - and V -statistics. The important issue of bootstrapping M -estimators and other smooth statistical functionals was addressed in [59]. This whole set of research directions and problems culminated in Evarist's St. Flour Lecture notes on asymptotic theory for bootstrap resampling. These notes continue to serve as an important reference and touchstone for current research.

2.5. U -statistics and U -processes

The notion of U -statistic is a natural extension of the most classical object in probability, the sum of independent random variables. Given an i.i.d. sequence X_1, \dots, X_n, \dots of random variables in a measurable space S and a measurable function $h : S \times \dots \times S \mapsto \mathbb{R}$ (a kernel), the U -statistic of order k is defined as

$$U_n(h) := \frac{(n-k)!}{n!} \sum_{i_1, \dots, i_k} h(X_{i_1}, \dots, X_{i_k})$$

with the sum being extended to all $1 \leq i_1, \dots, i_k \leq n$ such that $i_l \neq i_{l'}, l \neq l'$. This notion originated in the work of Halmos [13] on unbiased estimation and von Mises [35] on expansions of smooth statistical functionals in the late 40s. U -statistics were formally introduced and studied by Hoeffding in 1948. Evarist started being interested in the asymptotic theory of U -statistics in the early 90s, when this theory was already relatively well developed, both in the case of U -statistics based on i.i.d. observations and in more general cases. Moreover, Nolan and Pollard [29,30] initiated the study of U -processes (empirical processes of U -statistic structure indexed by their kernels). However, by the early 90s, many of these results had not reached the same degree of finality as the classical limit theorems for the sums of independent random variables and a number of hard and challenging problems remained open. Many of these problems were solved in the 90s in a series of papers by Evarist with several co-authors, including Joel Zinn, Miguel Arcones, Stanislaw Kwapien and Rafal Latala. The results obtained by Evarist and his collaborators included Marcinkiewicz type laws of large numbers for U -statistics (Giné and Zinn [61]) and the necessity of finiteness of the second moment and degeneracy of the kernel for the Central Limit Theorem for U -statistics (Giné and Zinn [65]). They also included important results on the central limit theorem for U -processes indexed by Vapnik–Chervonenkis classes of functions (Arcones and Giné [64]) and striking applications of these results to asymptotics of M -estimators based on U -statistics, in particular, a beautiful proof of asymptotic normality of the simplicial median (Arcones, Chen, and Giné [67]). These developments were based on new and powerful technical tools, such as the Hoffmann–Jørgensen inequality for U -processes (Giné and Zinn [61]) and decoupling inequalities by de la Peña [5], de la Peña and Montgomery-Smith [6,7]. The method of decoupling became particularly important and it gave the name to the (1999) book ‘Decoupling’ written by Evarist with Victor de la Peña that remains to be the most important reference on the modern theory of U -statistics. However, one of the most spectacular results of this theory was obtained after this book was published. In several papers written in the later 90s, Evarist and his co-authors were trying to find a necessary and sufficient condition for the law of iterated logarithms (LILs) for degenerate U -statistics, a problem that happened to be extremely hard. The sufficiency of finiteness of the second moment of the kernel for the LIL was known since the late 80s [8]. Giné and Zhang [71] showed that there are degenerate kernels with infinite second moment for which the LIL does hold. They provided sufficient conditions on the kernel that did not imply the finiteness of the second moment, but these conditions were still not necessary. This challenging problem was solved for

the second order U -statistics in a remarkable paper by Giné, Kwapień, Latala and Zinn [82] who proved the following result. Suppose X, Y, X_1, X_2, \dots are i.i.d. random variables with values in a measurable space (S, \mathcal{A}) and let $h : S \times S \mapsto \mathbb{R}$ be a measurable symmetric kernel. Then,

$$\limsup_n \frac{1}{n \log \log n} \left| \sum_{1 \leq i \neq j \leq n} h(X_i, X_j) \right| < \infty \quad \text{a.s.}$$

if and only if the following conditions hold for some constant $C < \infty$:

