Lecture 21 (Ch. 8)

Last time: Belief: a statement NOT based on data ("a priori")

Example Data says: $n = 64$, $\bar{x}_{obs} = 34.4$, $S = 1.1$.
Does the data provide evidence to support $\mu_x > 34$?
So, The a priori belief is $\mu_x < 34$.

Let's assume The belief is true

Formally, Let $H_o: \mu_x < 34$. Assume H_o = True.

Evidence, from data, against (Contrary to) This assumption

p-value = $pr(\bar{x} > \bar{x}_{obs} \mid \mu_x = 34)$ ← sufficient to test equality. No need to do $\mu_x < 34$ (See The FYI fig, below).

against (Contrary to) H_o

$$= pr\left(\frac{\bar{x} - \mu_x}{s/\sqrt{n}} > \underbrace{\frac{\bar{x}_{obs} - \mu_x}{s/\sqrt{n}}}_{= t_{obs}}\right) = pr(t > t_{obs}) \simeq .0025$$

$$= t_{obs} = \frac{34.4 - 34}{1.1/\sqrt{64}} = 2.91$$

This p-value is small!
It's really easy to misinterpret This small prob.
But The best way to Think of it is That it suggests our assumption was bad, and so, we should reject it.

In short: small p-value $\Rightarrow$ large evidence against H_o.

$\Downarrow$

Reject H_o

What determines "small"? α does = 1 − conf. level.

And you choose α.

Dropping subscript x.

<u>Example</u>

$\mu_x, \pi_x, \dots$

Here is the generalization of the above example:

1) Decide the pop. parameter being tested

2) Set-up H_0 (Null hyp.) and H_1 (Alternative hyp.)

H_0:	$\mu > \mu_0$	$\mu < \mu_0$	$\mu = \mu_0$
H_1:	$\mu < \mu_0$	$\mu > \mu_0$	$\mu \neq \mu_0$
H_0:	$\mu = \mu_0$	$\mu = \mu_0$	$\mu = \mu_0$
H_1:	$\mu < \mu_0$	$\mu > \mu_0$	$\mu \neq \mu_0$

$\mu < 34$, $\mu > 34$ — μ_0

sufficient to test equality in H_0.

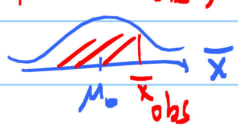
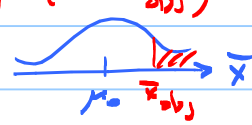
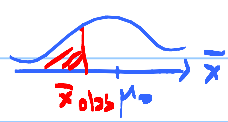
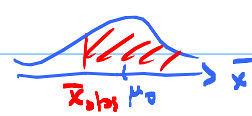
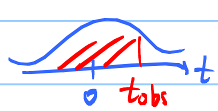
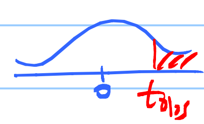
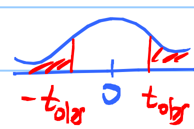
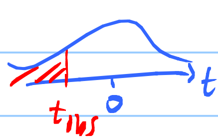
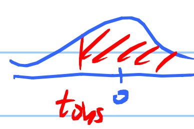
4) Choose appropriate statistic. $z, t, \dots$

5) Assume $H_0 = $ TRUE. set $\mu = \mu_0$

6) Compute test statistic for observed data/sample. $t_{obs} = \dfrac{\bar{x}_{obs} - \mu_0}{s/\sqrt{n}}$

7) Find p-value, i.e. prob of getting a random test statistic more extreme (contrary to H_0) than the observed one,

mnemonic

p-value =	$pr(\bar{X} < \bar{X}_{obs})$	$pr(\bar{X} > \bar{X}_{obs})$	... (see below)
			...
			...
	$pr(t < t_{obs})$	$pr(t > t_{obs})$	...
	left area	right area	sum of tail areas
			
			

prob$(t > 2.91)$

8) If p-value $< \alpha$, reject H_0 in favor of H_1; else cannot reject H_0 in favor of H_1

Note: If p-value $< \alpha$, Then reject H_0 in favor of H_1.

In English: There is evidence from data in favor of H_1 ...
(against H_0).

Else, cannot reject H_0 in favor of H_1.
(Not The same Thing as "Accept H_0")

In English: There is NO evidence from data in favor of H_1 ...
(against H_0).
(Not The same Thing as "There is evidence for H_0")

We cannot Accept H_0, because we assumed it was true! When p-value is large, There is simply no evidence from data for anyThing - not for H_1, not for H_0!

As I said, There are lots of similarities between The C.I and The p-value approach, but The differences are very important. For example There is no t^* (or z^*) here. from Tables.
But There is t_{obs} (or z_{obs}) from data

$$\boxed{z^* \neq z_{obs},\ t^* \neq t_{obs}}$$

Now, let's repeat our example, but now following The general procedure.

