Algorithm SpectralClustering

Input Similarity matrix S, number of clusters K

1. Transform S:
   Set $D_i = \sum_{j=1}^{n} S_{ij}$, $j = 1 : n$ the node degrees.
   Form the transition matrix $P = [P_{ij}]_{ij=1}^{n}$ with

$$P_{ij} \leftarrow S_{ij}/D_i, \text{ for } i, j = 1 : n$$

2. Spectral embedding
   Compute the largest K eigenvalues $\lambda_1 = 1 \geq \lambda_2 \geq \ldots \geq \lambda_K$ and eigenvectors $\mathbf{v}_1, \ldots \mathbf{v}_K$ of P.

3. Let $V = [\, \mathbf{v}_2 \; \mathbf{v}_3 \; \ldots \; \mathbf{v}_K \,] \in \mathbb{R}^{n \times K}$, $\mathbf{x}_i \leftarrow i$-th row of V.
   Optionally, normalize each x_i by $x_i \leftarrow \frac{x_i}{||x_i||}$

4. Orthogonal initialization Find K initial centers by

   (a) take μ_1 randomly from $\mathbf{x}_1, \ldots \mathbf{x}_n$

   (b) for $k = 2, \ldots K$ set $\mu_k = \text{argmin}_{\mathbf{x}_i} \max_{k' < k} \mu_{k'}^T \mathbf{x}_i$.

5. Run K-means on "data" $\mathbf{x}_{1:n}$ starting from centers $\mu_{1:K}$.