Lecture Notes IV – Continuous distributions. Parametric density estimation.

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CDF and PDF. Sampling

Examples of continuous distributions

ML estimation for continuous distributions

ML estimation by gradient ascent

Reading: Ch.5, 6

CDF and PDF refresher

Cumulative distribution function (CDF)

$$F(x) = P[X \leq x] \quad (1)$$

1. $F \geq 0$ positivity.
2. $\lim_{x \to -\infty} F = 0$
3. $\lim_{x \to \infty} F = 1$
4. F is an increasing function

Probability density [function] (PDF)

$$f = \frac{dF}{dx} \quad (2)$$

$$P(a, b) = P[a, b] = F(b) - F(a) = \int_a^b f(x)dx \quad (3)$$

normalization condition

$$\int_{-\infty}^{\infty} f(x)dx = 1 \quad (4)$$

Examples of continuous distributions

$$\mathcal{F}_1 = \{u_{[a,b]},\ a < b\} \quad \text{uniform} \tag{5}$$

$$f(x; a, b) = \begin{cases} \frac{1}{b-a}, & x \in [a, b] \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

$$\mathcal{F}_2 = \{N(.; \mu, \sigma^2)\} \quad \text{normal} \tag{7}$$

$$f(x; \mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{8}$$

$$F(x; a, b) = \frac{1}{1 + e^{-ax-b}}, \quad a > 0 \quad \text{logistic} \tag{9}$$

$$f(x; a, b) = \frac{ae^{-ax-b}}{(1 + e^{-ax-b})^2} \tag{10}$$

ML estimation for continuous distributions

ML estimation by gradient ascent

$$l(a,b) = n\ln a - a\sum_i x_i - nb - 2\sum_{i=1}^{n}\ln(1+e^{-ax_i-b})$$

$$\frac{\partial l}{\partial a} = \frac{n}{a} - \sum_{i=1}^{n} x_i \frac{1-e^{-ax_i-b}}{1+e^{-ax_i-b}} = 0$$

$$\frac{\partial l}{\partial b} = -\sum_{i=1}^{n} \frac{1-e^{-ax_i-b}}{1+e^{-ax_i-b}} = 0$$