Errata

1 Introduction

Since publication of our book *Empirical Processes with Applications to Statistics* in 1986, we have become aware of several mathematical errors and a number of typographical and other minor errors. Although we would now do many things differently, our purpose here is only to give corrections of the errors of which we are currently aware.

We encourage readers finding further errors to let us know of them.

We owe thanks to the following friends, colleagues, reviewers, and users of the book for telling us about errors, difficulties, and shortcomings: N. H. Bingham, M. Csörgő, S. Csörgő, Kjell Doksum, Peter Gaenssler, Richard Gill, Paul Janssen, Keith Knight, Ivan Mizera, D. W. Muller, David Pollard, Peter Sasieni, and Ben Winter.

We owe special thanks to Peter Gaenssler for providing us with a long list of typographical errors which provided the starting point for section 3 here.

The corrections of Chapters 7, 21, and 23 given in section 2 were aided by discussions and correspondence with Richard Gill and Ben Winter (in the case of Chapter 7), I. Bomze and E. Reschenhofer, and W.D. Kaigh (in the case of Chapter 21), and Keith Knight (in the case of section 23.3).

2 Major changes and revisions

Here we give substantial corrections and revisions of section 7.3 (pages 304–306) and section 23.3 (pages 767–771).

2.1 Revision and correction of section 7.3

The last two lines (pages 305, lines -7 and -6) of the proof of (1) of Theorem 1 (page 304) are false. Hence there are also difficulties in the cases (i)–(v) on pages 305–306. The following revision of section 7.3 should replace that entire section. As indicated in the following text, these results are due to Peterson (1977), Gill (1981), and Wang (1987).

We owe thanks to Richard Gill and Ben Winter for pointing out these difficulties and for correspondence concerning their solution.

Section 7.3*, pages* 304–306*, should be replaced by the following:*

3. CONSISTENCY OF $\widehat{\Lambda}_n$ and $\widehat{\mathbb{F}}_n$.

In this section we use the representations of Theorem 7.2.1 and continuity of the product integral map $\mathcal{E}$ which takes Λ to F (see section B.6 and especially example B.6.1, page 898) to establish weak and strong consistency of $\widehat{\Lambda}_n$ and $\widehat{\mathbb{F}}_n$. Our first result gives strong consistency of both $\widehat{\Lambda}_n$ and $\widehat{\mathbb{F}}_n$ on any interval $[0, \theta]$ with $\theta < \tau \equiv \tau_H \equiv H^{-1}(1)$.

Theorem 1. Suppose that F and G are arbitrary df's on $[0, \infty)$. Recall $\tau \equiv \tau_H \equiv H^{-1}(1)$ where $1 - H \equiv (1 - F)(1 - G)$. Then for any fixed $\theta < \tau$

$$\|\widehat{\mathbb{F}}_n - F\|_0^\theta \to_{a.s.} 0 \qquad \text{as } n \to \infty \tag{1}$$

and

$$\|\widehat{\Lambda}_n - \Lambda\|_0^\theta \to_{a.s.} 0 \qquad \text{as } n \to \infty. \tag{2}$$

The following theorems strengthen (1) of Theorem 1 in different directions.

Theorem 2. Suppose F and G are df's on $[0, \infty)$ with $\tau \equiv \tau_H \equiv H^{-1}(1)$ satisfying either $H(\tau-) < 1$ or $F(\tau-) = 1$ (where $\tau = \infty$ is allowed). Then

$$\sup_{0 \le t \le \tau} |\widehat{\mathbb{F}}_n(t) - F(t)| = \|\widehat{\mathbb{F}}_n - F\|_0^\tau \to_{a.s.} 0 \qquad \text{as } n \to \infty, \tag{3}$$

and, with $T \equiv Z_{n:n}$,

$$\sup_{0 \le t \le T} |\widehat{\mathbb{F}}_n(t) - F(t)| = \|\widehat{\mathbb{F}}_n - F\|_0^T \to_{a.s.} 0 \qquad \text{as } n \to \infty. \tag{4}$$

The following theorem is more satisfactory since F and G are completely arbitrary; the price is that the consistency is in probability (and the supremum in (5) is just over the interval $[0, \tau)$).

Theorem 3 (Wang). Suppose that F and G are completely arbitrary. Then

$$\sup_{0 \le t < \tau} |\widehat{\mathbb{F}}_n(t) - F(t)| \to_p 0 \qquad \text{as } n \to \infty, \tag{5}$$

and, with $T \equiv Z_{n:n}$,

$$\sup_{0 \le t \le T} |\widehat{\mathbb{F}}_n(t) - F(t)| \to_p 0 \qquad \text{as } n \to \infty. \tag{6}$$

Open Question 1. Does Wang's Theorem 3 continue to hold with $\to_p$ replaced by $\to_{a.s.}$? (The hard case not covered by Theorem 2 is $F(\tau-) < 1$, $G(\tau-) = 1$.)

