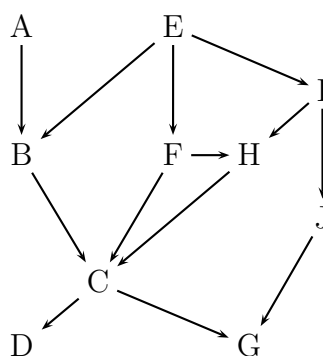


STAT 535 Homework 3  
 Out October 18, 2011  
 Due October 25, 2011  
 ©Marina Meilă  
 mmp@stat.washington.edu

**Problem 1 - D-separation as undirected separation. Factorization**

This is the same graph from Homework 2.



- a. Write a topological ordering of the nodes in  $V$ .
- b. Verify the following D-separation statements by
  - constructing the respective ancestral graph
  - moralizing the obtained ancestral graph
  - checking undirected separation in the moral ancestral graph

$B \perp H \mid E$   
 $B \not\perp I \mid A, D, E$

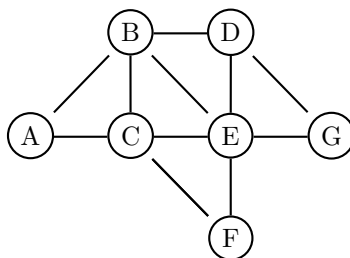
- c. Write the factored expression of a joint distribution  $P_V$  over  $V$  for which this graph is an I-map.

- d. Verify that  $B \perp I \mid EH$  in  $P_V$  using marginalization in the factored form of  $P_V$ .

**Problem 2 – A graph of tree-width 2**

The graph  $\mathcal{G}$  below has **treewidth** 2. The treewidth of a graph is one less than the size of the maximum clique in that graph.

Graph  $\mathcal{G}$



- a. Find an orientation for the graph  $\mathcal{G}$ , which produces no V-structures. Denote the resulting DAG by  $\mathcal{G}'$ .
- b. Write the general factored form of a distribution  $P$  for which the undirected graph is an I-map.
- c. Write the general factored form of a distribution  $P'$  of which the DAG you found in question a. is an I-map.
- d. The two factorizations in b,c must be equal. Find a way to group the factors in  $P'$  to obtain the factorization in  $P$ . The grouping may not be unique.

Using the grouping you found, show that the potentials of  $P$  have a probabilistic interpretation. Since there will be many potentials, it is sufficient to find a probabilistic interpretation for one potential containing variable  $A$  and for one containing the variable  $E$ .

- e. Verify that  $\mathcal{G}$  is chordal by the Tarjan elimination algorithm.
- f. Construct a junction tree for the graph  $\mathcal{G}$ . Is this tree unique? List its separators (with multiplicities).