TWO-STEP EM **Algorithm**

Assumes K spherical gaussians, separation $||\mu_k^{true} - \mu_{k'}^{true} \geq C\sqrt{d}\sigma_k$

1. Pick $K' = \mathcal{O}(K \ln K)$ centers μ_k^0 at random from the data
2. Set $\sigma_k^0 = \frac{d}{2} \min_{k \neq k'} ||\mu_k^0 - \mu_{k'}^0||^2$, $\pi_k^0 = 1/K'$
3. Run one E step and one M step $\Longrightarrow$ $\{\pi_k^1, \mu_k^1, \sigma_k^1\}_{k=1:K'}$
4. Compute "distances" $d(\mu_k^1, \mu_{k'}^1) = \frac{||\mu_k^1 - \mu_{k'}^1||}{\sigma_k^1 - \sigma_{k'}^1}$
5. Prune all clusters with $\pi_k^1 \leq 1/4K'$
6. Run Fastest First Traversal with distances $d(\mu_k^1, \mu_{k'}^1)$ to select K of the remaining centers. Set $\pi_k^1 = 1/K$.
7. Run one E step and one M step $\Longrightarrow$ $\{\pi_k^2, \mu_k^2, \sigma_k^2\}_{k=1:K}$

Theorem For any $\delta, \epsilon > 0$ if d large, n large enough, separation $C \geq d^{1/4}$ the Two step EM algorithm obtains centers μ_k so that

$$||\mu_k - \mu_k^{true}|| \leq ||\text{mean}(C_k^{true}) - \mu_k^{true}|| + \epsilon\sigma_k\sqrt{d}$$