

# Lecture Notes I – (Discrete) Sample spaces and the Multinomial

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Sample space, outcome, event, probability, . . .

Discrete sample spaces

Repeated independent trials, Binomial, Multinomial

**Reading:** Ch. 2, 3

# Basic vocabulary

- ▶  $S$  sample space (outcome space)
- ▶  $x \in S$  outcome
- ▶  $E \subseteq S$  event
- ▶  $2^S = \{E \mid E \subseteq S\} \equiv \mathcal{P}(S)$
- ▶  $P : 2^S \rightarrow [0, 1]$  probability distribution
- ▶  $f : S \rightarrow \mathbb{R}$  random variable
  
- ▶  $f : S \rightarrow \mathbb{R}$  statistic

# Discrete sample spaces

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## Repeated independent trials, Binomial, Multinomial

A coin is tossed 4 times, and the probability of 1 is  $p > 0.5$ . The outcomes, their probability and their counts are (in order of decreasing probability):

outcome $x$	$n_0$	$n_1$	$P(x)$	event
1111	0	4	$p^4$	$E_{0,4}$
1110	1	3	$p^3(1-p)^1$	$E_{1,3}$
1101	1	3	$p^3(1-p)^1$	
1011	1	3	$p^3(1-p)^1$	
0111	1	3	$p^3(1-p)^1$	
1100	2	2	$p^2(1-p)^2$	$E_{2,2}$
1010	2	2	$p^2(1-p)^2$	
1001	2	2	$p^2(1-p)^2$	
0110	2	2	$p^2(1-p)^2$	
0101	2	2	$p^2(1-p)^2$	
0011	2	2	$p^2(1-p)^2$	
0100	3	1	$p^1(1-p)^3$	$E_{3,1}$
1000	3	1	$p^1(1-p)^3$	
0010	3	1	$p^1(1-p)^3$	
0001	3	1	$p^1(1-p)^3$	
0000	4	0	$(1-p)^4$	$E_{4,0}$

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