Semiparametric Regression Models for Panel Count Data: Comparing Two Estimators

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- joint work with Ying Zhang,
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- Talk at IMS-WNAR meeting,
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- Efficiency comparisons: when should we avoid the pseudo-MLE
- Cautions: identifiability issues and conditions for the theory
- Further work and Open Problems

1. Introduction: a Semiparametric Regression Model for Panel Count Data

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- B Poisson process assumption: $(\mathbb{N}|Z) \sim$ non-homogeneous Poisson process.
- Parameters of interest: (θ, Λ) (or just θ).
- Study estimators when the Poisson assumption B fails, but the conditional mean model given by A holds.

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- No assumptions about G or H

III. Data and Primary Goal:

- Data:

$$\begin{aligned} X &= (Z, K, \underline{T}_K, \mathbb{N}(T_{K,1}), \ldots, \mathbb{N}(T_{K,K})) \\ &\equiv (Z, K, \underline{T}_K, \underline{\mathbb{N}}_K) \end{aligned}$$

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- Based on $X_1, \ldots, X_n$ i.i.d. as X, estimate (θ, Λ)

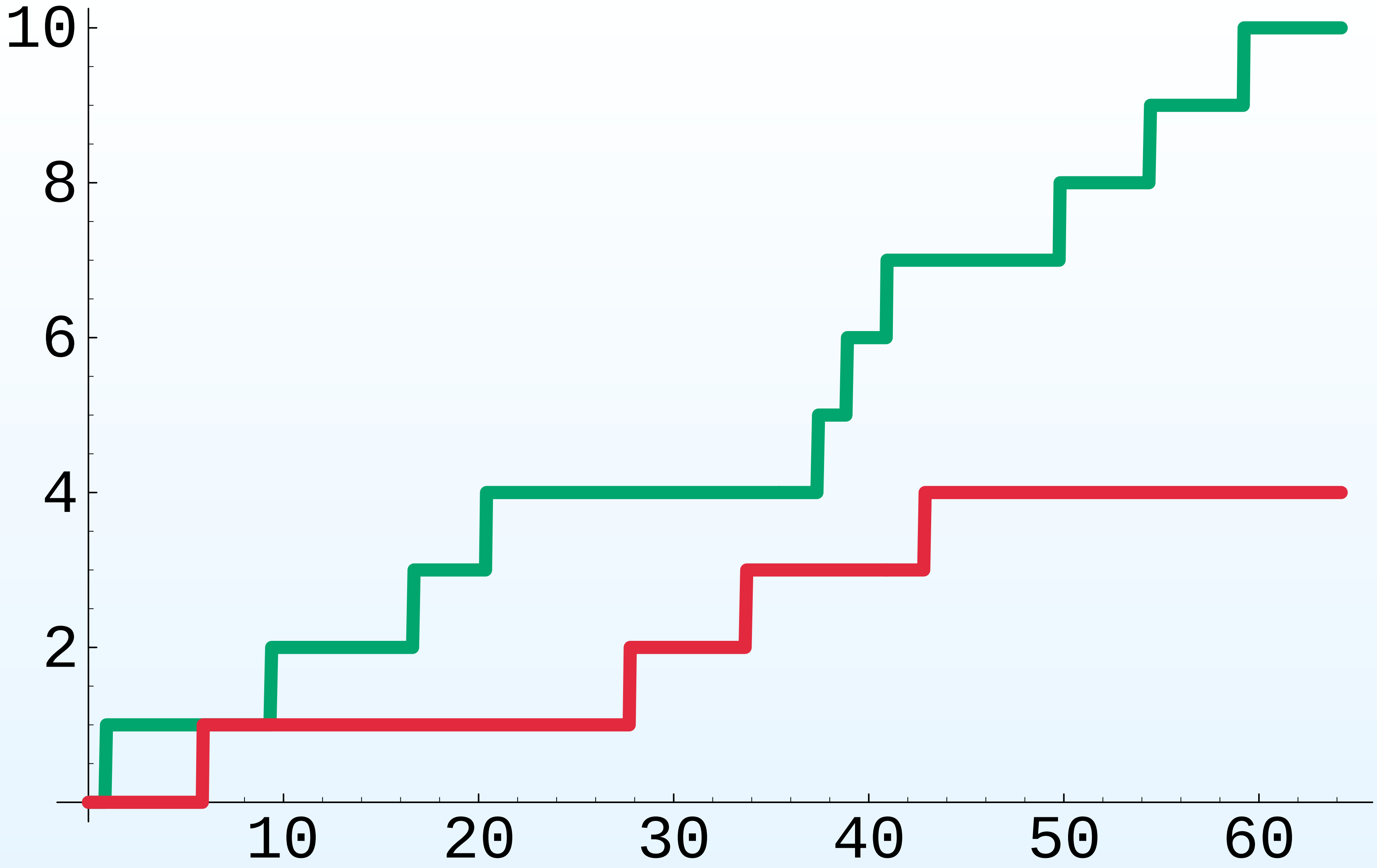
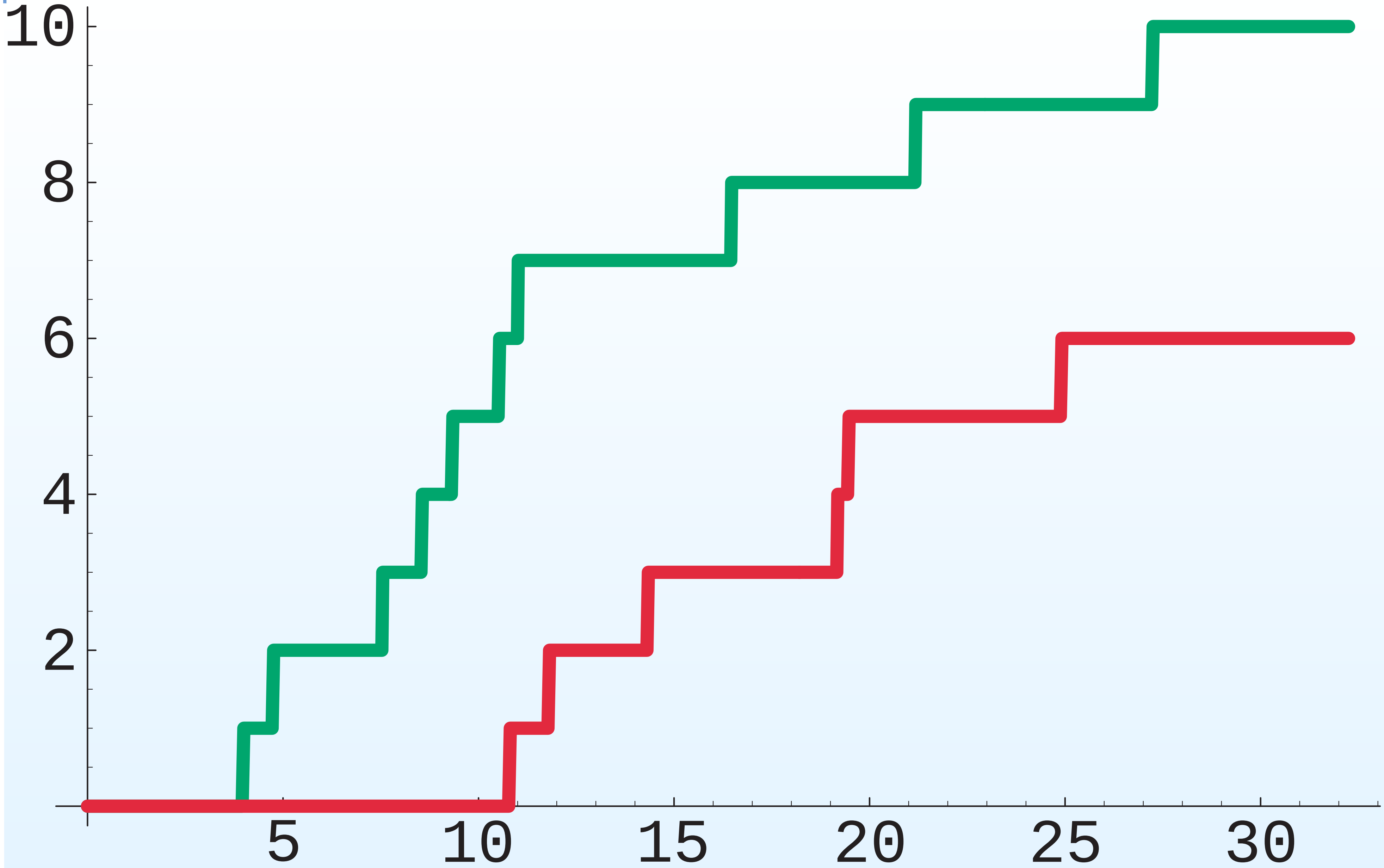