Recall that for an arbitrary hazard function Λ (of a df F on R^+), the (product integral) or exponential map $\mathcal{E}(-\Lambda)$ recovers $1 - F$:

$$\begin{aligned} 1 - F(t) &= \mathcal{E}(-\Lambda)(t) \equiv \prod_{0 \le s \le t} (1 - d\Lambda) \\ &= \exp(-\Lambda^c(t)) \prod_{0 \le s \le t} (1 - \Delta\Lambda(s)); \end{aligned}$$

see section B.6 and Example B.6.1. Our proofs of Theorems 1–3 will use the following basic lemma which is due to Peterson (1977), Gill (1981), and, in the present form, Wang (1987).

Lemma 2.1 (Continuity of the product integral map $\mathcal{E}$) *Suppose that* $\{g_n\}_{n \ge 0}$ *is a sequence of nondecreasing functions on* $A = [0, \tau]$ *or* $[0, \tau)$ *satisfying* $\Delta g_0 < 1$*, and set* $h_n \equiv \mathcal{E}(-g_n)$, $n = 0, 1, \ldots$. *If*

(7) $$\sup_{t \in A} |g_n(t) - g_0(t)| \to 0 \qquad \textit{as } n \to \infty,$$

then

(8) $$\sup_{t \in A} |h_n(t) - h_0(t)| \to 0 \qquad \textit{as } n \to \infty.$$

Proof of Theorem 1. Now $\|\mathbb{H}_n - H\| \to_{a.s.} 0$ by Glivenko–Cantelli, so that $\|\mathbb{H}_n(\cdot -) - H_-\| \to_{a.s.} 0$ also. Thus for any fixed $t \le \theta$ we have a.s. that

$$\begin{aligned} |\widehat{\Lambda}_n(t) - \Lambda(t)| \le \int_0^t &|(1 - \mathbb{H}_{n-})^{-1} - (1 - H_-)^{-1}| d\mathbb{H}_n^1 \\ &+ \left| \int_0^t (1 - H_-)^{-1} d(\mathbb{H}_n^1 - H^1) \right| \end{aligned}$$

(a) $$\to_{a.s.} 0 + 0 = 0$$

by the Glivenko–Cantelli theorem and $H(t-) \le H(\theta) < 1$ for the first term, and by the SLLN for the second term. Since $\widehat{\Lambda}_n$ and Λ are $\uparrow$, the standard argument of (3.1.83) improves (a) to (2).

But then (1) follows from (2) and continuity of the product integral map $\mathcal{E}$ given in Lemma 2.1. □

Proof of Theorem 2. First suppose $H(\tau-) < 1$. Then as in (a) of the proof of theorem 1,

$$\begin{aligned} |\widehat{\Lambda}_n(t) - \Lambda(t)| \le &\left| \int_0^t \{(1 - \mathbb{H}_{n-})^{-1} - (1 - H_-)^{-1}\} d\mathbb{H}_n^1 \right| \\ &+ \left| \int_0^t (1 - H_-)^{-1} d(\mathbb{H}_n^1 - H^1) \right|, \end{aligned}$$

where the first term converges to zero uniformly on $[0,\tau]$ by the Glivenko–Cantelli theorem since $1-H(\tau-)>0$ and $\widehat{\mathbb{H}}_n^1(\tau)\le 1$. Now the second term: for $0\le t\le \tau$,

$$\begin{aligned}
&\left|\int_0^t \frac{1}{1-H_-}d(\mathbb{H}_n^1-H^1)\right| \\
&\quad\le \left|\frac{\widehat{\mathbb{H}}_n^1(t)-H^1(t)}{1-H(t-)}-\int_0^t(\widehat{\mathbb{H}}_n^1(s)-H^1(s))d\left(\frac{1}{1-H(s-)}\right)\right| \\
&\qquad\qquad + \left|\frac{\Delta\widehat{\mathbb{H}}_n^1(\tau)-\Delta H^1(\tau)}{1-H(\tau-)}\right| \\
&\quad\le 2\frac{\|\widehat{\mathbb{H}}_n^1-H^1\|_0^\tau}{1-H(\tau-)}+\left|\frac{\Delta\widehat{\mathbb{H}}_n^1(\tau)-\Delta H^1(\tau)}{1-H(\tau-)}\right| \\
&\quad\to_{a.s.} 0+0=0,
\end{aligned}$$

so the second term converges to zero a.s. uniformly in $t\in[0,\tau]$. Hence

(a) $$\|\widehat{\Lambda}_n-\Lambda\|_0^\tau\equiv\sup_{0\le t\le\tau}|\widehat{\Lambda}_n(t)-\Lambda(t)|\to_{a.s.}0.$$

If $\Delta\Lambda(\tau)<1$, then (3) follows from Lemma 1. If $\Delta\Lambda(\tau)=1$ (so $F(\tau)=1$), then lemma 1 and (a) imply that

$$\sup_{0\le t<\tau}|\widehat{\mathbb{F}}_n(t)-F(t)|\to_{a.s.}0$$

and

$$\begin{aligned}
0\le 1-\widehat{\mathbb{F}}_n(\tau) &\le 1-\Delta\widehat{\Lambda}_n(\tau)\\
&\to_{a.s.} 1-\Delta\Lambda(\tau)=0=1-F(\tau),
\end{aligned}$$

so again (3) holds.

