Design of Engineering Experiments The Blocking Principle

- Montgomery text Reference, Chapter 4
- **Blocking** and **nuisance factors**
- The randomized complete block design or the **RCBD**
- Extension of the ANOVA to the RCBD
- Other blocking scenarios…Latin square designs

The Blocking Principle

- **Blocking** is a technique for dealing with **nuisance factors**
- A **nuisance** factor is a factor that probably has some effect on the response, but it's of no interest to the experimenter…however, the variability it transmits to the response needs to be minimized
- Typical nuisance factors include batches of raw material, operators, pieces of test equipment, time (shifts, days, etc.), different experimental units
- **Many** industrial experiments involve blocking (or should)
- Failure to block is a common flaw in designing an experiment (consequences?)

The Blocking Principle

- If the nuisance variable is **known** and **controllable**, we use **blocking**

but not "measureable"

- If the nuisance factor is **known** and **uncontrollable**, sometimes we can use the **analysis of covariance** (see Chapter 15) to remove the effect of the nuisance factor from the analysis

but "measureable"

- If the nuisance factor is **unknown** and **uncontrollable** (a **"lurking" variable**), we hope that **randomization** balances out its impact across the experiment
- Sometimes several sources of variability are **combined** in a block, so the block becomes an aggregate variable

The Hardness Testing Example

- Text reference, pg 120
- We wish to determine whether 4 different tips produce different (mean) hardness reading on a Rockwell hardness tester
- Gauge & measurement systems capability studies are frequent areas for applying DOX
- Assignment of the tips to an **experimental unit**; that is, a test coupon

tip is pressed into a metal test "coupon"; depth of depression yields hardness

- Structure of a completely randomized experiment
- The test coupons are a source of **nuisance variability**
- Alternatively, the experimenter may want to test the tips across coupons of various hardness levels
- The need for blocking

The Hardness Testing Example

- To conduct this experiment as a RCBD, assign all 4 tips to each coupon
- Each coupon is called a "**block**"; that is, it's a more homogenous experimental unit on which to test the tips
- Variability **between** blocks can be large, variability **within** a block should be relatively small
- In general, a **block** is a specific level of the nuisance factor
- A complete replicate of the basic experiment is conducted in each block
- A block represents a **restriction on randomization**
- All runs **within** a block are **randomized**

The Hardness Testing Example

- Suppose that we use $b = 4$ blocks:

Table 4-1 Randomized Complete Block Design for the Hardness Testing Experiment

Test Coupon (Block)			
1	2	3	4
Tip 3	Tip 3	Tip 2	Tip 1
Tip 1	Tip 4	Tip 1	Tip 4
Tip 4	Tip 2	Tip 3	Tip 2
Tip 2	Tip 1	Tip 4	Tip 3

- Notice the **two-way structure** of the experiment
- Once again, we are interested in testing the equality of treatment means, but now we have to remove the variability associated with the nuisance factor (the blocks)

Extension of the ANOVA to the RCBD

- Suppose that there are *a* treatments (factor levels) and *b* blocks
- A **statistical model** (effects model) for the RCBD is

$$y_{ij} = \mu + \tau_i + \beta_j + \varepsilon_{ij} \begin{cases} i = 1, 2, \ldots, a \\ j = 1, 2, \ldots, b \end{cases}$$

- The relevant (fixed effects) hypotheses are

$$H_0 : \mu_1 = \mu_2 = \cdots = \mu_a \text{ where } \mu_i = (1/b)\sum_{j=1}^{b}(\mu + \tau_i + \beta_j) = \mu + \tau_i$$

