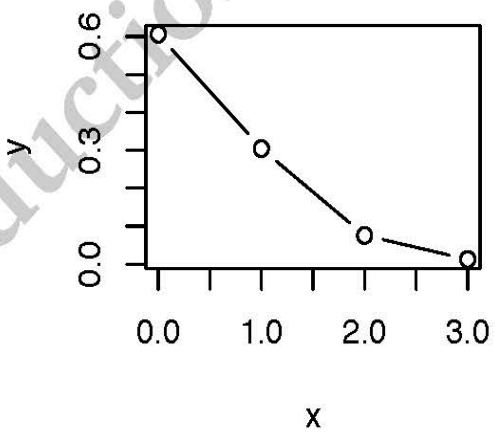
2 Distributions

2.1 Binomial and Poisson Distribution

```
# The format is dbinom(x, n, pi), where x = number of heads out of n tosses of a
# coin, and pi = prob of head. For example,
dbinom(0, 100, 0.005)  # returns the value of the distribution (pmf) itself.

[1] 0.6058

dbinom(0:3, 100, 0.005)  # running dbinom() for multiple values of x in one sweep.

[1] 0.60577 0.30441 0.07572 0.01243

sum(dbinom(0:3, 100, 0.005))  # summing up the above probabilities.

[1] 0.9983
```

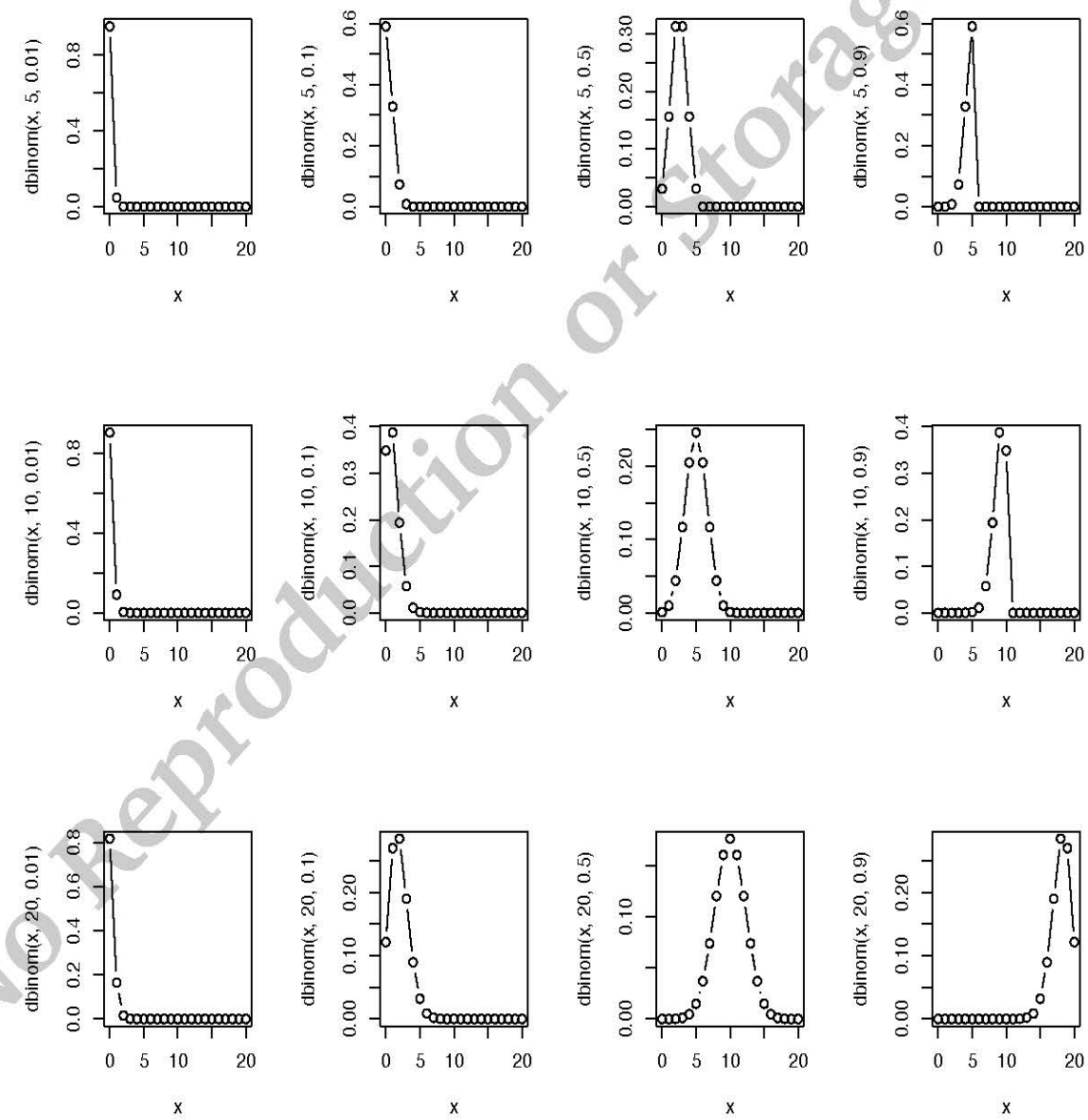
2.1.1 Plotting

```
x <- 0:3
y <- dbinom(0:3, 100, 0.005)
plot(x, y, type = "b")  # "b" (for "both") connects the points with lines.
                        # See ?plot for more options for line types
```




```
# Plotting the mass function for different values of n and pi.
# Note the n and pi values that produce normal-looking distributions,
# and those that produce Poisson-looking distributions.
par(mfrow = c(3, 4))  # A 3 by 4 matrix of figures.
x <- 0:20
plot(x, dbinom(x, 5, 0.01), type = "b")  # n=5, pi=0.01
```

