Lecture Notes I – (Discrete) Sample spaces and the Multinomial

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January 2025

Sample space, outcome, event, probability,. . .

Discrete sample spaces

Repeated independent trials, Binomial, Multinomial

Reading: Ch. 2, 3

Basic vocabulary

- S sample space (outcome space)
- $x \in S$ outcome
- $E \subseteq S$ event
- $2^S = \{E \mid E \subseteq S\} \equiv \mathcal{P}(S)$
- $P : 2^S \to [0, 1]$ probability distribution
- $f : S \to \mathbb{R}$ random variable

- $f : S \to \mathbb{R}$ statistic

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A coin is tossed 4 times, and the probability of 1 is $p > 0.5$. The outcomes, their probability and their counts are (in order of decreasing probability):

outcome x	n_0	n_1	$P(x)$	event
1111	0	4	p^4	$E_{0,4}$
1110	1	3	$p^3(1-p)^1$	$E_{1,3}$
1101	1	3	$p^3(1-p)^1$	
1011	1	3	$p^3(1-p)^1$	
0111	1	3	$p^3(1-p)^1$	
1100	2	2	$p^2(1-p)^2$	$E_{2,2}$
1010	2	2	$p^2(1-p)^2$	
1001	2	2	$p^2(1-p)^2$	
0110	2	2	$p^2(1-p)^2$	
0101	2	2	$p^2(1-p)^2$	
0011	2	2	$p^2(1-p)^2$	
0100	3	1	$p^1(1-p)^3$	$E_{3,1}$
1000	3	1	$p^1(1-p)^3$	
0010	3	1	$p^1(1-p)^3$	
0001	3	1	$p^1(1-p)^3$	
0000	4	0	$(1-p)^4$	$E_{4,0}$

Repeated independent trials, Binomial, Multinomial