Fig. 1: Counting process (green) and sampling process (red)



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2. Maximum Pseudo-likelihood and Maximum Likelihood Estimators

A. Maximum pseudo-likelihood.

- use the Poisson marginal distributions of $\mathbb{N}$,

$$P(\mathbb{N}(t) = k|Z) = \frac{\Lambda(t|Z)^k}{k!} \exp(-\Lambda(t|Z))$$

and *ignore dependence between* $\mathbb{N}(t_1)$ and $\mathbb{N}(t_2)$ to obtain the pseudo-likelihood

$$l_n^{ps}(\theta, \Lambda) = \sum_{i=1}^{n} \sum_{j=1}^{K_i} \Big\{ \mathbb{N}^{(i)}(T_{K_i,j}^{(i)}) \log \Lambda(T_{K_i,j}^{(i)}) + \mathbb{N}^{(i)}(T_{K_i,j}^{(i)})\theta' Z_i - e^{\theta' Z_i} \Lambda(T_{K_i,j}^{(i)}) \Big\}.$$

Then

$$(\widehat{\theta}_n^{ps}, \widehat{\Lambda}_n^{ps}) \equiv \text{argmax}_{\theta,\Lambda}\, l_n^{ps}(\theta, \Lambda)\,.$$

Implement in two steps:

$$\widehat{\Lambda}_n^{ps}(\cdot, \theta) \equiv \text{argmax}_{\Lambda}\, l_n^{ps}(\theta, \Lambda)\,,$$

and define

$$l_n^{ps,profile}(\theta) \equiv l_n^{ps}(\theta, \widehat{\Lambda}_n^{ps}(\cdot, \theta))\,.$$

Then

$$\widehat{\theta}_n^{ps} = \text{argmax}_{\theta}\, l_n^{ps,profile}(\theta)\,,$$

and

$$\widehat{\Lambda}_n^{ps} = \widehat{\Lambda}_n^{ps}(\cdot, \widehat{\theta}_n^{ps})\,.$$

Let $t_1 < \ldots < t_m$ denote the ordered distinct observation time points in the collection of all observations times, $\{T^{(i)}_{K_i,j},\ j = 1, \ldots, K_i,\ i = 1, \ldots, n\}$, and set

$$w_l = \sum_{i=1}^{n} \sum_{j=1}^{K_i} 1_{[T^{(i)}_{K_i,j} = t_l]}\,, \quad \overline{N}_l = \frac{1}{w_l} \sum_{i=1}^{n} \sum_{j=1}^{K_i} \mathbb{N}^{(i)}_{K_i,j} 1_{[T^{(i)}_{K_i,j} = t_l]}\,,$$

$$\overline{A}_l(\theta, Z) = \frac{1}{w_l} \sum_{i=1}^{n} \sum_{j=1}^{K_i} \exp(\theta' Z^{(i)})\, 1_{[T^{(i)}_{K_i,j} = t_l]}\,.$$

Then the cumulative sum diagram is given by

$$\{(\sum_{l \le i} w_l \overline{A}_l(\theta, Z), \sum_{l \le i} w_l \overline{N}_l)\}_{i=1}^{m}$$

$$
\begin{aligned}
\widehat{\Lambda}_n^{ps}(\cdot,\theta) &= \text{left-derivative of} \\
& \quad \text{Greatest Convex Minorant} \\
& \quad \quad \text{of } \{(\sum_{l \le i} w_l \overline{A}_l(\theta, Z), \sum_{l \le i} w_l \overline{N}_l)\}_{i=1}^{m} \\
&= \max_{i \le l} \min_{j \ge l} \frac{\sum_{i \le p} w_p \overline{N}_p}{\sum_{i \le p} w_p \overline{A}_p(\theta, Z)} \text{ at } t_l \,,
\end{aligned}
$$

which is easy to compute.

B. Maximum likelihood: use the independence of the increments $\Delta\mathbb{N}(s,t] \equiv \mathbb{N}(t) - \mathbb{N}(s)$, and the Poisson distribution of these increments of $\mathbb{N}$,

$$P(\Delta\mathbb{N}(s,t] = k|Z) = \frac{[\Delta\Lambda((s,t]|Z)]^k}{k!}\exp(-\Delta\Lambda((s,t]|Z))$$

to obtain the log-likelihood:

$$\begin{aligned} l_n(\theta,\Lambda) &= \sum_{i=1}^{n}\sum_{j=1}^{K_i}\Big\{\Delta\mathbb{N}^{(i)}((T^{(i)}_{K_i,j-1},T^{(i)}_{K_i,j}])\cdot\log\Delta\Lambda((T^{(i)}_{K_i,j-1},T^{(i)}_{K_i,j}]) \\ &+ \Delta\mathbb{N}^{(i)}((T^{(i)}_{K_i,j-1},T^{(i)}_{K_i,j}])\theta' Z_i \\ &- \exp(\theta' Z_i)\Lambda((T^{(i)}_{K_i,j-1},T^{(i)}_{K_i,j}])\Big\} \end{aligned}$$

Then the MLE is $(\widehat{\theta}_n, \widehat{\Lambda}_n) \equiv \text{argmax}_{\theta,\Lambda}\, l_n(\theta, \Lambda)$.
Implement this maximization in two steps (profile likelihood):

$$\widehat{\Lambda}_n(\cdot, \theta) \equiv \text{argmax}_{\Lambda}\, l_n(\theta, \Lambda)\,,$$

and define $l_n^{profile}(\theta) \equiv l_n(\theta, \widehat{\Lambda}_n(\cdot, \theta))$. Then

$$\widehat{\theta}_n = \text{argmax}_{\theta}\, l_n^{profile}(\theta)\,, \quad \widehat{\Lambda}_n = \widehat{\Lambda}_n(\cdot, \widehat{\theta}_n)\,.$$

Computation of the (profile) “estimator” $\widehat{\Lambda}_n(\cdot, \theta)$ is hard, but possible: iterative convex minorant algorithm.