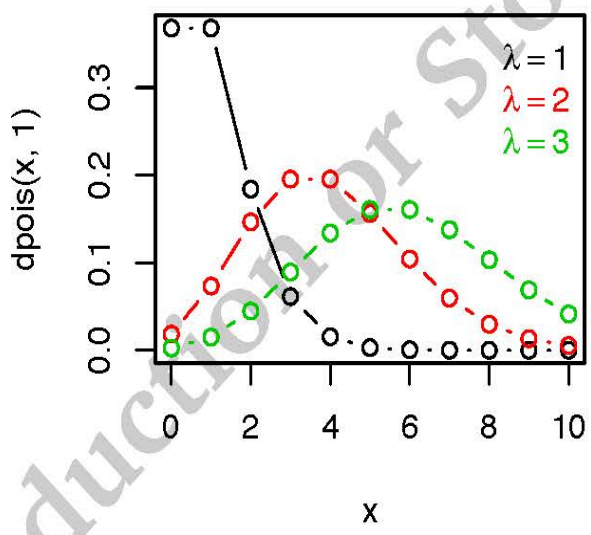
```r
plot(x, dbinom(x, 5, 0.1), type = "b")  # n=5, pi=0.1. Use UP-ARROW to get
                                        # most recent run commend
plot(x, dbinom(x, 5, 0.5), type = "b")  # n=5, pi=0.5
plot(x, dbinom(x, 5, 0.9), type = "b")  # n=5, pi=0.9

plot(x, dbinom(x, 10, 0.01), type = "b")  # n=10, pi=0.01  USE UP-ARROW
plot(x, dbinom(x, 10, 0.1), type = "b")  # n=10, pi=0.1
plot(x, dbinom(x, 10, 0.5), type = "b")  # n=10, pi=0.5
plot(x, dbinom(x, 10, 0.9), type = "b")  # n=10, pi=0.9

plot(x, dbinom(x, 20, 0.01), type = "b")  # n=20, pi=0.01
plot(x, dbinom(x, 20, 0.1), type = "b")  # n=20, pi=0.1
plot(x, dbinom(x, 20, 0.5), type = "b")  # n=20, pi=0.5
plot(x, dbinom(x, 20, 0.9), type = "b")  # n=20, pi=0.9
```



Note that we can approximate the binomial distribution with the Poisson distriution (when π is small and n is large) or the normal distribution (when π is mid-range and n is large).

The shape of the Poisson distribution depends on the parameter λ.

```
par(mfrow = c(1, 1))  # One figure on whole page.
x <- 0:10
plot (x, dpois(x, 1), type = "b")  # "b" stands for "both"
                                                    # points and lines.
lines(x, dpois(x, 4), type = "b", col=2, main = 'lambda = 4')  # USE UP-ARROW
lines(x, dpois(x, 6), type = "b", col=3, main = 'lambda = 6')  # lines() adds lines
                                                               # on existing plot.

legend('topright', c(expression(lambda == 1), expression(lambda == 2),
                     expression(lambda == 3)), text.col = c(1, 2, 3), bty = 'n')
# Similarly, dnorm(x, mu, sigma) produces the density function Normal(mu,sigma) .
# See ?dnorm() for required format.
```