Extension of the ANOVA to the RCBD

ANOVA partitioning of total variability:

$$\sum_{i=1}^{a}\sum_{j=1}^{b}(y_{ij}-\bar{y}_{..})^2 = \sum_{i=1}^{a}\sum_{j=1}^{b}[(\bar{y}_{i.}-\bar{y}_{..})+(\bar{y}_{.j}-\bar{y}_{..})$$

$$+(y_{ij}-\bar{y}_{i.}-\bar{y}_{.j}+\bar{y}_{..})]^2$$

$$= b\sum_{i=1}^{a}(\bar{y}_{i.}-\bar{y}_{..})^2 + a\sum_{j=1}^{b}(\bar{y}_{.j}-\bar{y}_{..})^2$$

$$+\sum_{i=1}^{a}\sum_{j=1}^{b}(y_{ij}-\bar{y}_{i.}-\bar{y}_{.j}+\bar{y}_{..})^2$$

$$SS_T = SS_{Treatments} + SS_{Blocks} + SS_E$$

Extension of the ANOVA to the RCBD

The degrees of freedom for the sums of squares in

$$SS_T = SS_{Treatments} + SS_{Blocks} + SS_E$$

are as follows:

$$ab - 1 = a - 1 + b - 1 + (a-1)(b-1)$$

Therefore, ratios of sums of squares to their degrees of freedom result in mean squares and the ratio of the mean square for treatments to the error mean square is an *F* statistic that can be used to test the hypothesis of equal treatment means

ANOVA Display for the RCBD

Table 4-2 Analysis of Variance for a Randomized Complete Block Design

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Treatments	$SS_{\text{Treatments}}$	$a - 1$	$\dfrac{SS_{\text{Treatments}}}{a-1}$	$\dfrac{MS_{\text{Treatments}}}{MS_E}$
Blocks	SS_{Blocks}	$b - 1$	$\dfrac{SS_{\text{Blocks}}}{b-1}$	
Error	SS_E	$(a-1)(b-1)$	$\dfrac{SS_E}{(a-1)(b-1)}$	
Total	SS_T	$N - 1$		

Vascular Graft Example (pg. 124)

- To conduct this experiment as a RCBD, assign all 4 pressures to each of the 6 batches of resin
- Each batch of resin is called a "**block**"; that is, it's a more homogenous experimental unit on which to test the extrusion pressures

Table 4-3 Randomized Complete Block Design for the Vascular Graft Experiment

Extrusion Pressure (PSI)	Batch of Resin (Block) 1	2	3	4	5	6	Treatment Total
8500	90.3	89.2	98.2	93.9	87.4	97.9	556.9
8700	92.5	89.5	90.6	94.7	87.0	95.8	550.1
8900	85.5	90.8	89.6	86.2	88.0	93.4	533.5
9100	82.5	89.5	85.6	87.4	78.9	90.7	514.6
Block Totals	350.8	359.0	364.0	362.2	341.3	377.8	$y_{..} = 2155.1$

Vascular Graft Example

"Design-Expert" Output

Response: Yield
ANOVA for Selected Factorial Model
Analysis of variance table [Partial sum of squares]

Source	Sum of Squares	DF	Mean Square	F Value	Prob > F
Block	192.25	5	38.45		
Model	178.17	3	59.39	8.11	0.0019
A	*178.17*	*3*	*59.39*	*8.11*	*0.0019*
Residual	109.89	15	7.33		
Cor Total	480.31	23			

Std. Dev.	2.71	R-Squared	0.6185
Mean	89.80	Adj R-Squared	0.5422
C.V.	3.01	Pred R-Squared	0.0234
PRESS	281.31	Adeq Precision	9.759

