Consider a Bernoulli dist with parameter $\pi$, (i.e. a population consisting of two types of objects denoted $x = 0,1$, with the proportion of 1s in the population given by $\pi$). Take samples of size $n=3$.

a) What's the probability that the maximum of the three numbers is 1?

b) What's the probability that the maximum of the three numbers is 0?

Hint: repeat the derivation of the binomial distribution, i.e. writing out all possibilities, etc. but with $X$ (i.e., the number of heads out of $n$) replaced with Max (i.e., the maximum of the three numbers).

\[
\begin{array}{cccc}
\text{possible samples} & X = \text{Max of the 3 numbers} & \text{prob} \\
\hline
0,0,0 & 0 & (1-\pi)^3 \\
0,0,1 & 1 & (1-\pi)^2 \pi \\
0,1,0 & 1 & (1-\pi)^2 \pi \\
0,1,1 & 1 & (1-\pi)^2 \pi \\
1,0,0 & 1 & (1-\pi)^2 \pi \\
1,0,1 & 1 & (1-\pi)^2 \pi \\
1,1,0 & 1 & (1-\pi)^2 \pi \\
1,1,1 & 1 & 7\pi^3 \\
\end{array}
\]

\[a) \quad P(X=1) = 2 \text{ ways} = \left\{ \begin{array}{l} 
3(1-\pi)^2 \pi + 3(1-\pi)^2 \pi^2 + \pi^3 = 3\pi(1-\pi) + \pi^3 \\
1 - (1-\pi)^3 
\end{array} \right. \]

\[b) \quad P(X=0) = (1-\pi)^3 \]

Note that you just derived the dist. of maximum of 3 numbers $(0/1)$:

\[
\begin{array}{c|c|c}
X & 0 & 1 \\
\hline
p(x) & (1-\pi)^2 & 1 - (1-\pi)^3 \\
\end{array}
\]
For a period of 10 hours, we observe the number of cars that went through a stop sign (without stopping), per hour. Here is my data: 2, 2, 3, 2, 4, 2, 0, 1, 3, 2. What's the prob that, for a random hour, all cars will stop at the stop sign? Note: What we are given here is data, and so, we can only approximate the distribution parameter(s).

Let $X =$ # of cars that go thru the stop sign, per hour.
Assume $X$ = poisson with param $\lambda$.
$\lambda$ is the parameter of the distribution, i.e. something we do not know. But its meaning is the average # of cars that go thru the stop sign, per hour. So, we can approximate it with the average of the 10 counts we have, i.e. $\lambda = 0.21$.

Then prob all will stop $= P(X=0) = \frac{e^{-\lambda} \lambda^0}{0!} = e^{-0.21} = 0.12$.

Make sure you realize how amazing this result is! You can actually find the prob of $X=0$, with almost nothing at all related to $X=0$!
In the prev. hw problem, you have to approximate the parameter of a distribution based on some observed data. In some problems, however, the value of the parameter can be obtained from knowledge of the problem itself. For example, suppose \( x \sim \text{Poisson}(\lambda) \).

a) If \( p_x(x=0) \) is known, what's the value of \( \lambda \)?

\[
p(x) = \frac{e^{-\lambda} \lambda^x}{x!} \implies p(x=0) = e^{-\lambda} \implies \lambda = -\log p(x=0)
\]

b) If the ratio \( \frac{p(x=2)}{p(x=1)} \) is known, what's the value of \( \lambda \)?

\[
p(x=1) = e^{-\lambda} \frac{\lambda}{1} \\
p(x=2) = e^{-\lambda} \frac{\lambda^2}{2}
\]

\[
\implies \frac{p(x=2)}{p(x=1)} = \frac{2}{\lambda} \implies \lambda = 2 \frac{p(x=2)}{p(x=1)}
\]

c) Suppose \( p(x=1) \) is known. Then \( p(x=1) = A e^{-\lambda} \).

Plot (by hand) \( A e^{-\lambda} \) as a function \( \lambda \), where \( 0 < \lambda < \infty \).

Clearly mark the maximum value of \( A e^{-\lambda} \); call it \( P \).

\[
y = A e^{-\lambda} \\
y' = e^{-\lambda} - A e^{-\lambda} \\
    = (1-A) e^{-\lambda}
\]

\[
\therefore \lambda = 1 \implies \max_y y(\lambda = 1) = e^{-1}
\]

d) Finally, suppose in a problem involving some random variable \( x \), whose distribution is not known, we find \( p(x=1) > P \). What does that say about using Poisson distribution to describe \( x \)?

Then the Poisson distribution cannot be used, because for a variable that does follow Poisson, \( p(x=1) < P \).

Big Moral: learn to manipulate the formulas for distributions!