- (a) h is canonical (degenerate) for the law of X (that is, $\mathbb{E}h(X, y) = 0$ for almost all y)
- (b) for all $u \geq 10$,

$$\mathbb{E}(h^2(X, Y) \wedge u) \leq C \log \log u$$

- (c) for some $C > 0$,

$$\sup \mathbb{E} \left\{ h(X, Y) f(X) g(Y) : \max(\mathbb{E}f^2(X), \mathbb{E}g^2(X)) \leq 1; f, g \in L^\infty \right\} \leq C.$$

The proof of this completely unexpected result was a masterpiece of technique and it relied on a variety of tools of U -statistics theory (many of them developed by the authors such as exponential bounds for Rademacher chaos due to Latala) and on rather sophisticated truncation arguments. A related result is a new and final version of a Bernstein type concentration inequality for U -statistics (Giné, Latala and Zinn [84]) that is one of the most important and useful inequalities in this area of probability. This inequality was proved for U -statistics of order 2, but later extended to higher orders by Adamczak [1]. Adamczak and Latala [2] obtained necessary and sufficient conditions for bounded LIL for higher order U -statistics.

2.6. The asymptotic distribution of the t -statistic

Student’s one-sample t -statistic

$$T_n = \frac{\sum_{i=1}^n X_i / n^{1/2}}{\left\{ \sum_{i=1}^n (X_i - \bar{X}_n)^2 / (n - 1) \right\}^{1/2}} = \frac{S_n / V_n}{\sqrt{\frac{n - (S_n / V_n)^2}{n - 1}}}$$

where $S_n = \sum_{i=1}^n X_i$, $V_n = \sum_{i=1}^n X_i^2$ and X_1, \dots, X_n are i.i.d. random variables, plays a key role in basic applied statistics. While its exact distribution is well-known under Gaussian sampling theory, it is important to understand the properties of T_n under non-Gaussian (or other non-standard) assumptions. Efron [11] reviewed early studies of T_n under non-standard conditions (including [16,14], and others), and studied the limiting behavior of T_n and self-normalized sums. Logan et al. [23] showed that if X is in the domain of attraction of an α -stable law, $0 < \alpha \leq 2$, centered if $\alpha > 1$ and symmetric if $\alpha = 1$, then $S_n / V_n \rightarrow_d Z_\alpha$ where Z_α is sub-Gaussian. They conjectured that “ S_n / V_n is asymptotically normal if (and perhaps only if) X is in the domain of attraction of the normal law” and X is centered. The direct part of this conjecture follows fairly easily from standard results, see [24]. The converse or “only if” part was proved in [72] in 1997 by Evarist in collaboration with David Mason and Friedrich Götze. This beautiful paper shows Evarist’s complete mastery of the Paley–Zygmund inequality and the use thereof to show that if $\{S_n / V_n\}$ is stochastically bounded, then it is L_1 -bounded as well.

Evarist returned to this theme in at least two other papers: in [73] (with David Mason) he studied laws of the iterated logarithm for self-normalized sums; in [95] (with Friedrich Götze) he established asymptotic normality of multivariate t -statistics under non-standard conditions.

2.7. Nonparametric statistics

In the 21st century, Evarist started to work on problems in nonparametric statistics, an area within mathematical statistics of high activity since the mid 90s. Evarist's interest was sparked by the broad applicability of empirical process tools to the area. A main insight was that Talagrand's [34] deep inequality for empirical processes could be used with great effect, particularly to deal with problems that involve risk bounds in supremum-norm loss in density estimation, see the papers [87,107]. For example consider a kernel estimator of the form

$$f_n(x) = \frac{1}{nh} \sum_{i=1}^n K\left(\frac{x - X_i}{h}\right), \quad X_i \sim \text{i.i.d. } P,$$

with K a suitable kernel function. If we ignore the estimation 'bias', the uniform risk can be viewed as the empirical process

$$\|f_n - Ef_n\|_\infty = \frac{1}{h} \sup_{g \in \mathcal{G}} |(P_n - P)g|, \quad \mathcal{G} = \left\{ g = K\left(\frac{x - \cdot}{h}\right) : x \in \mathbb{R}^d \right\}.$$