Now suppose that $F(\tau-)=1$. Given $\epsilon>0$, choose $\theta<\tau$ such that $F(\theta)>1-\epsilon$. For $\theta\le t\le\tau$ both

$$\widehat{\mathbb{F}}_n(\theta)\le\widehat{\mathbb{F}}_n(t)\le 1$$

and

$$1-\epsilon\le F(\tau)\le 1.$$

Hence

(b) $$\|\widehat{\mathbb{F}}_n-F\|_\theta^\tau\le\max\{\epsilon,1-\widehat{\mathbb{F}}_n(\theta)\}\to_{a.s.}\max\{\epsilon,1-F(\theta)\}=\epsilon$$

by (1). Since ϵ is arbitrary, (1) and (b) imply (3) in this case ($F(\tau-) = 1$), too.

Since $T \equiv Z_{n:n} \leq \tau$ a.s., (4) follows from (3) . $\square$

Proof of Theorem 3. We first suppose $\theta \leq \tau$ with $F(\theta-) < 1$ and show that

(a) $$\sup_{0 \leq t < \theta} |\widehat{\Lambda}_n(t) - \Lambda(t)| \to_p 0 \qquad \text{as } n \to \infty$$

and

(b) $$\sup_{0 \leq t < \theta} |\widehat{\mathbb{F}}_n(t) - F(t)| \to_p 0 \qquad \text{as } n \to \infty.$$

Let $\mathbb{D}_n \equiv \widehat{\Lambda}_n - \Lambda$. Then, with $T \equiv Z_{n:n}$, $\mathbb{D}_n^T \equiv \{\mathbb{D}_n(t \wedge T) \,:\, t \geq 0\}$ is a square integrable martingale with predictable variation process

(c) $$\langle \mathbb{D}_n^T \rangle(t) = \int_0^{t \wedge T} \frac{1 - \Delta\Lambda(s)}{n(1 - \mathbb{H}_n(s-))} d\Lambda(s).$$

Now

(d) $$\langle \mathbb{D}_n^T \rangle(\theta-) \to_{a.s.} 0.$$

To see this, let $\epsilon > 0$, and choose $\sigma < \theta$ so that $\Lambda(\theta-) - \Lambda(\sigma) < \epsilon$, and hence $H(\sigma) < 1$ also. Then

$$\begin{aligned} \langle \mathbb{D}_n^T \rangle(\theta-) - \langle \mathbb{D}_n^T \rangle(\sigma) &= \int_{(\sigma,\theta)} 1_{[T \geq s]} \frac{1 - \Delta\Lambda(s)}{n(1 - \mathbb{H}_n(s-))} d\Lambda(s) \\ &\leq \Lambda(\theta) - \Lambda(\sigma) < \epsilon, \end{aligned}$$

and, by the Glivenko–Cantelli theorem

$$\begin{aligned} n\langle \mathbb{D}_n^T \rangle(\sigma) &= \int_0^\sigma \frac{1 - \Delta\Lambda(s)}{1 - \mathbb{H}_n(s-)} d\Lambda(s) \\ &\to_{a.s.} \int_0^\sigma \frac{1 - \Delta\Lambda(s)}{1 - H(s-)} d\Lambda(s) < \infty. \end{aligned}$$

Therefore,

$$\langle \mathbb{D}_n^T \rangle(\sigma) \to_{a.s.} 0$$

and

$$\limsup_{n \to \infty} \langle \mathbb{D}_n^T \rangle(\theta-) \leq \epsilon \qquad \text{a.s.}$$

Since $\epsilon > 0$ is arbitrary, (d) holds.

By Lenglart's inequality B.4.1,

(e) $$\sup_{0\le t<\theta} |\mathbb{D}_n^T(t)| \to_p 0 \qquad \text{as } n \to \infty.$$

Since we also have (recall that $T \equiv Z_{n:n}$)

$$\{\Lambda(\theta-) - \Lambda(T)\}1_{[T<\theta]} \to_{a.s.} 0 \qquad \text{as } n \to \infty,$$

in view of $F(\theta-) < 1$, (a) holds.

