STAT 391
Exam Solutions
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Problem 1

1.1 $S = \{\emptyset, A, B, C, AB, BC, AC, ABC\}$

1.2 Multinomial distribution. $\hat{\theta}_A^{ML} = \frac{3}{9}, \hat{\theta}_B^{ML} = \frac{2}{9}, \hat{\theta}_C^{ML} = \frac{4}{9}$

1.3 Geometrically distributed. $\tilde{\theta}_A = \frac{3}{12}, \tilde{\theta}_B = \frac{2}{11}, \tilde{\theta}_C = \frac{4}{13}$

1.4 An example is AB or a customer getting both books A and B.

Problem 2

2.1 $h_{opt} = 0.15$

2.2 True, False, True, False, False

Problem 3

3.1 f_T because this results in the largest likelihood.

3.2 f_C because this results in the largest likelihood.

3.3

$$P(f_U|x^1) = \frac{P(x^1|f_U)P^0(f_U)}{P(x^1|f_U)P^0(f_U) + P(x^1|f_T)P^0(f_T) + P(x^1|f_C)P^0(f_C)}$$

$$P(f_C|x^1) = \frac{P(x^1|f_C)P^0(f_C)}{P(x^1|f_U)P^0(f_U) + P(x^1|f_T)P^0(f_T) + P(x^1|f_C)P^0(f_C)}$$

$$P(f_T|x^1) = \frac{P(x^1|f_T)P^0(f_T)}{P(x^1|f_U)P^0(f_U) + P(x^1|f_T)P^0(f_T) + P(x^1|f_C)P^0(f_C)}$$

3.4 True

3.5 We compare $P(f_U|x^1) = 0.5 * 0.1, P(f_C|x^1) = 0, P(f_T|x^1) = 0.12 * \frac{1}{6}$. We would choose f_U.

Problem 4

4.1

$$\begin{aligned}
P(y^i = 1) &= P(x^i \geq 1) && (1)\\
&= \sum_{x^i=1}^{\infty} (1-\gamma)\gamma^{x^i} && (2)\\
&= (1-\gamma) \sum_{x^i=1}^{\infty} \gamma\gamma^{x^i-1} && (3)\\
&= (1-\gamma)\frac{\gamma}{1-\gamma} && (4)\\
&= \gamma && (5)
\end{aligned}$$

4.2

$$\begin{aligned}
\ell(\gamma) &= \log\left[\binom{n}{\sum_i y_i}\gamma^{\sum_i y_i}(1-\gamma)^{n-\sum_i y_i}\right] && (6)\\
&= \log\binom{n}{\sum_i y_i} + (\sum_i y_i)\log\gamma + (n - \sum_i y_i)\log(1-\gamma) && (7)\\
&= C + \log(\gamma)\sum_{i=1}^{n} y_i + (n - \sum_{i=1}^{n} y_i)\log(1-\gamma) && (8)
\end{aligned}$$

4.3 Set $\partial\ell(\gamma)/\partial\gamma = 0$ we obtain that

$$\frac{\sum_{i=1}^n y_i}{\gamma} - \frac{(n - \sum_{i=1}^n y_i)}{1-\gamma} = 0 \quad (9)$$

Therefore $\gamma^{ML} = \frac{\sum_{i=1}^{n} y_i}{n}$

4.4 Yes, $\sum_{i=1}^{n} y_i$ is the sufficient statistics.

Problem 5

5.1 β, γ

5.2 $L(y^{1:n}|\beta, \gamma, x^{1:n}) = \Pi_{i=1}^{n} P(y^i|\beta, \gamma, x^i) = \Pi_{i=1}^{n} \frac{\gamma}{2} e^{-\gamma|y^i - \beta x^i|}$

5.3 $\ell(y^{1:n}|\beta, \gamma, x^{1:n}) = \log(\Pi_{i=1}^{n} \frac{\gamma}{2} e^{-\gamma|y^i - \beta x^i|}) = n\log(\frac{\gamma}{2}) - \gamma \sum_{i=1}^{n} |y^i - \beta x^i|$

5.5 Set $\partial\ell(\gamma, \beta)/\partial\gamma = 0$ we obtain that

$$\frac{n}{\gamma} - \sum_{i=1}^{n} |y^i - \beta x^i| = 0 \tag{10}$$

Therefore $\gamma^{ML} = \frac{n}{\sum_{i=1}^{n} |y^i - \beta x^i|} = \frac{n}{\sum_{i=1}^{n} \hat{\epsilon}^i}$

5.6
Model 1:
$AIC(\ell) = -101 - 2 = -103$

Model 2:
$AIC(\tilde{\ell}) = -100.1 - 3 = -103.1$

We would choose Model 1 based on AIC.