hw_lect26_1

In a simple regression problem, we have $n = 16$, $\bar{x} = 10$, $S_x = \frac{1}{\sqrt{8}}$, and $S_e = 4$. At $x = 11$, what is the value of T_0 such that prob(<u>prediction</u> error $> T_0$) $= 0.01$?
(Hint: how do we standardize <u>prediction</u> error?)

$$\left.\begin{array}{l} \text{prob}(\text{ pred. err.} > T_0) = .01 \\ \text{prob}\left(\frac{\text{pred err}}{S_{\text{pred err}}} > \frac{T_0}{\sqrt{S_{\hat{y}}^2 + S_e^2}} \right) = .01 \end{array}\right\} \text{standardize}$$

$$S_{\hat{y}} = S_e \sqrt{\frac{1}{n} + \frac{(x-\bar{x})^2}{S_{xx}}} = S_e \sqrt{\frac{1}{n} + \frac{(x-\bar{x})^2}{(n-1) S_x^2}} = 4 \sqrt{\frac{1}{16} + \frac{(11-10)^2}{15(\frac{1}{8})}}$$

$$\simeq 4 \sqrt{\frac{1}{16} + \underbrace{\frac{8}{15}}_{\sim \frac{8}{16}}} \simeq \cancel{4} \sqrt{\frac{9}{\cancel{16}}} = 3$$

$$\text{prob}\left(t > \frac{T_0}{\underbrace{\sqrt{9+16}}} \right) = .01 \Rightarrow \underline{T_0 = 2.6\,(5)}$$

$2.6 \leftarrow df = n - 2 = 14$, Table $\underline{\text{VI}}$

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Here is the regression version of a problem we have seen many times before.

a) Show that, at a given x, the prob that a random y would fall into the obs CI for $y(x)$ is

$$pr\left(t_{obs} - t^*\sqrt{\frac{1}{n}+\frac{(x-\bar{x})^2}{S_{xx}}} < t < t_{obs} + t^*\sqrt{\frac{1}{n}+\frac{(x-\bar{x})^2}{S_{xx}}}\right)$$

where $t_{obs} = \dfrac{\hat{y}_{obs}(x) - y(x)}{S_e}$

(Hint: How do you standardize observation error?)

Obs C.I for $y(x)$: $\hat{y}_{obs}(x) \pm t^* S_{est.err}$

$$pr\left(\hat{y}_{obs}(x) - t^* S_{est.err} < y < \hat{y}_{obs}(x) + t^* S_{est.err}\right)$$ standardize

$$= pr\left(\underbrace{\frac{\hat{y}_{obs}(x) - y(x) - t^* S_{est.err}}{S_e}}_{t_{obs} - t^* \frac{\cancel{S_e}\sqrt{\frac{1}{n}+\frac{(x-\bar{x})^2}{S_{xx}}}}{\cancel{S_e}}} < \frac{\boxed{y - y(x)}}{S_e} < \frac{\hat{y}_{obs}(x) - y(x) + t^* S_{est.err}}{S_e}\right)$$

$$= pr\left(t_{obs} - t^*\sqrt{\frac{1}{n}+\frac{(x-\bar{x})^2}{S_{xx}}} < t < \cdots + \cdots\right)$$

b) Show that, at a given x, the prob that a random $\hat{y}(x)$ would fall into the obs CI for $y(x)$ is $pr(t_{obs} - t^* < t < t_{obs} + t^*)$

where $t_{obs} = \dfrac{\hat{y}_{obs}(x) - y(x)}{S_{est.err.}}$.

(Hint: how do you standardize estimation error?)

$$pr\left(\hat{y}_{obs}(x) - t^* S_{est.err} < \hat{y}(x) < \hat{y}_{obs} + t^* S_{est.err}\right)$$

$$= pr\left(\frac{\hat{y}_{obs}(x) - y(x) - t^* S_{est.err}}{S_{est.err}} < \frac{\boxed{\hat{y}(x) - y(x)}}{S_{est.err}} < \frac{+}{S_{est.err}}\right)$$

est.err

$$= pr\left(t_{obs} - t^* < t < t_{obs} + t^*\right)$$