Now (a) implies that for every subsequence $\{n'\}$ there is a further subsequence $\{n''\} \subset \{n'\}$ so that

(f) $$\sup_{t<\theta} |\widehat{\Lambda}_{n''}(t) - \Lambda(t)| \to_{a.s.} 0 \qquad \text{as } n \to \infty.$$

But by continuity of $\mathcal{E}$ given by Lemma 1, it follows from (f) that

(g) $$\sup_{0\le t<\theta} |\widehat{\mathbb{F}}_{n''}(t) - F(t)| \to_{a.s.} 0,$$

and hence (b) holds when $F(\theta-) \le 1$.

To complete the proof of (5), it remains only to consider the case $F(\tau-) = 1$. But then (5) follows from (3).

To prove (6), consider the two cases $H(\tau-) = 1$ and $H(\tau-) < 1$: If $H(\tau-) = 1$, then $T \equiv Z_{n:n} < \tau$ a.s., and hence (6) follows from (5). If $H(\tau-) < 1$, then (6) follows from (4). □

Proof of Lemma 1. By (7) and $\Delta g_0 < 1$ we can assume that

(a) $$\Delta g_n < 1 \qquad \text{for } n = 1, 2, \ldots.$$

Since g_n are nondecreasing and finite and (a) holds, it is easy to verify that $h_n > 0$, $n = 0, 1, 2, \ldots$. For $t \in A$ and $\epsilon > 0$, define (note (B.5.3))

(b) $$\underline{g}_n^\epsilon(t) \equiv g_n^c(t) - \sum_{s\le t} \log(1 - \Delta g_n(s))1_{[|\Delta g_n(s)|\le\epsilon]}$$

and

(c) $$\overline{g}_n^\epsilon(t) \equiv -\sum_{s\le t} \log(1 - \Delta g_n(s))1_{[|\Delta g_n(s)|>\epsilon]}$$

so that

(d) $$\underline{g}_n^\epsilon(t) + \overline{g}_n^\epsilon(t) = -\log h_n(t).$$

Now $\overline{g}_n^\epsilon(t)$ is the sum of at most a finite number of terms. Thus by (7) for every $\epsilon > 0$ with

$$\text{(e)} \qquad \epsilon \in \{a < 1/2 :\ \Delta g_0(t) \neq a \ \text{ for all } t \in A\}$$

it follows that

$$\text{(f)} \qquad \sup_{t\in A} \left| \sum_{s\le t} \Delta g_n(s) 1_{[|\Delta g_n(s)|>\epsilon]} - \sum_{s\le t} \Delta g_0(s) 1_{[|\Delta g_0(s)|>\epsilon]} \right| \to 0$$

as $n \to \infty$ and

$$\text{(2.1)} \qquad \sup_{t\in A} \left| \overline{g}_n^\epsilon(t) - \overline{g}_0^\epsilon(t) \right| \to 0 \qquad \text{as } \ n \to \infty.$$

But note that

$$\begin{aligned}
&\left|\underline{g}_n^\epsilon(t) - \underline{g}_0^\epsilon(t)\right| \\
&\quad \le \left|\underline{g}_n^\epsilon(t) - g_n^c(t) - \sum_{s\le t} \Delta g_n(s) 1_{[|\Delta g_n(s)|\le\epsilon]}\right| \\
&\qquad + \left|g_n^c(t) + \sum_{s\le t} \Delta g_n(s) 1_{[|\Delta g_n(s)|\le\epsilon]} - g_0^c(t) - \sum_{s\le t} \Delta g_0(s) 1_{[|\Delta g_0(s)|\le\epsilon]}\right| \\
&\qquad + \left|\underline{g}_0^\epsilon(t) - g_0^c(t) - \sum_{s\le t} \Delta g_0(s) 1_{[|\Delta g_0(s)|\le\epsilon]}\right| \\
&\quad \le \Big| \sum_{s\le t} \{\log(1-\Delta g_n(s)) + \Delta g_n(s)\}\, 1_{[|\Delta g_n(s)\le\epsilon]} \Big| \\
&\qquad\quad + \Big| \sum_{s\le t} \Delta g_n(s) 1_{[|\Delta g_n(s)|>\epsilon]} - \sum_{s\le t} \Delta g_0(s) 1_{[|\Delta g_0(s)|>\epsilon]} \Big| \\
&\qquad\quad + |g_n(t) - g_0(t)| \\
&\qquad\quad + \Big| \sum_{s\le t} \{\log(1-\Delta g_0(s)) + \Delta g_0(s)\}\, 1_{[|\Delta g_0(s)|\le\epsilon]} \Big| \\
&\qquad \le \epsilon(|g_n(t)| + |g_0(t)|) \\
&\qquad\quad + \Big| \sum_{s\le t} \Delta g_n(s) 1_{[|\Delta g_n(s)|>\epsilon]} - \sum_{s\le t} \Delta g_0(s) 1_{[|\Delta g_0(s)|>\epsilon]} \Big| \\
&\qquad\quad + |g_n(t) - g_0(t)|.
\end{aligned}$$