3. Properties of the Estimators

when the Poisson Assumption Fails

Theorem 1. If assumption A holds, then (under further integrability, boundedness, and identifiability hypotheses):

$$\sqrt{n}(\widehat{\theta}_n - \theta_0) \to_d Z \sim N_d\left(0, A^{-1}B\left(A^{-1}\right)'\right),$$

and

$$\sqrt{n}(\widehat{\theta}_n^{ps} - \theta_0) \to_d Z^{ps} \sim N_d\left(0, (A^{ps})^{-1}B^{ps}\left((A^{ps})^{-1}\right)'\right)$$

where A, B, A^{ps}, and B^{ps} are given by:

$$
\begin{aligned}
B &= Em^*(\theta_0, \Lambda_0; X)^{\otimes 2} \\
&= E\left\{ \sum_{j,j'=1}^{K} C_{j,j'}(Z)\left[Z - \frac{E\left(Ze^{\theta_0' Z}|K, \underline{T}_{K,j,j'}\right)}{E\left(e^{\theta_0' Z}|K, \underline{T}_{K,j,j'}\right)} \right]^{\otimes 2} \right\} \\
A &= E\left\{ \sum_{j=1}^{K} \Delta\Lambda_{0Kj} e^{\theta_0' Z} \left[Z - \frac{E\left(Ze^{\theta_0' Z}|K, \underline{T}_{K,j,j-1}\right)}{E\left(e^{\theta_0' Z}|K, \underline{T}_{K,j,j-1}\right)} \right]^{\otimes 2} \right\}, \\
C_{j,j'}(Z) &= \text{Cov}\left[\Delta N_{Kj}, \Delta N_{Kj'} | Z, K, \underline{T}_K\right].
\end{aligned}
$$

$$
\begin{aligned}
B^{ps} &= Em^{*ps}(\theta_0, \Lambda_0; X)^{\otimes 2} \\
&= E\left\{ \sum_{j,j'=1}^{K} C^{ps}_{j,j'}(Z) \left[Z - \frac{E\left(Ze^{\theta_0' Z} | K, T_{K,j}\right)}{E\left(e^{\theta_0' Z} | K, T_{K,j}\right)} \right]^{\otimes 2} \right\}, \\
A^{ps} &= E\left\{ \sum_{j=1}^{K} \Lambda_{0Kj} e^{\theta_0' Z} \left[Z - \frac{E\left(Ze^{\theta_0' Z} | K, T_{K,j}\right)}{E\left(e^{\theta_0' Z} | K, T_{K,j}\right)} \right]^{\otimes 2} \right\}, \\
C^{ps}_{j,j'}(Z) &= \text{Cov}\left[N_{Kj}, N_{Kj'} | Z, K, T_{K,j,j'} \right],
\end{aligned}
$$

If the Poisson process assumption B holds,

$$
A = B = I(\theta),
$$

and $\widehat{\theta}_n$ is (asymptotically) efficient.

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$$ARE(pseudo, mle) = \frac{[E(K/2)]^2}{E\left\{\frac{K}{K+1}\right\} E\left\{\frac{K(2K+1)}{6}\right\}}$$

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- **Case (c):**

$$ARE(pseudo, mle)(\alpha)$$
$$= \frac{3}{2}\frac{\zeta(\alpha-1)}{\{2\zeta(\alpha-2) + \zeta(\alpha-1)\}E_\alpha\{\frac{K}{K+1}\}}.$$