2.2 Simulation from Mass and Density Functions

In this section, we will present how to generate data that follow the binomial distribution; i.e., simulate the tossing of a coin, without actually tossing coins. For example, shown below is a way to generate 200 numbers from a binomial:

```
rbinom(200, 10, 0.5)  # format = rbinom(number of tosses, n, pi).
                      # See ?rbinom for more.
```

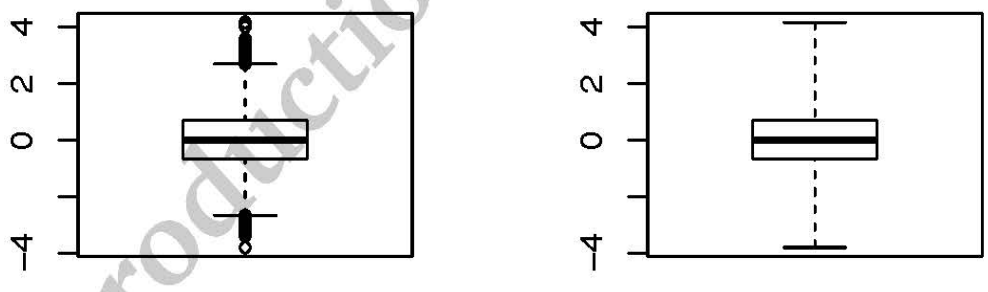
Effectively, you just tossed 10 fair coins, 200 times, each time noting the number of heads out of 10. This way, you can do a lot of experiments on the computer, without actually doing the experiment! If the coin is not fair, then just change the parameter π.

```r
# Putting an "r" before the name is R's way of generating the numbers.
# For example, consider the Possion distribution, which is often used to
# model the number of some event, per unit time, or space, etc. Then,
# rpois(100,4) generates 100 numbers from the poisson distribution. So, each of these
# 100 numbers could be the "number of people arriving at a teller, per hour",
# if the average number of people arriving per hour is 4.
rpois(100, 4) # generates 100 numbers from the Poisson distribution.

# Similarly, the following draws a single sample of size 10000 from a normal
# distribution with mu=0 and sigma=1.
x <- rnorm(10000, 0, 1)
hist(x, breaks = 200)  # Checks the histogram and it looks pretty normal
```

2.3 Boxplots

A boxplot of data is a way of summarizing the data into five numbers that capture the shape of the histogram. The five numbers are the minimum, 25th percentile, median, 75th percentile, and maximum.

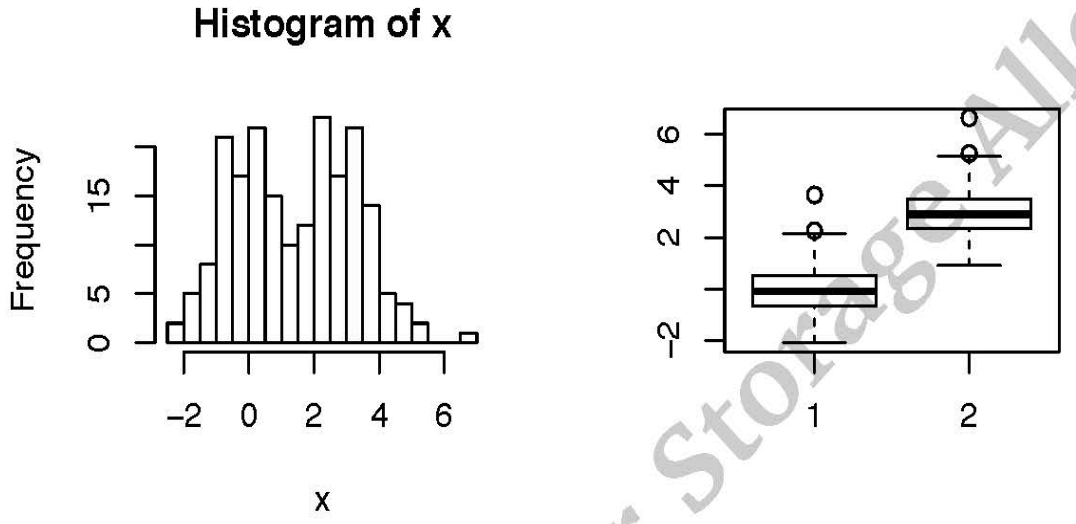
```r
x <- rnorm(10000, 0, 1)
par(mfrow = c(1, 2))
boxplot(x, cex = 0.7)  # Circles at the end of boxplot are outliers according to some
           # criterion.
boxplot(x, range = 0)  # Suppresses outliers.
```