Therefore, for every ϵ satisfying (e), (f) yields

$$\limsup_{n\to\infty} \sup_{t\in A} |\underline{g}_n^\epsilon - \underline{g}_0^\epsilon(t)| \le 2\epsilon g_0(\tau),$$

and hence, by (d) and (g),

(h) $$\limsup_{n\to\infty} \sup_{t\in A} |\log h_n(t) - \log h_0(t)| \le 2\epsilon g_0(\tau).$$

Since ϵ is arbitrary, (h) implies (8). □

2.2 Revision and correction of Section 7.7. Weak convergence $\Rightarrow$ of $\mathbb{B}_n$ and $\mathbb{X}_n$ in $\|\cdot/q\|_0^T$-metrics

On page 325 in Exercise 3, the displayed equation should read as follows:

$$(1-K)/(1-F) = \left(1 + \int_0^{\cdot} \widetilde{C}\, dF\right)^{-1},$$

and then "Hence $(1-K)/(1-F)$ is $\searrow$."

2.3 Revision and correction of Section 19.4

Page 689, *lines* 9–12, *should be replaced by the following:*

$$\begin{aligned}
&\left|\int_0^1 [1_{[\xi\le t]} - t] J(t)\, dg(t)\right| \\
&\quad\le \int_0^{\xi} tB(t)\, dg(t) + \int_{\xi}^1 (1-t) B(t)\, dg(t) \\
&\quad\le tB(t)D(t)\Big|_0^{\xi} + \int_0^{\xi} D(t)\, d(tB(t)) \\
&\qquad\quad + (1-t)B(t)D(t)\Big|_{\xi}^1 + \int_{\xi}^1 D(t)\, d(tB(t)) \\
&\quad\le M'[\xi(1-\xi)]^{-a} \qquad \text{where } \ a < 1/2.
\end{aligned}$$

2.4 Revision and correction of Section 23.3. The Shorth

There is an error here in the grouping of the $n^{1/6}$ factor leading to (i) on page 768; and exercise 1 on page 771) is not correct. The following correction is perhaps the simplest. A different, somewhat longer correction, was suggested to us by Keith Knight. Knight's alternative correction changes the "centering" in the definition of M_n in (6) from $2F^{-1}(1-a)$ to $\mathbb{F}_n^{-1}(1-a) - \mathbb{F}_n^{-1}(a)$.)

Beginning on page 768 *just after* (g):

Moreover, since g' exists and is continous,

$$\begin{aligned}
&\sup_{|t|\le K}\left|g\left(1-a+\frac{At}{n^{1/3}}\right)n^{1/6}\mathbb{B}_n\left(1-a+\frac{At}{n^{1/3}}\right)\right.\\
&\qquad\left.-\,g(1-a)n^{1/6}\mathbb{B}_n\left(1-a+\frac{At}{n^{1/3}}\right)\right|\\
&\le\left\{n^{1/6}\sup_{|t|\le K}\left|g\left(1-a+\frac{At}{n^{1/3}}\right)-g(1-a)\right|\right\}\\
&\qquad\cdot\left\{\sup_{|t|\le K}\left|\mathbb{B}_n\left(1-a+\frac{At}{n^{1/3}}\right)\right|\right\}\\
&\le\left\{n^{1/4}\sup_{|t|\le K}\left|g\left(1-a+\frac{At}{n^{1/3}}\right)-g(1-a)\right|\right\}\\
&\qquad\cdot\left\{n^{-1/12}\sup_{|t|\le K}\left|\mathbb{B}_n\left(1-a+\frac{At}{n^{1/3}}\right)\right|\right\}\\
&=o(1)O(1)\qquad\text{a.s.}\\
&=o(1)\qquad\text{a.s.}
\end{aligned}$$

Continue on page 769, *line* 1.

Correction of Exercise 23.3.1, page 771. Replace Exercise 1 by the following:

Exercise 1. Show that for any $0 \le K < \infty$ and $0 \le A < \infty$ and $0 \le a < 1$ we have

$$n^{-1/12}\sup_{|t|\le K}\left|\mathbb{B}_n(a+tA/n^{1/3})\right| = O(1) \qquad \text{a.s.}$$

Knight's alternative correction for this section involves the following alternative exercise.