Data says: $n = 64$, $\bar{x}_{obs} = 34.4$, $S_{obs} = 1.1$.

Does the data provide evidence to support $\mu > 34$?

1) The param. of interest : μ (dropping subscript x).

2,3) $H_0 : \mu < 34$ (or $\mu = 34$) $\mu_0 = 34$

$H_1 : \mu > 34$

[Setting-up H_0/H_1 is The hardest part of These problems. See below for guidance.

4) Appropriate test statistic : z, t

5) Assume $H_0 = T$. (I.e. set $\mu = 34$)

6) Compute statistic assuming $H_0 = $ True (ie. $\mu = \mu_0$) $t_{obs} = \frac{34.4 - 34}{1.1/\sqrt{64}} = 2.91$

Contrary to H_0 (ie. in The direction of H_1)

7) p-value = $\text{prob}(\bar{x} > \bar{x}_{obs}) = \text{prob}(t > 2.91) \simeq .0025$

8) Conclusion : At $\alpha = .05$, p-value $< \alpha$.

Therefore,

Data provide sufficient evidence to reject H_0 ($\mu < 34$) in favor of H_1 ($\mu > 34$).

"In English": Data provide suff. evidence in favor of $\mu > 34$.

At $\alpha = 0.001$, p-value $> \alpha$ ($\mu < 34$) ($\mu > 34$)

Therefore, we cannot reject H_0 in favor of H_1.

"In English": there is no support for $\mu > 34$

[Note: This conclusion is ~~NOT~~ NOT The same thing as "There is support for $\mu < 34$." All we can say is that we cannot reject $\mu < 34$.

Important.

The hardest part of hypothesis testing is setting-up H0/H1 correctly. Here is some guidance:

Four ways I go about for deciding what H0/H1 should be:

1) Don't assume what DATA are supposed to test.
The question asks "Does data provide evidence for claim X?" Meanwhile, the hypothesis testing procedure begins by assuming whatever you put under H0 is True. So, it makes no logical sense to assume X is true, even before data. So, put the complement/opposite of X under H0.

2) Ask yourself what statement you should be left with if there is NO DATA at all. The answer to that question tells you what H0 should be. Then, the complement of that goes under H1.
The data provide evidence for H1 (against H0), because of the way the whole procedure is set-up.
Then, if the evidence is weak (eg when there is no data at all), then the procedure leaves you with H0, as it should. In our example, if there is no data at all, then we should not reject the belief that mu < 34, and so, H0 should be mu < 34.

3) Some problems ask you to test some prior belief (i.e., some claim based on something other than data). Then that belief should go under H0.

4) Another way of deciding on H0/H1 will be discussed later, when we learn the meaning of alpha, and Type I and Type II errors.

Further comments:

H0 and H1 are statements about some pop. param, and so, they have no probability.

The p-value is the quantity that represents the evidence from data against H0, in favor of H1. But note that smaller p-value means more evidence against H0 (in favor of H1). This is so because we are giving the benefit of our doubt to H0; so, if H0 is true, and the prob of getting data more extreme than the observed data is large, then there is no evidence for rejecting H0.