Residual Analysis for the Vascular Graft Example

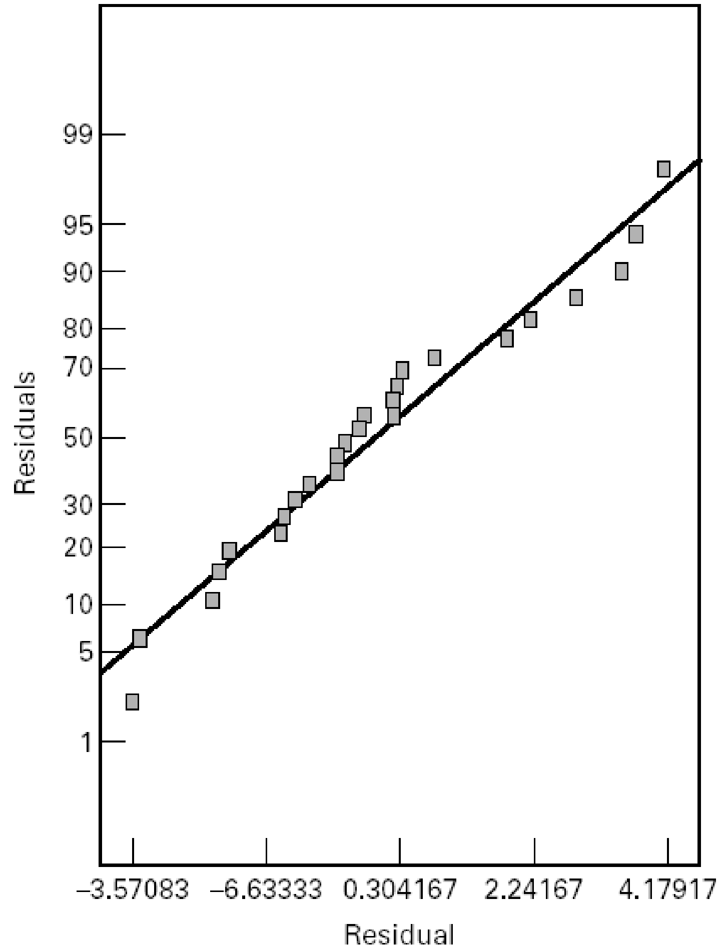

Figure 4-4 Normal probability plot of residuals for Example 4-1.

Residual Analysis for the Vascular Graft Example

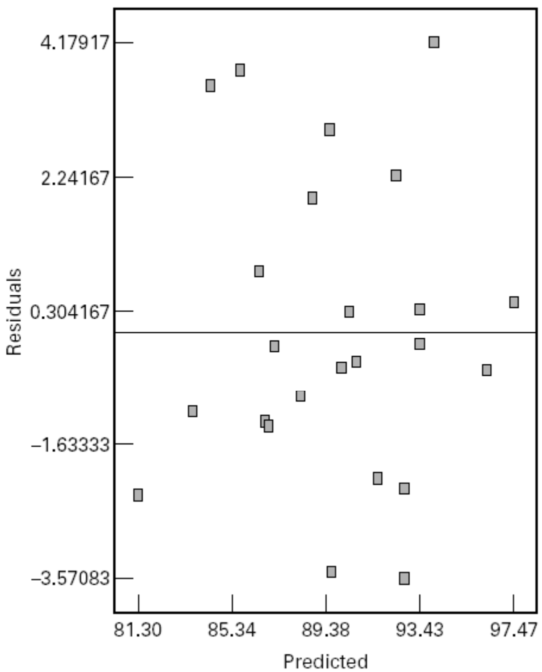

Figure 4-5 Plot of residuals versus $\hat{y}_{ij}$ for Example 4-1.