Exercise $1'$**.** Show that for any $0 \le K < \infty$ and $0 \le A < \infty$ and $0 \le a < 1$ we have

$$\sup_{|t|\le K} n^{1/2}\left|\mathbb{B}_n(a+tA/n^{1/3}) - \mathbb{B}_n(a)\right| = O(1) \qquad \text{a.s.}$$

3 Typographical errors, spelling errors, and minor changes

Page	Line or equation	Error or change
12	(10)	factor of $(-1)^{k+1}$ is missing
14	(7)	factor of $(-1)^{k+1}$ is missing
15	(14)+1	$\sum_{j=1}^{\infty} \chi_i^2 \to \sum_{j=1}^{\infty} \chi_j^2$
16	-1	(2.2.11) → (2.2.13)
25	Exercise 4	replace G on the LHS by g (lower case)
25	-4	left continuous inverse K^{-1}
27	(11)	$d(x, y)$ is the $d_0(x, y)$ of Billingsley (1968, pp. 112–115)
28	(15)	$X \to x$ (is needed in 5 places)
29	(18)+1	$x \to X$
29	(18)+5	the set of continuous
30	(5) +1	$(s_1 \wedge s_2)(t_1 \wedge t_2 - t_1 t_2)$
37	(j)+1	5.9 → 9.9
37	(15)-4	5.6 → 9.6
47	-12	replace “to then” by “then to”
47	Theorem 4	referred to on p. 113 as “Skorokhod’s theorem”
47	(16)+1	M_S δ-separable implies M_S is $\mathcal{M}_\delta^B$-measurable (cf. lemma in Gaenssler)
49	(24)-1	$\|Z - A_m Z\| \to \|Z - A_m \circ Z\|$
52	(26′)+1	insert “with $\mu_n([0, 1]) \to$ some $\# \in (0, \infty)$ ”
59	4	change to: . . . independent of $\mathbb{S}$
59	5	$dF(a) \ \to \ dF(-a)$
61	Exercise 8	Keifer → Kiefer
61	-2	(23) → (30)
63	-8	(1.1.31) → (0.1.31)
65	12	$(E\|X - Y\|)^{1/p} \ \to \ (E\|X - Y\|^p)^{1/p}$
69	(2)–(3)	$b \to b_n$
70	16	Brieman → Breiman
73	-8	$\overline{\lim} S_n \ \to \ \overline{\lim} \|S_n\|$
83	-3	$E\|X\| = . \ \to \ E\|X\| = \infty.$
88	(21)	$\xi_{ni} \to \xi_i$
90	(33) +1	$X \to \xi$ twice
90	(35)	$= \to \equiv$
90	(35)+1	identify → identity
92	(54)	Σ
112	2	Lemma 2.3.1 → Lemma 1.3.1
124	-6	$1_{(t_{j-1}, t_j]} \ \to \ 1_{(t_{j-1}, t_j]}(t)$

Page	Line or equation	Error or change
126	7	$\nu^n \to \nu_n$
135	(a)	$E\mathbb{V}_n \ \to\ E\mathbb{V}_n^2$
138	-5	martingale → submartingale
140	(37)	$x_{ni} \to X_{ni}$
150	-8, -9	replace "with $m = n$" by "with $m = m' = n$"
151	(2)	$\sum_{i=1} \to \sum_{i=1}^{n}$
153	-2	$X_{ni} = F_n^{-1}(\xi_{ni}) \to X_{ni} = F_{ni}^{-1}(\xi_{ni})$
154	(16)+1	$X_{ni} \equiv F_{ni}^{-1}(\xi_{ni})$ again
156	6	Theorem 1 → Theorem 3
160	(d)	$\int \ \to\ \int_{-\infty}^{x}$
160	(e) +1	(the second occurrence of $a'a$) $\ \to\ \sqrt{a'a}$
163	-2	$\|[\cdot]\| \ \to \|[\cdot]\|^2$
168	Corollary 1	$F_0 \to F$
168	(5)-1	3.6 → 3.8
169	1	Theorem 14.1.4 → Theorem 4.1.1
169	-12	Theorem 4.1.2 → Theorem 4.1.5
169	-1	$\tilde{P} \to \overline{P}$; Theorem 4.1.2 → Theorem 4.1.5
193	-6	$X_n \ \to\ X_j$
195	1	vector → matrix; constant → constants
195	-10	$X_1' \ \to\ X_i'$
224	(32)	$G_n^2 \to G^2$
224	(32)+1	change $G_n \to_d G$ to $G_n^2 \to_d G^2$
224	(33)	change $P(G > \lambda)$ to $P(G^2 > \lambda)$
262	(25)+3	$d\Lambda(x) = \to\ d\Lambda(x) \equiv$
264	(6)	$1_{[X_i \le y]} \ \to\ 1_{[X_{ni} \le y]}$
264	(6)	$1 \le i \le n. \ \to\ 1 \le i \le n_j$
265	(14)	$X_i \ \to\ X_{ni}$ twice
266	7-8, -2	$X_i \ \to\ X_{ni}$ throughout
270	(32)-1	change (A.9.6) to (A.9.16)
272	(40)	$X_i \ \to\ X_{ni}$ twice
273	(1)	$X_i \ \to\ X_{ni}$ twice
274	-3	$\psi(x) = x^2 \ \to\ \psi(x) = x$
275	(9)	$\|\cdot\|_0^1,\ \|\cdot\| \ \to\ \|\ \|_0^1,\ \|\ \|_0^1$
275	-1	$\int_{-\infty}^{\theta} \ \to\ \int_0^{\theta}$
276	(1)	$X_i \ \to\ X_{ni}$ (twice)
279	(9)	$\mathbb{N} \ \to\ \mathbb{K}$ on RHS
282	(21)	delete nonsymbol before $=$
288	-4	$[\mathbb{K}_n - K] \ \to\ [K_n - \mathbb{K}]$