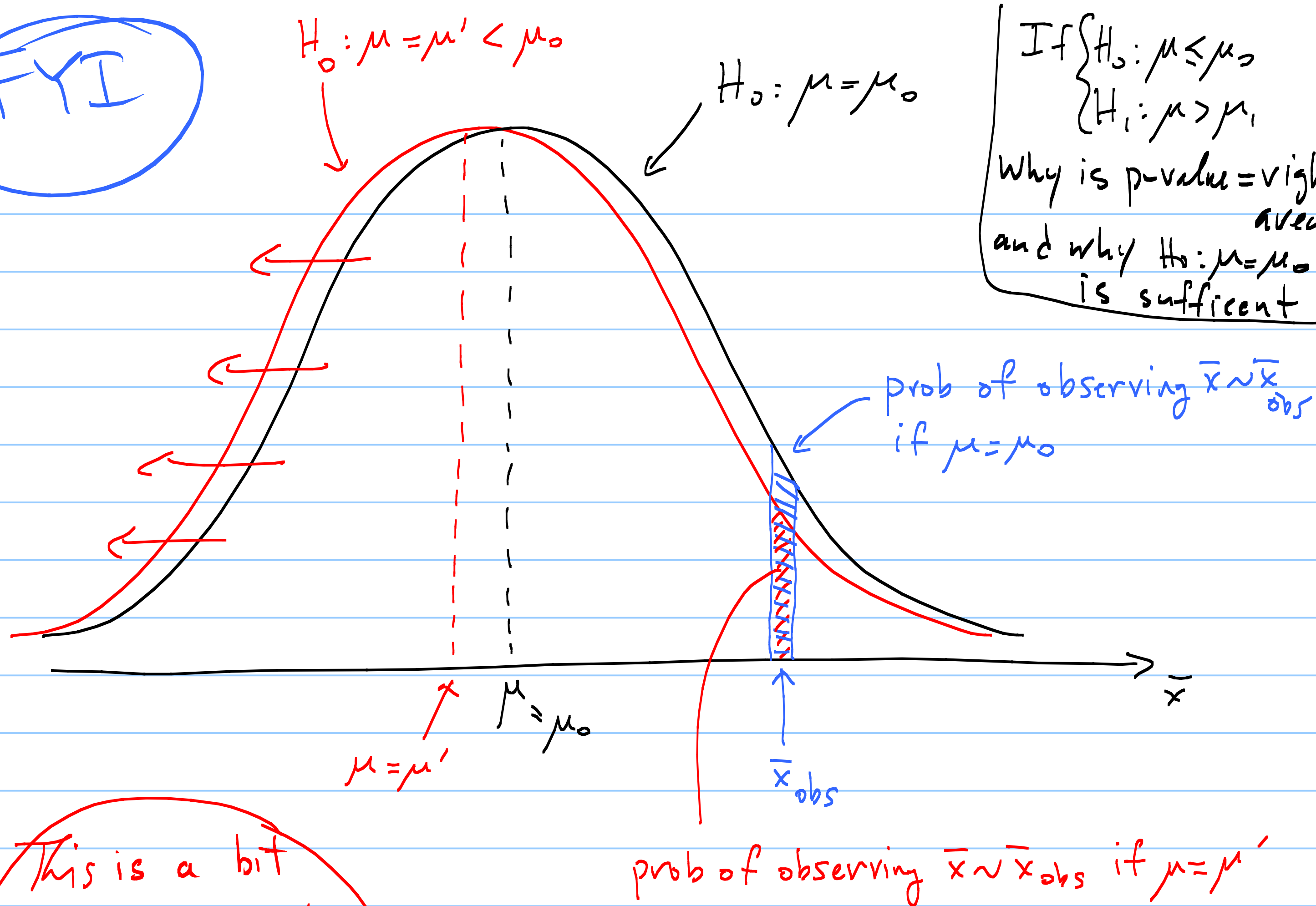
If we cannot reject H0 in favor of H1, then we don't know anything. Not rejecting H0 is not the same thing as accepting H0. Making that mistake of interpreting the lack of evidence for H1 as support for H0 is the source of much confusion in science.

In general, we cannot accept a belief about an unknown pop. parameter (eg. mu < 34). All we can to is either reject it,or not, based on evidence from data. And that evidence comes through the p-value; the mathematical way to see this is to note that the p-value is a conditional prob. i.e. it already assumes H0 is true.