A first key idea used in the paper [87] is that under simple conditions on K , such as that it is of bounded variation, the class \mathcal{G} can be shown to be a Vapnik–Chervonenkis type class and hence, by the usual chaining argument and if P has a bounded density,

$$E \sup_{g \in \mathcal{G}} |(P_n - P)g| \lesssim \sqrt{\frac{h \log(1/h)}{n}}$$

for relevant choices of h . Moreover, by Talagrand's inequality the concentration of $\|f_n - Ef_n\|_\infty$ around its expectation is effectively Gaussian (again for relevant choices of h), and this can be used for various things: Initially Evarist derived from it the exact almost sure limiting constant of $\sqrt{nh/\log(1/h)}\|f_n - Ef_n\|_\infty$ as $n \rightarrow \infty, h \rightarrow 0$, both for the kernel estimator [87] and for wavelet estimators [107], where a slightly different scaling is required. In later work [106,108] these exponential inequalities were shown to be of great use also to construct adaptive estimators of densities that successfully deal with the bias $\|Ef_n - f\|_\infty$ too (by applying Lepski's [22] method). Related techniques were also used in the paper [97] to give empirical approximations (using the graph Laplacian) of the true Laplace operator on a Riemannian manifold (a result that has been used in the machine learning community), and also in the paper [113], where these concentration inequalities were used to give a new approach to derive contraction rates in Bayesian nonparametric function estimation, ideas which have since been used in Bayesian non-parametrics in the recent papers Ray [32] and Nickl and Söhl [28], among others.

In another influential paper [110] Evarist constructed adaptive confidence bands for unknown densities by deriving the exact Gumbel limit distribution of $\|f_n - f\|_\infty$, suitable scaled and centered, where f_n is a fully adaptive estimator (again based on Lepski's [22] method). This was the first exact limiting distribution result for any adaptive estimator, and required a subtle use of Gaussian approximation techniques and limit theory for non-stationary Gaussian processes. Next to the probabilistic challenges this required the introduction of some new qualitative assumptions on f , which have now become known under the name of 'self-similarity', and which were

shown in [110] to be effectively generic for Hölder spaces. These ‘self-similarity’ conditions have turned out to be more or less the ‘right’ conditions for the existence of adaptive nonparametric confidence sets, and have been further studied in several recent papers including Hoffmann and Nickl [15], Chetverikov et al. [4] and the recent discussion paper by Szabo et al. [33], all in the *Annals of Statistics*.

A further result that deserves mentioning here, and that is related to some techniques dating back to the famous 1984 paper [34] of Evarist with Joel Zinn, is the paper [103], where it is shown that certain pre-gaussian classes \mathcal{F} of functions exist that are (a) *not* P -Donsker for certain P but for which (b) the *smoothed empirical process* $\sqrt{n}(P_n * K_h - P)$ corresponding to a kernel density estimator *does* converge in distribution in $\ell_\infty(\mathcal{F})$ to the generalized Brownian bridge G_P —thus $P_n * K_h$ is strictly better than P_n in this case. These results have been instrumental in the recent study of statistical inference for the distribution function of Lévy measures and infinitely divisible distributions, see the articles Nickl and Reiß [26] and Nickl et al. [27].

The key relevance of probabilistic techniques for the foundations of nonparametric statistics has led to Evarist’s third monograph, of the title *Mathematical Foundations of Infinite-Dimensional Statistical Models*. It summarizes and contains much of Evarist’s work in the area, and demonstrates yet again the deep insight Evarist had into the mathematical foundations that underpin modern probability theory and statistics. In particular in the chapters on Gaussian and empirical processes in this book Evarist has left us a great intellectual monument that will be a reference for generations to come.

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