Page	Line or equation	Error or change
294	(4)+3	$\tau \equiv \tau_H = \tau_F \vee \tau_G \;\rightarrow\; \tau \equiv \tau_H = \tau_F \wedge \tau_G$
295	-4	change (10) to (12)
304-306		see the major revision in Section 2 of this Errata
323	2	change "Proof of (10)" to "Proof of (9)"
325	Exercise 3	see the major revision in Section 2 of this Errata
339	-1	$\mathbb{U}_n \rightarrow \mathbb{U}_{N_n}$
369	(38)	change $\frac{1}{n!}$ to $n!$ on the right side of this display
419	(4)	change $< \epsilon$. to $< M/\sqrt{n}$.
424	9	delete $a_n \downarrow 0$
425	-7	$\mathbb{G} - I \rightarrow \mathbb{G}_n - I$
425	(15) +3	Mason (1981) $\rightarrow$ Mason (1981b)
425	-1	Mason (1981) $\rightarrow$ Mason (1981b)
429	(a)	$= 0$ becomes $= 0$ a.s.
450	10	$]^{\epsilon} g^2(t) \rightarrow]^{\epsilon g^2(t)}$ (at the end of the line)
451	(16)+1	see Bretagnolle and Massart (1987) for $P(\Vert\mathbb{U}\Vert_0^b \geq \lambda) \leq \exp\left(-\frac{\lambda^2}{2b(1-b)}\right)$.
454	(7)	$\leq \rightarrow \geq$.
478	Exercise 4. +1	Anderson's
483	(13)	$\binom{n}{i} \rightarrow \binom{n}{i-1}$ twice
492	-1	Esseen $\rightarrow$ Esséen
545	(18)	$\#\mathbb{U}_n \rightarrow \mathbb{U}_n^{\#}$
558	section title	$\mathbb{K}_n \rightarrow \mathbb{K}$
584	(3) - 1	$((\log_2 n)^{1/4}\sqrt{\log n}/\sqrt{n}$ $\rightarrow ((\log_2 n)(\log n)^2/n)^{1/4}$
604	(2) + 10	$n \rightarrow t$
604	(2) + 11	$t \rightarrow n$
661	5	$\Psi \rightarrow \Psi_n$ twice
661	(5) +1	would $\rightarrow$ might
661	(9) + 1	in the next section $\rightarrow$ in section 4
662	(12)	$= \rightarrow \doteq$
688	(1) -1	"since the ..." $\rightarrow$ "since for the ..."
695	(3) + 2	$\frac{k}{n} \rightarrow \frac{k}{n+1}$
696	(3) + 1	(3.7.4) $\rightarrow$ (3.6.4)
697	(15)	$0 \leq t \leq 1$
698	(21)	$\int_0^{P_{n,i+1}} \rightarrow \int_0^{p_{n,i+1}}$
699	(7)	$\int_0^1 \rightarrow \int_0^t$
731	(22)	$f \circ F^{-1} \rightarrow f \circ F^{-1}\rho_n(1,c)$

