STAT 535 Homework 6
Out November 9, 2010
Due November 16, 2010

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Problem 1 - J.T algorithm implementation

Submit code for all question of this problem through the Assignments web site. You can write a program that does everything below step by step (preferred, but please comment where every step starts) or a separate program for each question, or any other combination.

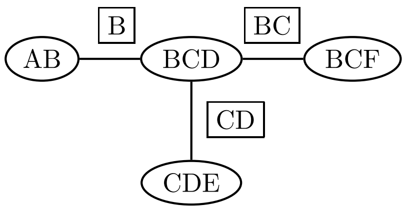
In the problem, you will be required to implement the function ABSORB *and then various propagation algorithms. You are required to implement* ABSORB *as a general purpose algorithm taking tables as parameters, in a representation of your choice.*

It is OK to special case the rest of the assignment by hand-coding calls to ABSORB. *Please comment or write "readable" code.*

In addition to submitting the program, the problem calls for answers to a few questions and printing of results. These should be returned on paper or in a separate .pdf file.

a. Implement the function ABSORB($\phi_{\text{source}}, \phi_S, \phi_{\text{destination}}$). (Only code required here.)

b. Consider the following junction tree:



The tables P_{AB}, P_{BCD}, P_{BCF}, P_{CDE} are given below, and can also be found in the file `hw6-ptables.dat` in plain text.

	A		CD				CF			
B	.6	.05	.4	.05	.05	.15	.4	.05	.15	.05
	.05	.3	.05	.05	.05	.2	.0	.1	.25	.0
E			.35	.0	.05	.1				
			.1	.1	.05	.25				

The rows correspond to $B = 0$, $B = 1$, $E = 0$, $E = 1$, and the columns correspond respectively to: $A = 0, A = 1$, for P_{AB}; $CD = 00$, 01, 10, 11 for P_{BCD} and P_{ECD}, $CF = 00$, 01, 10, 11 for P_{CDF}.

Read the tables and create a calibrated junction tree structure representing this distribution. Do all this in code, and do not print anything for this question. Okay to specialize for this problem.

c. Assume $A = 0$ is observed. Write code that implements PROPAGATION ALGORITHM 3 for this case.

Print out the clique potentials for cliques AB and ECD after running the algorithm. Please use the same format as for the input (e.g. E on rows, CD on columns), or label your tables carefully. What probability distributions do the clique potentials represent?

Print the normalization constant that you obtain in the NORMALIZE step. What probability does it represent?

d. Now assume that $A = 0, F = 0$ are observed. Run PROPAGATION ALGORITHM 4, the junction tree algorithm with this evidence to obtain the posterior of the variables given the evidentce. Choose the root at clique CDE. Print the clique potentials for AB, ECD twice: first, after COLLECT EVIDENCE, next and at the end of PROPAGATION ALGORITHM 4.

Print the normalization constant that you obtain in the NORMALIZE step. What probability does it represent?

[**Optional, not graded but recommended**] Print all clique potentials.

e. Now, taking as prior the junction tree potentials obtained in question **c.**, assume me make a new observation $F = 0$. Run ALGORITHM 3 with this evidence and print the resulting clique potentials for AB, ECD.

Compare these potentials with the ones obtained in **d.**. Are they equal? Explain why or why not.

[Optional, not graded but recommended] Print all clique potentials.

[f. Optional] Explain the relationship you observe between each of the clique potentials in **d.** before and after DISTRIBUTE EVIDENCE.

Problem 2. - Inference with "uncertain evidence"

This problem uses the junction tree and implementation in **Problem 1.**. Assume that the variables represent the state of a patient, with A being "appendicitis". In this problem we assume that A cannot be observed directly, but only through the variable $O = o$"observation", or "data". Intuitively P_{ABCDEF} is a doctor's prior about patient states; a patient comes in and the doctor, after some questioning and tests ("gathering data") concludes that the likelihood of A given the observation is $P^*(O|A)$ with $P^*(O = o|A = 0) = 0.1$, $P^*(O = o|A = 1) = 0.01$. We assume that the observations O depend only on A, that is, $O \perp BCDEF \mid A$.

a. Use the junction tree algorithm to find the posterior of all the variables given O in the following way:

1. calculate P_A the prior probability of A.
2. Run the junction tree algorithm as in Problem **1,c** for $A = 0$. (this is already done, no need to print or show anything) and obtain $P_{ABCDEF|A=0}$.
3. Run the junction tree algorithm again for $A = 1$ and obtain $P_{ABCDEF|A=1}$. Print the posterior potentials for AB, ECD.
4. Show that $P_{ABCDEF|O}$ can be written as a combination of the two above cases, and that this combination can itself be stored in a junction tree data structure (i.e it is factored like the original junction tree). Calculate and print the clique potentials corresponding to this posterior for AB, ECD.

   [Optional, not graded but recommended] Print all clique potentials.

b. Obtain the same posterior as in **a.** with a single junction tree propagation (and possibly other operations). Explain how it should be done, write the code and print the posterior potentials for AB, ECD.

[Problem 3 – Optional: Extra credit]

Solve Problem 1 and 2 by the Message Passing algorithm.

Implementation: Clearly comment your implementation

On paper or separate .pdf file: Give an outline of the algorithm you implemented for each question. I am interested in: (a) what are the messages that you calculate (in generic form, once at the beginning) and (b) for each inference, the sequence in which messages are sent between cliques.

Print results of inference, in the same way as for Problems 1, 2.