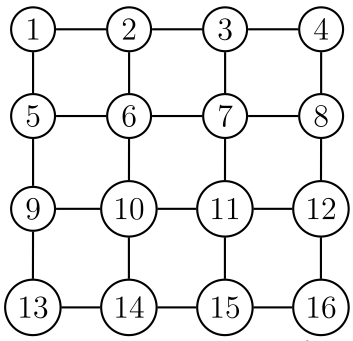
STAT 534 Homework 6
Out Thursday, May 30, 2019
Due Monday, June 10, 2019 (noon)

©Marina Meilă
mmp@stat.washington.edu

Problem 1 – Gibbs sampling

Submit your code through the homework web site. Do all the implementations in one file. For questions **b, c, e, f, g** *provide the answers on paper.*

Consider the following 16 nodes Markov network



where all variables are binary, $x_i \in \{\pm 1\}$ and all clique potentials are of the form

$$\phi_{ij}(x_i x_j) \;=\; \begin{cases} 1 & \text{if } x_i = x_j \\ e^{-2J} & \text{if } x_i \neq x_j \end{cases}$$

a. Implement Gibbs sampling for this Markov network, by sampling variables $1, 2, 3, \ldots 16$ in turn.

b. Initialize $x_{1:16} = 1$. Run the Gibbs sampling algorithm for 100 iterations (a cycle in which all variables have their state sampled once is consindered an iteration) for $J = 0.2$ and $J = 1$. Plot the states of the variable vector over the iteration (see example below).

c. Calculate the mean of x_1, x_2, x_3 over the 100 iterations.

d. Now implement Gibbs sampling for the same Markov network, by sampling the groups of variables $1, 3, 6, 8, 9, 11, 14, 16$, and $2, 4, 5, 7, 10, 12, 13, 15$ alternatively.

e. Repeat **b,c** for the new sampling procedure.

f. Because states x and $-x$ have the same probability, the true marginal of any singe variable x_j in this model is $(1/2, 1/2)$, with expectation 0. Compare the numerical results of the two algorithms with the true expectations or true marginals; does it appear that one algorithm converges faster than the other?

Example: Below is displayed the state $x_{1:16}$ over 50 iterations of a Gibbs sampling algorithm, with ascii characters.

```
++++++++++++++++
--+-+++-+--+++-+
-+++++-++--+++--
-++---++++--+---
+----+----+-++-+
++--++++++++--++
+++-++----++-++-
-++-++--++++++-+
+-+++-+++---+--+
-++++-+--++-----
+-++-++-+---+++-
+-++--++-+--+--+
--+-++++-+--++++
-++-+++-++----++
+++-+--+--+-++--
+--+--+--+-++-+-
---+--+-++--+++-
-+-+++-+---+-+--
-++---++--+++---
-+----+--+-++-+-
+---+---++--++-+
+--------++--+++
--+-+-+-++--+-++
...
```

Alternatively, you can display the same 50×16 states as a black and white image. Either way is satisfactory, as long as the patterns in the state-time are visible enough.