Example 1

Now, recall the bimodal histogram we saw before in hist data. It was bimodal because two separate data files were joined, each one with 100 cases in it. We can separate the two and boxplot them, separately:

```r
dat <- read.table('hist_dat.txt', header = F)
x <- dat[, 1]  # All of x.
x_1 <- x[1:100]  # Put the 1st 100 cases of x in x_1,
x_2 <- x[101:200]  # Put the remainder in x_2.
par(mfrow = c(1, 2))
hist(x, breaks = 20)  # Draw a histogram
boxplot(x_1, x_2)  # Draw boxplots
```

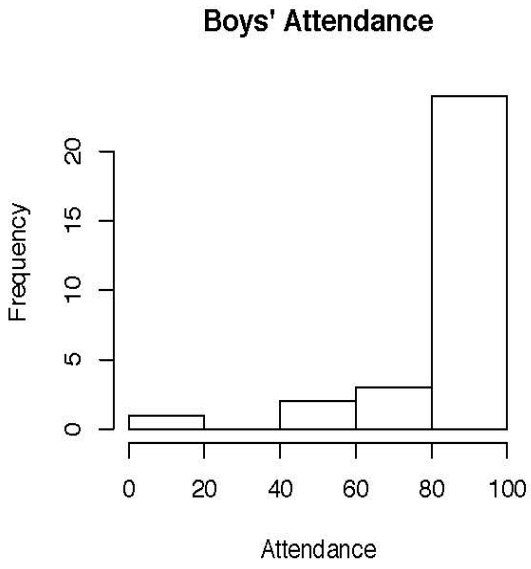


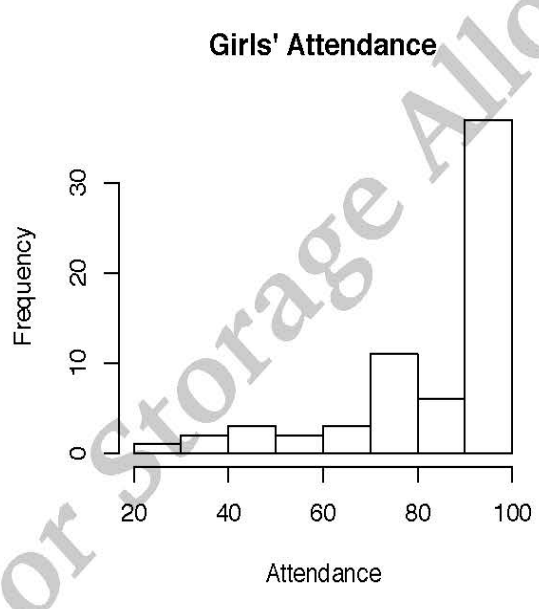
Example 2: Attendance Data

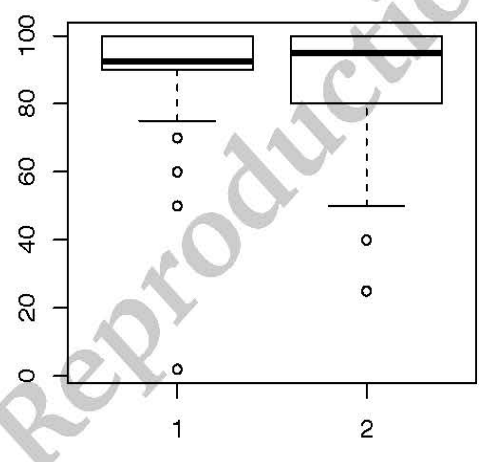
The variable of interest is the "percentage of time student attends lectures", and the two groups are boys and girls.

```r
dat <- read.table('attend_dat.txt', header = T)
x <- dat$attendance
y <- dat$Gender

par(mfrow = c(2, 2))
# A way of selecting cases in x that correspond to some value of y.
hist(x[y == 0], main = "Boys' Attendance", xlab = 'Attendance')
hist(x[y == 1], main = "Girls' Attendance", xlab = 'Attendance')
boxplot(x[y == 0], x[y == 1])

# Look at the two sample means to see if there is a difference between
# boys and girls with respect to their attendance.
mean(x[y == 0])  # Sample mean attendance for girls.

[1] 87.57

mean(x[y == 1])  # Sample mean attendance for boys.

[1] 86.4
```

```
# The y=0 group (girls) has the higher sample mean than the y=1 group (boys);
# (87.6 vs. 86.4). But the medians are reversed (92.5 vs. 95):
median(x[y == 0])  # Sample median attendance for girls.

[1] 92.5

median(x[y == 1])  # Sample median attendanceand for boys.

[1] 95
```







There are several sources of complexity in comparing two groups:

1. Sample mean or median measure only "center" or "location" of data.
2. They measure 2 different notions of "center;" and there are many others.

