STAT 403 4/21/25

Lecture 9

Course overview

Bayesian optimization

OH 1:30-2:25

This Wed only

Overview

- EDF $\hat{F}$
  and consistency of means
- MC integration $\leftarrow$ Application
- How to sample
  - F^{-1}
  - importance s.
  - rejection s.
- Bootstrap $\leftarrow$ $Var(\hat{\theta}) \approx ?$
- Imputation (fill missing data)
- MCMC (Markov Chain MC)

"Bayesian" Optimization

Bayesian Estimation ← sampling

Active Learning

$\mathcal{D} = \{ (x_1, y_1), (x_2, y_2), \dots \}$ $\quad |\mathcal{D}| = n$ ← standard

1. start with $\mathcal{D}_0$, $|\mathcal{D}_0| = n_0$, learn (train) model $\hat{f}$ from $\mathcal{D}_0$ (Active learning)
2. for $i = n_0+1, \dots$
   - sample $x_i \sim P_{active}(\mathcal{D})$
   - get y_i
   - $\mathcal{D} \leftarrow \mathcal{D} \cup \{(x_i, y_i)\}$
   - update model f (a predictor)

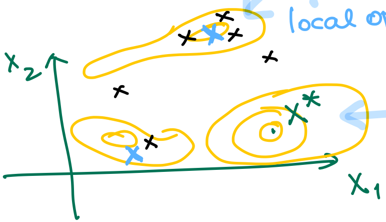
Bayesian Opt.

unknown

Pb Wanted $x^* = \arg\max f(x)$

Ex: cost of industrial process f = efficiency, x = process parameters

promising local optimum



unknown?

- new battery efficient, enviromental, cheap = f
  x = materials, technology

next x_i

Alg 1 (greedy)

for $i = n_0+1, \dots$

$x_i = \underset{x}{\operatorname{argmax}} \hat{f}(x)$ estimated

get $y_i = \underline{f}(x_i)$ + noise
true

re-estimate (update) $\hat{f}$

Alg 2 Naive sampling

for $i = n_0+1, \dots$

$x_i \sim g(x)$ $\quad g(x) \propto \hat{f}(x)$

get $y_i = \underline{f}(x_i)$ + noise
true

re-estimate (update) $\hat{f}$

Alg 3 B.O

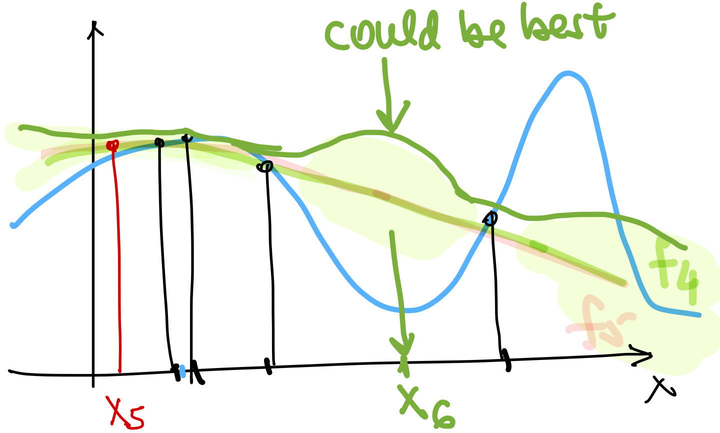
....

..

$x_i \sim g_{BO}(x)$ $\quad g_{BO}(x) \propto \hat{f}_{(x)} + \hat{\sigma}_{(x)} \cdot c$

get y_i

update $\hat{f}$, $\hat{\sigma}_{(x)}$ = uncertainty
$\hat{\text{Var}}\hat{f}(x)$



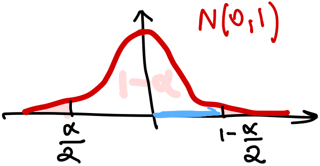
~~Example: Variance of the Median~~

Bootstrap = resampling method

estimates $\widehat{Var}\,\hat{\theta}$ param of interest

Assume $\hat{\theta} \sim N(\theta, \widehat{Var}\,\hat{\theta})$

Want $1-\alpha$ CI for $\hat{\theta}$



Bootstrap

CI for θ: $[\hat{\theta} \pm z_{1-\alpha/2}\sqrt{\widehat{\text{Var}}\hat{\theta}}]$

$1-\alpha$ = confidence level (e.g 95%)