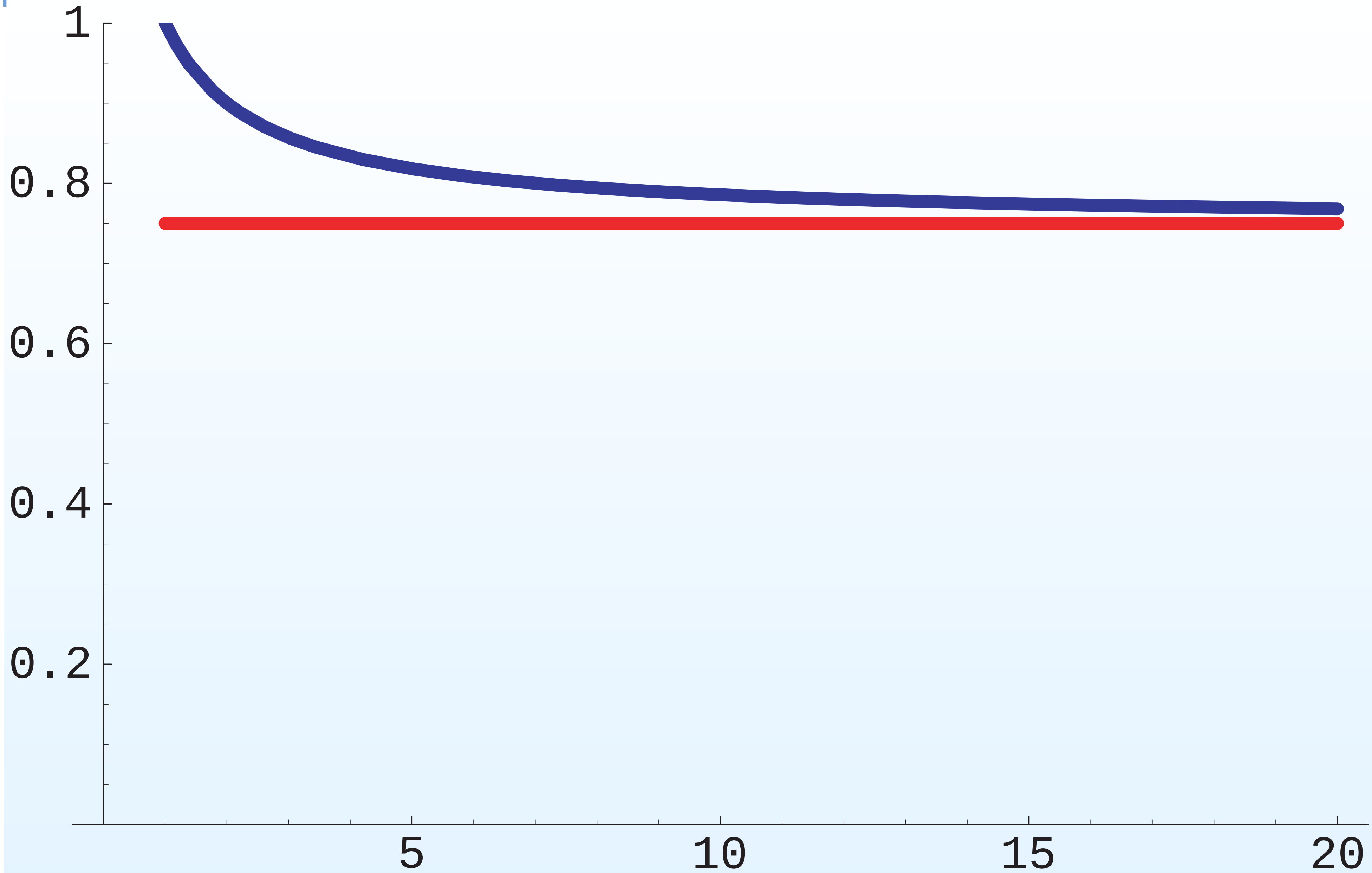

Figure 3. Relative efficiency, scenario 1(a): K degenerate at k_0 as a function of k_0

