STAT 535 Homework 1
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Problem 1

All the "$\perp$" statements in this problem should be interpreted as *probabilistic independence* statements in a joint distribution.

a. Assume that A, B, C, D are random variables, and that

$$A \perp CD \,|\, B \tag{1}$$
$$B \perp C \tag{2}$$

Prove that $D \perp A \,|\, BC$.

b. Under the same assumptions as in **a.**, show that $C \perp A$

c. Under the same assumptions as in **a.,b.**, show that

$$\sum_b P_{B|A} P_{D|BC} = P_{D|AC} \tag{3}$$

d. Prove without using the graphoid axioms that for any random variables A, B, C, D, E

$$AB \perp DEF \,|\, C \quad \Rightarrow \quad D \perp BA \,|\, FCE \tag{4}$$

Problem 2

Under the same assumptions as in **Problem 1, a., b.,c.** let

P_B

0	1
0.2	0.8

$P_{A|B}$

$A:$	0	1
$B = 0$	0.5	0.5
$B = 1$	0	1

$P_{D|BC}$

$D:$	0	1
$BC = 00$	0.4	0.6
$BC = 01$	0.5	0.5
$BC = 10$	0.2	0.8
$BC = 11$	0.7	0.3

Calculate symbolically and numerically $P(D = 1|A = 0, C = 0)$

Problem 3 – Proving the graphoid axioms [OPTIONAL, FOR EXTRA CREDIT]

Do only those proofs that weren't shown in the lecture.

Let X, Y, Z, W be disjoint subsets of discrete variables from V. Prove that for any probability distribution P over V the following relationships hold.

a. $X \perp YW \,|\, Z \;\Rightarrow\; X \perp Y \,|\, Z$ *(Decomposition)*

b. $X \perp YW \,|\, Z \;\Rightarrow\; X \perp Y \,|\, WZ$ *(Weak union)*

c. $X \perp Y \,|\, Z$ and $X \perp W \,|\, YZ \;\Rightarrow\; X \perp YW \,|\, Z$ *(Contraction)*

d. Prove that if all the variables are discrete and P is strictly positive for all instantiations of the variables, then the following relationship also holds.

$X \perp Y \,|\, WZ$ and $X \perp W \,|\, YZ \;\Rightarrow\; X \perp YW \,|\, Z$ *(Intersection)*

e. Find a counterexample to the Intersection property for a P that is not strictly positive.

Hints: Try the proofs first for $Z = \emptyset$. Use the "asymmetric" definition of independence.

Problem 4 - Explaining away (after J.Pearl)

This problem is a warmup in using conditional probabilities to reason about events. It also gives you a quantitative grasp of the phenomenon called "explaining away".

Your alarm A can be triggered ($A = 1$) either when a burglar B enters your house, or when an earthquake E strikes. In other words, we have:

$P_B(1)$	0.01	(probability of burglar present)
$P_E(1)$	0.001	(probability of an earthquake occuring)
$P_{A\|BE}(1\|1,1)$	1	(the alarm always sounds if both burglar and earthquke are present)
$P_{A\|BE}(1\|1,0)$	0.95	(the alarm almost always sounds if burglar enters, no earthquake)
$P_{A\|BE}(1\|0,1)$	0.85	(the alarm almost always sounds if earthquake, no burglar)
$P_{A\|BE}(1\|0,0)$	0.05	(the alarm may sound for other reasons)
$B \perp E$		(earthquakes and burglars strike independently)

a. Write the joint distribution P_{ABE} as a combination of the distributions given above. Do this in literal form only (do not plug in numbers), but show the full derivation of your result.

b. Compute the marginal distribution of the variable A. Give a formula based on the distributions in the table above, and a numerical answer.

c. Assume that the alarm sounds, i.e $A = 1$. Give the formula for $P_{B|A}(1|1)$ (the probability that a burglar is in the house) and the numeric result.

d. Assume that the alarm sounds and that an earthquake has just occured $A = 1, E = 1$. Give the formula for $P_{B|AE}(1|1, 1)$ and the numeric result.

Compare the values of $P_{B|AE}(1|1, 1)$ and $P_{B|A}(1|1)$. Are they the same? Which is higher? What can you conclude about the truth of "$B \perp E \,|\, A$"?

There are two phenomena to notice here. A qualitative one: B, E become dependent when A is observed; this is true for almost all values assigned to the initial probability tables in this problem. Then, a quantitative one: for the assigned values, the probability of a burglary decreases when we learn about the earthquake occuring. The earthquake is an alternative explanation for the alarm sounding, and learning about it decreases our belief in a burglary as explanation for the alarm. Therefore, this phenomenon is called *explaining away*.

e. We will now show that the two phenomena persist even when the common effect A is not directly observed. Assume that you are at UW, and a colleague C calls to tell you that at home your alarm is sounding ($C = 1$).

In addition to the information in the table above, you know that C is independent of anything else given A and

$P_{C|A}(1|1)$	0.9	(C calls you almost every time when the alarm sounds)
$P_{C|A}(1|0)$	0.05	(C may mistakenly believe that the alarm sounds) $C \perp BE \,|\, A$

Compute $P_{B|C}(1|1)$ and $P_{B|CE}(1|1, 1)$ (formula and numbers). Check that $B \not\perp E \,|\, C$ and that explaining away occurs.

f. [Optional–for extra credit] Can you show that the explaining away persists even if the information about E is indirect? Assume the same setting as in **e.** with the difference that E is not observed. Instead, you hear on the radio news the words "earthquake in Seattle" (call this event N). You know that $P(N|E = 1) = 0.95$ (almost every earthquake is announced on the news) and $P(N|E = 0) = 0.05$ (some times there is talk about "earthquake in Seattle" without one happening). Assume that, given E, N is independent of any other variables in the problem. Show that B, E are dependent given all the observations, and that $P_{B|C}(1|1) > P_{B|CN}(1|1, 1)$ (i.e explaining away occurs).