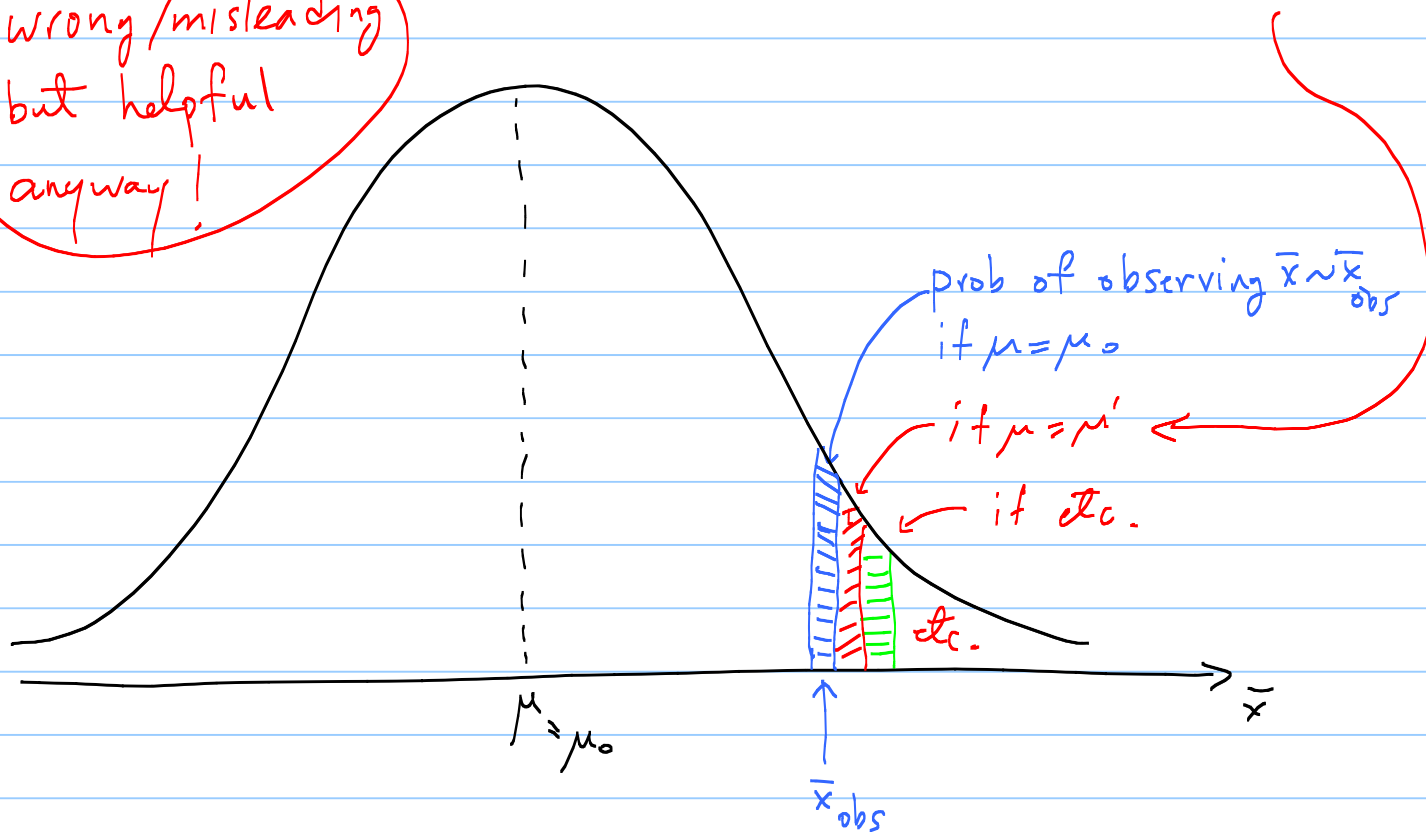
FYI

If $\begin{cases} H_0: \mu \le \mu_0 \\ H_1: \mu > \mu_1 \end{cases}$

Why is p-value = right area and why $H_0: \mu = \mu_0$ is sufficient?



This is a bit wrong/misleading but helpful anyway!



So, if $H_0: \mu \le \mu_0$, then the p-value $= \text{prob}(\bar{x} > \bar{x}_{obs} \mid \mu = 34)$

I.e. $H_1: \mu > \mu_0$ mnemonic $=$ right area, with $\mu = 34$

hw_lect21_1

Toothpaste tubes may be wasteful because there is always some amount of toothpaste that one cannot extract. To find out how much toothpaste is wasted, 5 discarded tubes are selected, cut open, and the amount of remaining toothpaste is recorded. The data are : 0.52, 0.65, 0.46, 0.50, 0.37 (in ounces). Is there evidence that the true average of the wasted toothpaste is less than 0.55 ounces? Apply the hypothesis testing procedure as follows:

a) what is the pop. parameter being tested? Write the symbol for it, AND explain it in words.

b) Restate the question as "Does data provide evidence --- "

c) which of the following pairs of hypotheses is appropriate?

Check the solns later to see the explanation/thinking.

$H_0: \mu_x < 0.55$	$H_0: \mu_x > 0.55$	$H_0: \mu_x = 0.55$
$H_1: \mu_x > 0.55$	$H_1: \mu_x < 0.55$	$H_1: \mu_x \neq 0.55$

d) In our procedure we must assume H0 = True. Assuming H0 =T, what is the "worse" value that mu_x can take? Hint: values in the direction of H1 are "worse " for H0.

e) Assuming the "worse-case" scenario of part d, compute the p-value. Hint; remember that the p-value measures evidence against (contrary to) H0, or in favor of H1; Use Table VI.

f) Is The p-value you have computed small (less than 0.05) or large (larger than 0.05)?

g) Based on your answer to part f should you reject H0 in favor of H1?

h) What is the conclusion (In English)?

hw_lect21-2

I) Suppose you are asked if There is evidence That $\mu_x > \bar{x}_{obs}$?

a) Set-up The appropriate H_0/H_1

b) Compute The p-value.

II) Suppose you are asked if There is evidence That $\mu_x > \bar{x}_{obs} - 1.645\frac{s}{\sqrt{n}}$?

FYI: The right-hand side is The 95% LCB (which we are skipping).

c) Set-up The appropriate H_0/H_1

d) Compute The p-value. Use $pr(t > 1.645) = 0.05$, i.e. $df = n-1 = \infty$

Look at The soln later to see the moral of This hw.