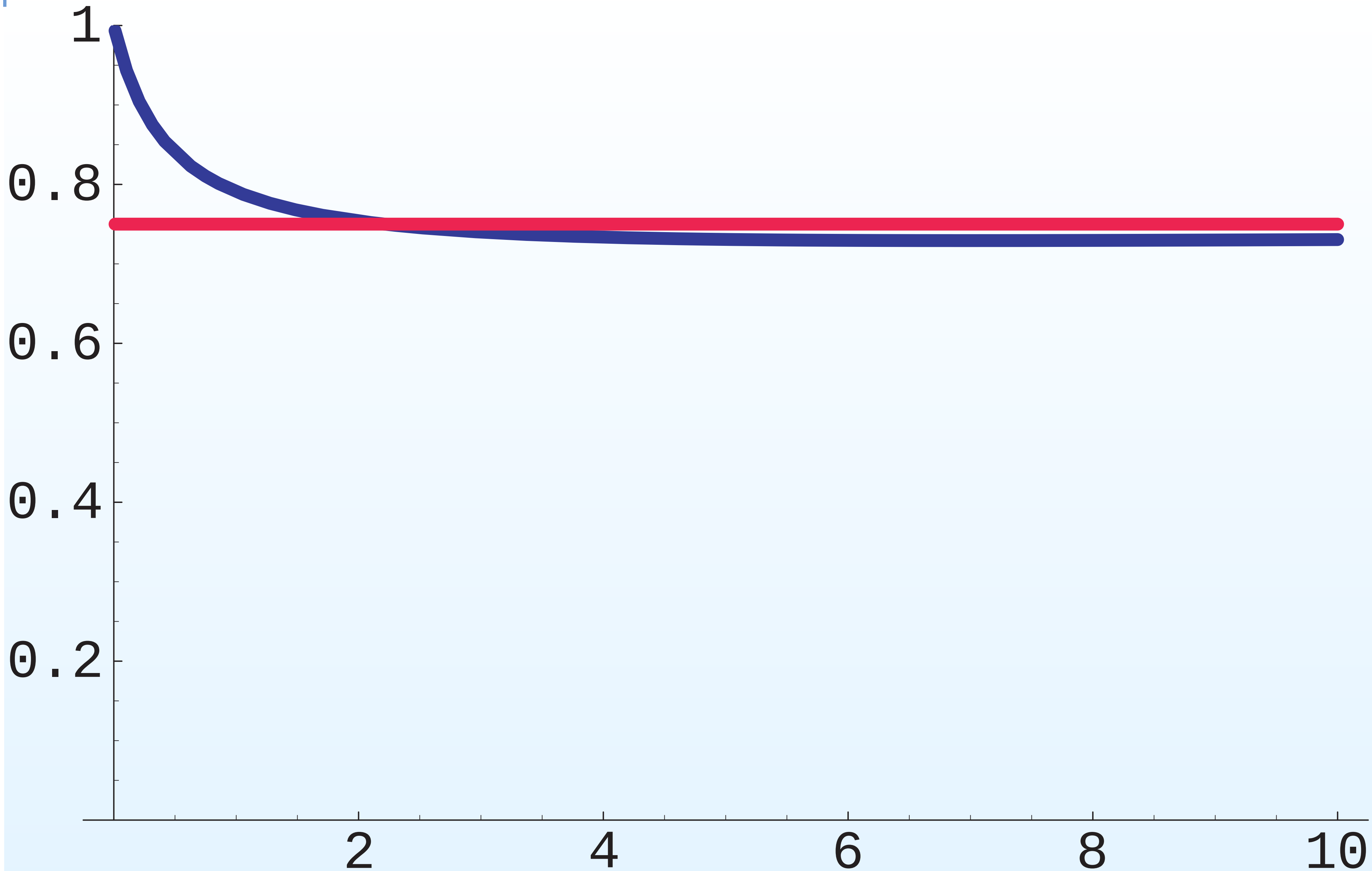

Figure 4. Relative efficiency, scenario 1(b): K shifted Poisson as a function of γ

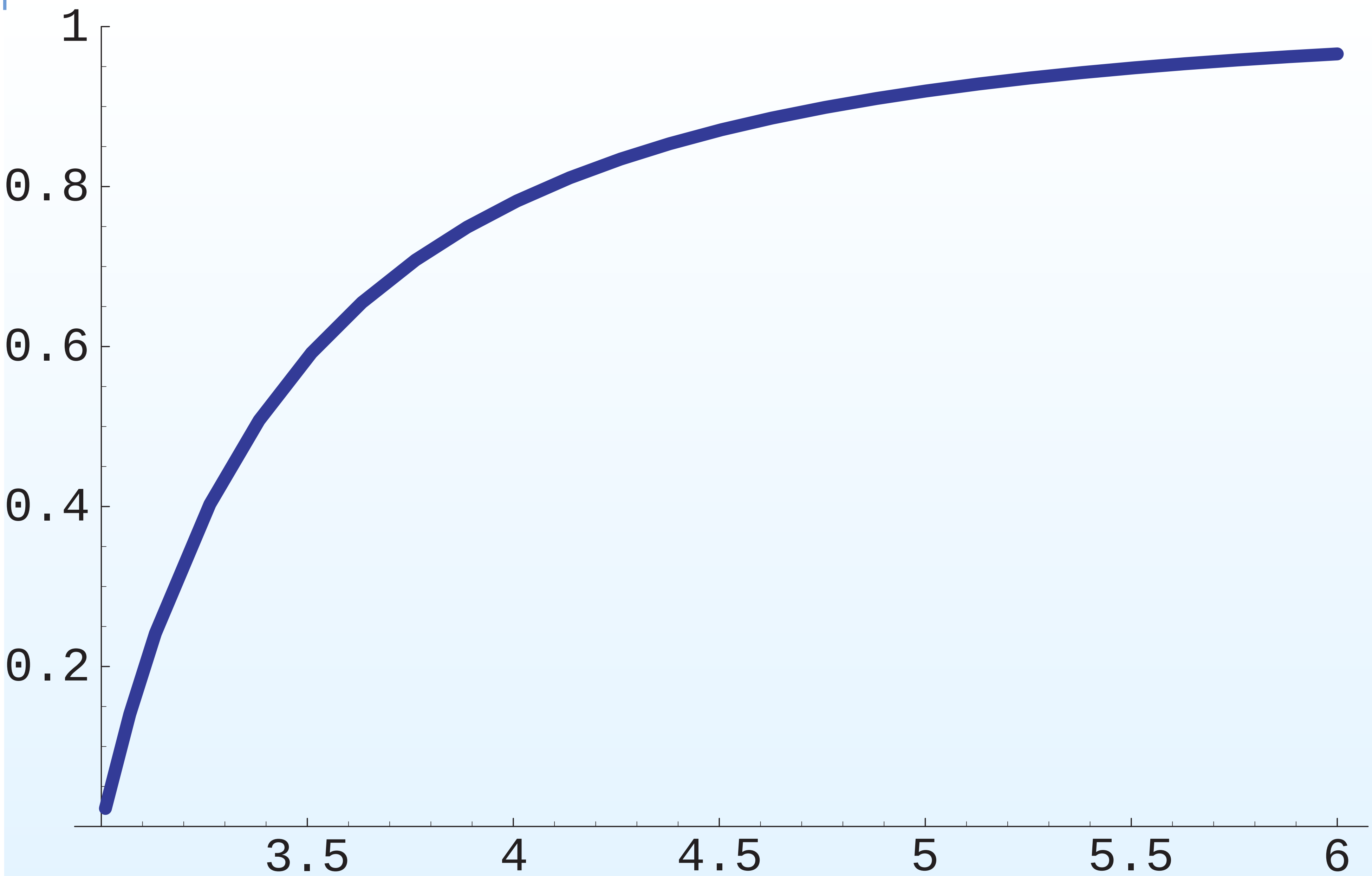

Figure 5. Relative efficiency, scenario 1(c): K discrete zeta

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$$
\begin{aligned}
&ARE(pseudo, mle)(NegBin) \\
&= \frac{\left(1 + a\frac{E\left(\frac{K}{K+2}\right)}{E\left(\frac{K}{K+1}\right)}\right)}{\left(1 + a\frac{E\left(\frac{K(3K+1)}{12}\right)}{E\left(\frac{K(2K+1)}{6}\right)}\right)} \cdot ARE(pseudo, mle)(Poisson)\,.
\end{aligned}
$$

where $a \equiv q/p = \lambda M/\gamma$.

