

# Lecture Notes I – (Discrete) Sample spaces and the Multinomial

Marina Meilă  
mmp@stat.washington.edu

Department of Statistics  
University of Washington

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Sample space, outcome, event, probability, . . .

Discrete sample spaces

Repeated independent trials, Binomial, Multinomial

**Reading:** Ch. 2, 3

## Basic vocabulary

- ▶  $S$  sample space (outcome space)
- ▶  $x \in S$  outcome
- ▶  $E \subseteq S$  event
- ▶  $2^S = \{E \mid E \subseteq S\} \equiv \mathcal{P}(S)$
- ▶  $P : 2^S \rightarrow [0, 1]$  probability distribution
- ▶  $f : S \rightarrow \mathbb{R}$  random variable
- ▶  $f : S \rightarrow \mathbb{R}$  statistic

# Discrete sample spaces

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## Repeated independent trials, Binomial, Multinomial

A coin is tossed 4 times, and the probability of 1 is  $p > 0.5$ . The outcomes, their probability and their counts are (in order of decreasing probability):

| outcome $x$ | $n_0$ | $n_1$ | $P(x)$       | event     |
|-------------|-------|-------|--------------|-----------|
| 1111        | 0     | 4     | $p^4$        | $E_{0,4}$ |
| 1110        | 1     | 3     | $p^3(1-p)^1$ | $E_{1,3}$ |
| 1101        | 1     | 3     | $p^3(1-p)^1$ |           |
| 1011        | 1     | 3     | $p^3(1-p)^1$ |           |
| 0111        | 1     | 3     | $p^3(1-p)^1$ |           |
| 1100        | 2     | 2     | $p^2(1-p)^2$ | $E_{2,2}$ |
| 1010        | 2     | 2     | $p^2(1-p)^2$ |           |
| 1001        | 2     | 2     | $p^2(1-p)^2$ |           |
| 0110        | 2     | 2     | $p^2(1-p)^2$ |           |
| 0101        | 2     | 2     | $p^2(1-p)^2$ |           |
| 0011        | 2     | 2     | $p^2(1-p)^2$ |           |
| 0100        | 3     | 1     | $p^1(1-p)^3$ | $E_{3,1}$ |
| 1000        | 3     | 1     | $p^1(1-p)^3$ |           |
| 0010        | 3     | 1     | $p^1(1-p)^3$ |           |
| 0001        | 3     | 1     | $p^1(1-p)^3$ |           |
| 0000        | 4     | 0     | $(1-p)^4$    | $E_{4,0}$ |

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