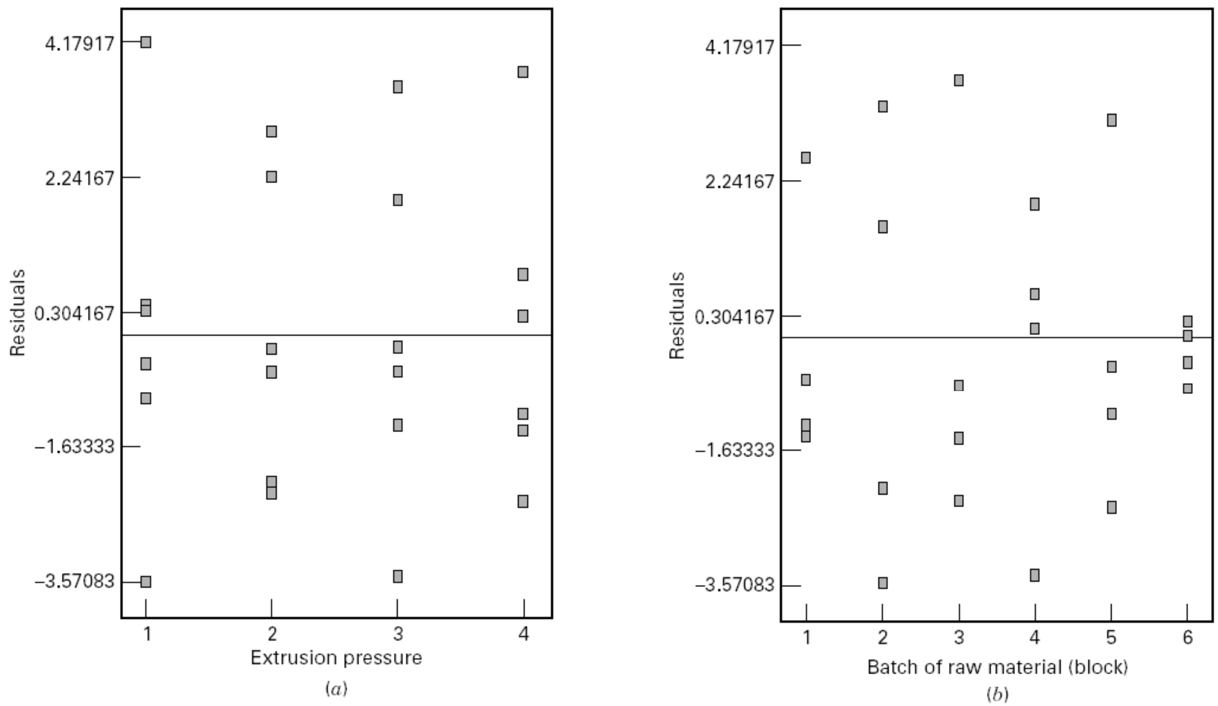

Figure 4-6 Plot of residuals by extrusion pressure (treatment) and by batches of resin (block) for Example 4-1.

Residual Analysis for the Vascular Graft Example

- Basic residual plots indicate that **normality**, **constant variance** assumptions are satisfied
- No obvious problems with **randomization**
- **No patterns in the residuals vs. block**
- Can also plot residuals versus the pressure (residuals by factor)
- These plots provide more information about the constant variance assumption, possible outliers

Multiple Comparisons for the Vascular Graft Example – Which Pressure is Different?

Treatment Means (Adjusted, If Necessary)

	Estimated Mean	Standard Error
1–8500	92.82	1.10
2-8700	91.68	1.10
3-8900	88.92	1.10
4-9100	85.77	1.10

Treatment	Mean Difference	DF	Standard Error	t for H_0 Coeff=0	Prob > |t|
1 vs 2	1.13	1	1.56	0.73	0.4795
1 vs 3	3.90	1	1.56	2.50	0.0247
1 vs 4	7.05	1	1.56	4.51	0.0004
2 vs 3	2.77	1	1.56	1.77	0.0970
2 vs 4	5.92	1	1.56	3.79	0.0018
3 vs 4	3.15	1	1.56	2.02	0.0621

Also see Figure 4-3, Pg. 128

Other Aspects of the RCBD
See Text, Section 4-1.3, pg. 130

- The RCBD utilizes an **additive model** – no interaction between treatments and blocks
- Treatments and/or blocks as random effects
- Missing values
- What are the **consequences** of **not blocking** if we should have?
- **Sample sizing** in the RCBD? The **OC curve** approach can be used to determine the number of blocks to run..see page 131