

Lecture Notes IV – The bootstrap

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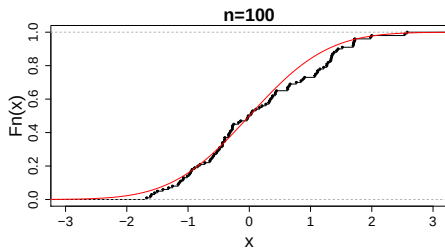
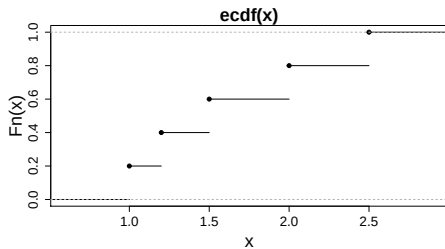
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Bootstrap: the idea

Example: Variance of the Median

Parametric bootstrap

Resampling



Example: Variance of the Median

CI Refresher

CI for parameter of interest θ :
$$[\hat{\theta} \pm z_{1-\alpha/2} \sqrt{\widehat{\text{Var}}\hat{\theta}}]$$

Parametric bootstrap for $\hat{\sigma}^2$

- ▶ $X_{1:n} \sim N(0, \sigma^2)$
- ▶ $\hat{\sigma}^2 = S_n^2$
- ▶ Bootstrap samples $X_{1:n}^{*(b)} \sim N(0, S_n^2)$
- ▶ $\hat{\text{Var}}_B(S_n^2) = \frac{1}{B-1} \sum_{b=1}^B \left(S_n^{*(b)} - \bar{S}_B^{2*} \right)^2$

Parametric bootstrap for $\exp(\lambda)$

- ▶ $X_{1:n} \sim \exp(\lambda)$
- ▶ $\hat{\lambda} = \frac{1}{\bar{X}}$
- ▶ Bootstrap samples $X_{1:n}^{*(b)} \sim \exp(\hat{\lambda})$
- ▶ $\hat{\text{Var}}_B(\hat{\lambda}_n^2) = \frac{1}{B-1} \sum_{b=1}^B \left(\hat{\lambda}_n^{*(b)} - \bar{\hat{\lambda}}_B^* \right)^2$