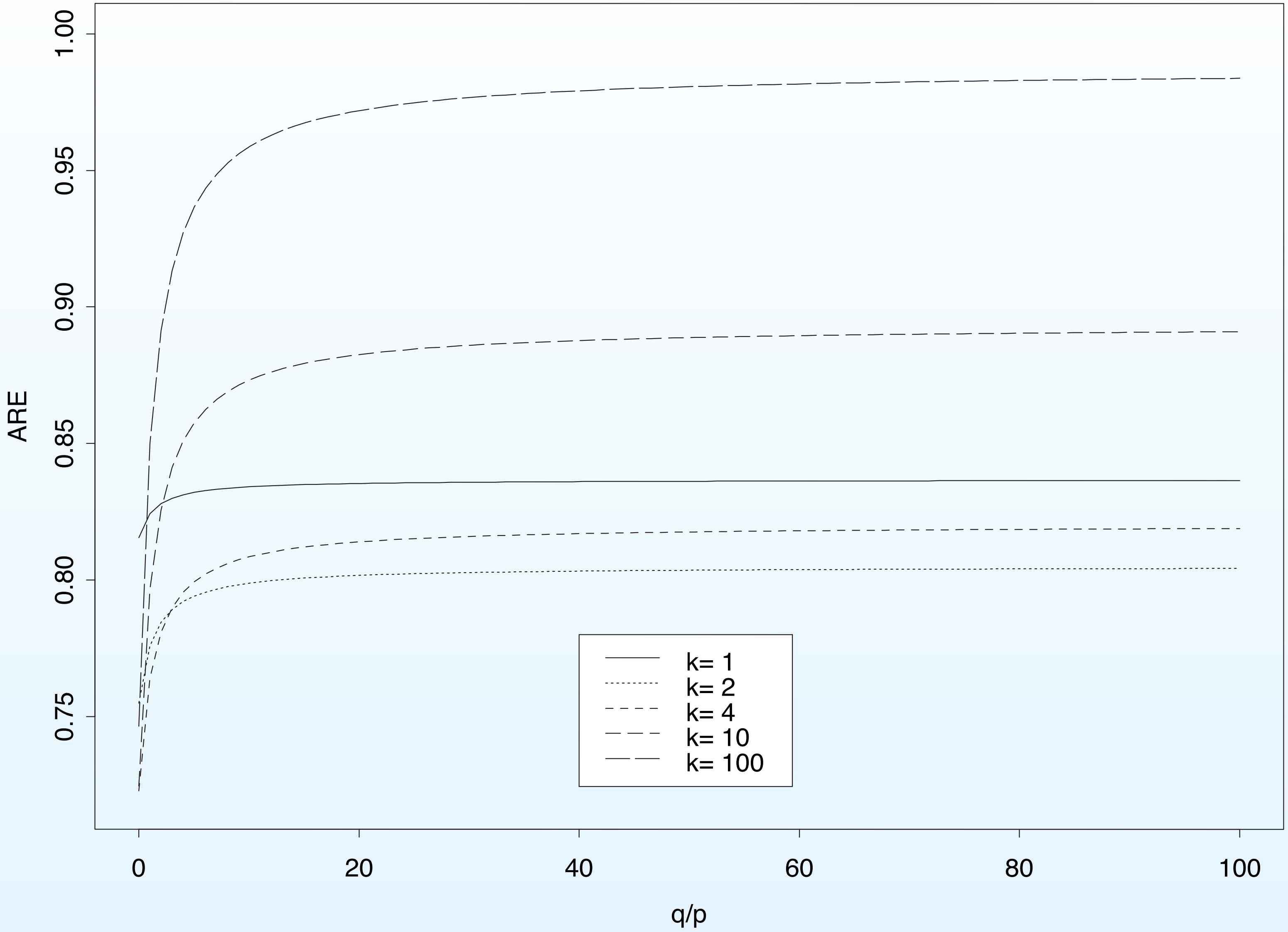

Figure 6. Relative efficiency, scenario 2, as a function of q/p

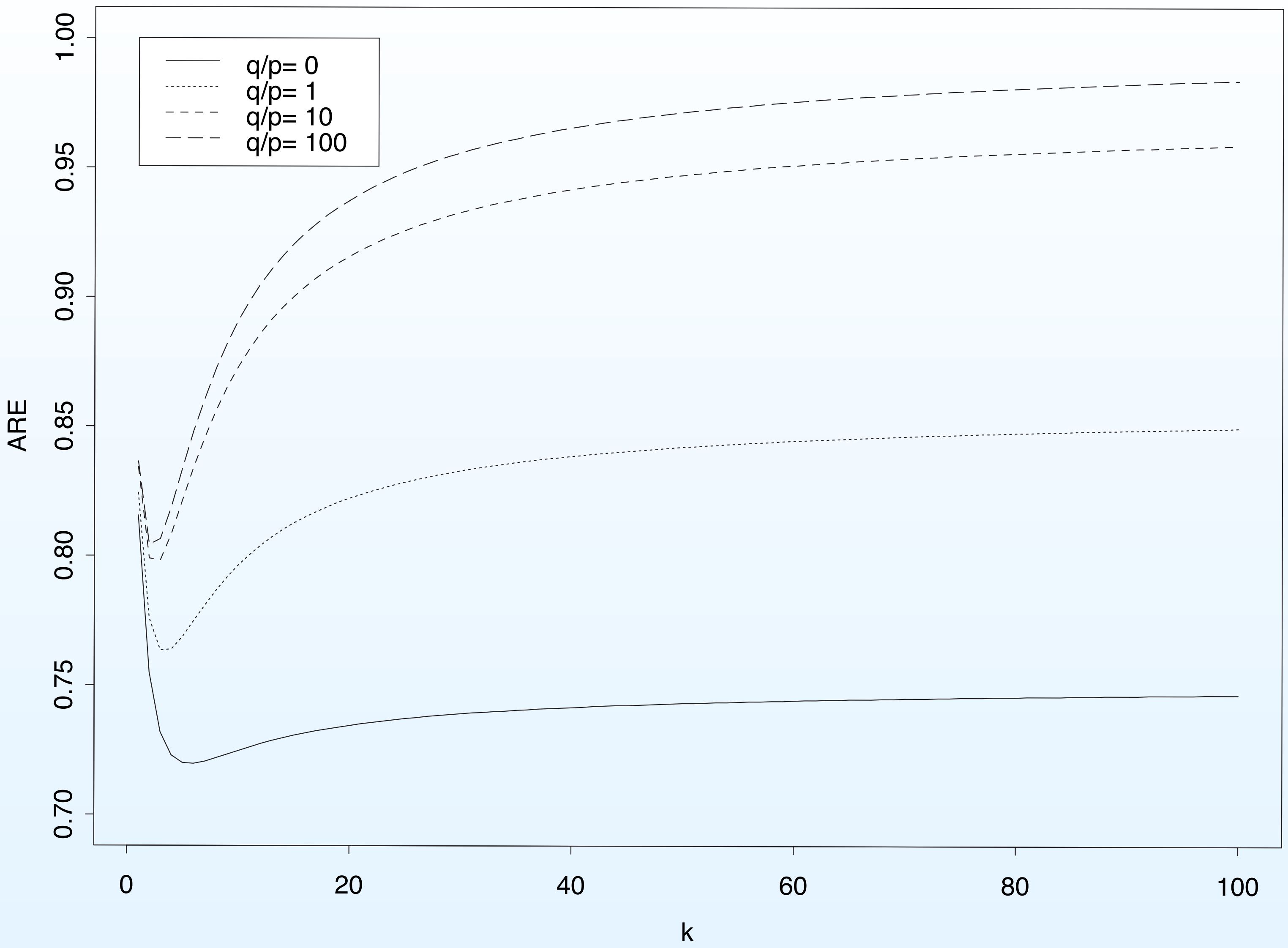

Figure 7. Relative efficiency, scenario 2, as a function of κ

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- Conditions needed to be able estimate both Λ and θ!

- Some measures:

$$\nu_1(B \times C)$$

$$= \int_C \sum_{k=1}^{\infty} P(K = k|Z = z) \sum_{j=1}^{k} P(T_{k,j} \in B|K = k, Z = z)dH(z),$$

$$\mu_1(B) = \nu_1(B \times \mathbb{R}^d)$$

$$\nu_2(B_1 \times B_2 \times C)$$

$$= \int_C \sum_{k=1}^{\infty} P(K = k|Z = z)$$

$$\cdot \sum_{j=1}^{k} P(T_{k,j-1} \in B_1, T_{k,j} \in B_2|K = k, Z = z)dH(z),$$

$$\mu_2(B_1 \times B_2) = \nu_2(B_1 \times B_2 \times \mathbb{R}^d)$$

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- Better algorithms for computation of the MLE $\widehat{\theta}$?
  - Pseudo MLE of Λ, no covariates: Sun and Kalbfleisch (1995)
  - MLE of Λ, no covariates: Zhang and Wellner (2000)

6. Further work and Open Problems

- Weaker hypotheses for asymptotic normality?
- Better understanding of identifiability issues?
- Further efficiency comparisons when $(K, \underline{T}_K)$ is dependent on Z?
- Large and small sample behavior of $\widehat{\Lambda}^{ps}$ and $\widehat{\Lambda}$?
- Better algorithms for computation of the MLE $\widehat{\theta}$?
  - Pseudo MLE of Λ, no covariates: Sun and Kalbfleisch (1995)
  - MLE of Λ, no covariates: Zhang and Wellner (2000)
  - pseudo MLE $\widehat{\theta}^{ps}$; Zhang (2000)

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