3. Measures of location (e.g., mean, median) do not conclude all characteristics of the sample. The spread is equally important.

```
# One measure of spread is the sample standard deviation:
sd(x[y == 0]) # Sample standard deviation of attendance for girls

[1] 20.41

sd(x[y == 1]) # Sample standard deviation of attendance for boys.

[1] 18.02
```

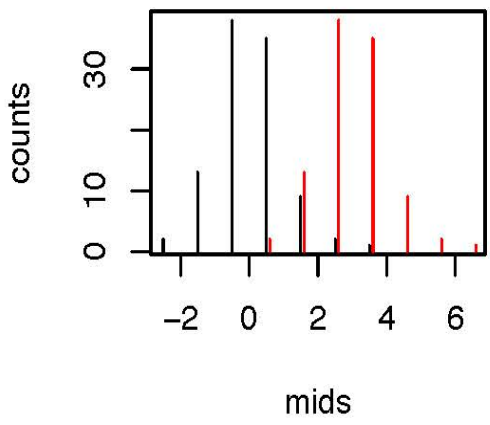
We can see that the spread is a bit wider for girls than for boys. In statistics, some interpretation is always important. For example, one might say that boys are more "consistent" across the sample.

Percentiles can also be used to assess spread. For example, the distance between the 25th percentile and the 75th percentile (the interquartile range) conveys a sense of the spread.

```
# To get percentiles, use quantile().
# The 25th percentile is simply the 0.25 quantile, etc.
# Quantiles of attendance for boys:
quantile(x[y == 1], prob = c(0, .25, .5, .75, 1))

  0%  25%  50%  75% 100%
  25   80   95  100  100

# Quantiles of attendance for girls.
quantile(x[y == 0], prob = c(0, .25, .5, .75, 1))

   0%   25%   50%   75%  100%
  2.0  90.0  92.5 100.0 100.0

# The interpretation of sample quartiles is as follows: Since the value
# corresponding to the 25th percentile is 90, it means that 25% of girls
# attended classes less than or equal to 90% of the time.
```

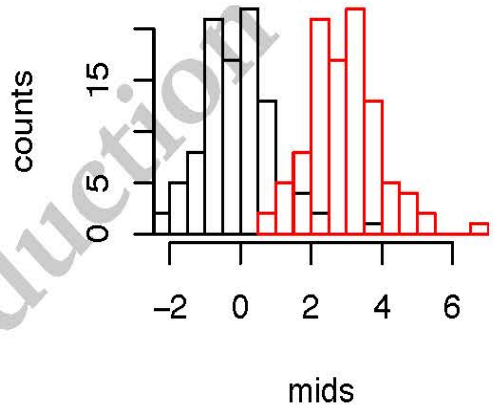
Overlaying two histograms:

```
dat <- read.table('hist_dat.txt', header = F)
x <- dat[, 1]  # Here is all of x.
x_1 <- x[1:100]  # Put the 1st 100 cases of x in x_1,
x_2 <- x[101:200]  # Put the remainder in x_2.
a <- hist(x_1, plot = F)
b <- hist(x_2, plot = F)
x.lim <- range(c(a$mids, b$mids))
plot(a$mids, a$counts, type = "h", xlim = x.lim, xlab = 'mids', ylab = 'counts')
lines(b$mids + 0.1, b$counts, type = "h", col = "red")  # The shift of 0.1 avoids
                                                        # overlapping histograms.
```



That was the hard way! But it shows the inner-workings of hist(). The easy way is:

```
hist(x_1, breaks = 20, xlim = range(x_1, x_2), xlab = 'mids', ylab = 'counts', main = '')
hist(x_2, breaks = 20, add = T, border = 2)
```



2.4 Binomial Distribution

Recall that the mean and variance of the binomial distribution are given by $n\pi$ and $n\pi(1-\pi)$. Note that they grow linearly with the sample size n. For example, if NG = number of girls in a random sample of size n, then as n increases the typical value of NG increases (obviously), and the variability (across samples) of NG also increases (not obvious). We will confirm this mathematical result with a simulated coin toss example.