Page	Line or equation	Error or change
732	(10)	$\log(1-t) \;\rightarrow\; (\log(1-t))\rho_n(1,c)$
732	(11)	$(1-2\rho_{1,c}) \;\rightarrow\; -\rho_{1,c}^2$
740	(32)	$(1-2\rho_{1,c}) \;\rightarrow\; -\rho_{1,c}^2$
746	5	change to: The definition of $\mathbb{S}_n$ is found first in Smirnov (1947); see also Butler (1969).
747	(11) + 2	change to: Smirnov (1947) and Butler (1969) give an expression for the exact distribution.
771	2	t missing just before K
778	(16)+3	Wang $\rightarrow$ Yang
790	(4)	$\pi \;\rightarrow\; \Pi$
802	(d)	$2.1_{[T_2,\infty)} \;\rightarrow\; 2\cdot 1_{[T_2,\infty)}$
804	(j)	$\underline{F}$ belongs with S_1 and S_m as part of the subscript
813	-4	by (A.14.7) $\rightarrow$ by (A.14.8)
819	-3	$(e^x-1-x^2) \;\rightarrow\; (e^x-1-x)$
820	9	(A.14.14) $\rightarrow$ (A.14.15)
821	-2	nonidentically $\rightarrow$ not identically
821	-3	combinations of $\rightarrow$ combinations of a function of
844	(6)	$\sqrt{2s_n} \;\rightarrow\; \sqrt{2}\, s_n$
850	3	$\gamma_1^3 \;\rightarrow\; \gamma_1^2$
850	-2	Mill's $\rightarrow$ Mills'
851	(5)	$\exp\left(-\frac{\lambda}{2\sigma^2}\psi\left(\frac{\lambda b}{\sigma^2\sqrt{n}}\right)\right) \;\rightarrow\; \exp\left(-\frac{\lambda^2}{2\sigma^2}\psi\left(\frac{\lambda b}{\sigma^2\sqrt{n}}\right)\right)$
852	(a)	$E\exp(\sum_1^n X_i) \;\rightarrow\; E\exp(r\sum_1^n X_i)$
853	7	$0<\lambda<1-\overline{\mu} \;\rightarrow\; 0<\lambda/\sqrt{n}<1-\overline{\mu}$
855	(12)	$\exp\left(-2\lambda^2/\sum_1^n(b_i-a_i)^2\right) \;\rightarrow\; \exp\left(-2n\lambda^2/\sum_1^n(b_i-a_i)^2\right)$
856	(21) +2	Steinback $\rightarrow$ Steinebach
859	-6	Renyi $\rightarrow$ Erdös and Rényi
863	4	$i-1/m \;\rightarrow\; (i-1)/m$
863	-2	$dt).\;\rightarrow\; dt)$ be monotone
868	-5	$U_+^iU_-^{r-i+1} \;\rightarrow\; U_+^iU_-^{k-i+1}$
868	-1	$r \rightarrow k$ twice
873	-12	$[0,\theta] \;\rightarrow\; (0,\theta]$
873	-2	$5 \;\rightarrow\; S$
879	13	$\max_{0\le j\le k} \rightarrow \max_{0\le j<k}$
890	(8) + 2	replace $A^c(t)-\sum_{s\le t}\Delta A(s)$ by $A^c(t)\equiv A(t)-\sum_{s\le t}\Delta A(s)$
896	(2)	$dX \;\rightarrow\; dX^i$
896	(3)	$dX \rightarrow dX^i$
897	(6)	$\int_{(0,t]} \;\rightarrow\; \int_{[0,t]}$
898	3 and (9)	$(0,t] \;\rightarrow\; [0,t]$

Page	Line or equation	Error or change
903	-5	enchantillon $\rightarrow$ echantillon
904	4	Burk $\rightarrow$ Burke
910	20	tall $\rightarrow$ tail
915	-6	Steinbach $\rightarrow$ Steinebach
916	-2	theroy $\rightarrow$ theory
925	-16, right	Wang $\rightarrow$ Yang
925	-7, left	Steinbach $\rightarrow$ Steinebach
929	-7, right	877 $\rightarrow$ 878
936	9, left	Rebelledo $\rightarrow$ Rebolledo
938	17, left	676 $\rightarrow$ 677

4 Accent mark revisions

Page	Line	Error or change
xxxiii	3.8.3	Rényi
xvii	-3	Rényi
16	8	Tusnády
19	-4	Lévy
223	-6, -3	Csörgő
559	4	Csörgő
274	-2	Horváth
492	-6	Horváth
903	-5	nonéquiréparti
904	4	Horváth
905	-6	Horváth
906	1, 3, 7	Horváth
923	23	Horváth
843	-7	Loève
844	-5	Loève
846	6	Loève
855	-10	Loève
861	1	Loève
913	13	Loève
924	24, left	Loève
924	4, left	Komlós
905	24	Csáki
905	24	Tusnády
908	-23, -21	Rejtő
913	1	Poincaré
270	9	Doléans-Dade
897	7, 12	Doléans-Dade
907	1	Doléans-Dade

5 Solutions of “Open Questions”

Problem	Page	Reference for solution
OQ 9.2.1	353	Götze (1985)
OQ 9.2.2	356	Massart (1990)
OQ 9.8.1	400	Khoshnevisan (1992)
OQ 9.8.2	400	Khoshnevisan (1992)
OQ 9.8.3	400	Bass and Khoshnevisan (1995)
OQ 10.6.1	428	
OQ 10.7.1	431	
OQ 12.1.1	495	
OQ 12.1.3	495	Deheuvels (1998, 1997)
OQ 13.4.1	526	Einmahl and Mason (1988)
OQ 13.5.1	530	Lifshits and Shi (2003)
OQ 13.6.1	530	
OQ 14.2.1	544	Deheuvels (1991)
OQ 14.2.2	545	Einmahl and Ruymgaart (1987)
OQ 15.2.1	596	
OQ 16.2.1	605	
OQ 16.4.1	616	Einmahl and Mason (1988)
OQ 17.2.1	628	
OQ 25.3.1	809	

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