Design and Analysis of Experiments

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Outline

- Basic Ideas
- Definitions
- Structures of an Experimental Design
  - ⇨ Design Structure
  - ⇨ Treatment Structure
- The Three R's of Experimental Design
- Examples

Basic Ideas

Questions:

- What is the scientific question?
- What are the sources of variation?
- How many treatments are to be studied?
- What are the *experimental units*?
- How does the experimenter apply the treatments to the available experimental units and then observe the responses?
- Can the resulting design be analyzed or can the desired comparisons be made?

Homilies

"To call in the statistician after the experiment is done may be no more than asking him to perform a postmortem examination…

… he may be able to say what the experiment died of."

R.A. Fisher, Indian Statistical Congress, Sankhya, ca 1938

Definitions

- **Factor** – A variable under the control of the experimenter. Factors are explanatory variables. A factor has 2 or more levels.
- **Treatment** - The combination of experimental conditions applied to an experimental unit.
- **Response** - The outcome being measured.
- **Experimental unit** - The unit to which the treatment is applied.
- **Observational unit** - The unit on which the response is measured. (This may not be the same as the experimental unit.)

Experimental Design Structures

- Design Structure
  - ⇨ The grouping of the experimental units into homogeneous *blocks*
    - ✓ E.g., twins, gender…
  - ⇨ Why might this be important?
    - ✓ To ensure a fair comparison when the number of experimental units is “small”

Experimental Design Structures

- Treatment Structure
  - ⇨ Consists of the set of treatments, treatment combinations or populations the experimenter has selected to study and/or compare.

- Combining the treatment structure and design structure forms an experimental design.

The Three R's of Experimental Design

- Randomization
- Replication
- Stratify (block)

The Three R's (cont.)

- Randomization – It is important to randomize because it averages out the effect of all other lurking variables - it doesn't remove their effects, but makes, on average, their effects equal in all groups.
  - ⇨ Proper randomization is crucial
    - ✓ Iron deficiency in rats experiment

The Three R's (cont.)

- Replication – A replication is an *independent* observation of a treatment. Two replications of a treatment must involve two experimental units.
  - ⇨ Important to have replication to insure you have power to detect differences
  - ⇨ Randomization helps to make fair or unbiased comparisons, but only in the sense of being fair or unbiased when ***averaged*** over a whole sequence of experiments.
  - ⇨ Beware of pseudo-replication (sub-sampling)
    - ✓ Pig myocardium experiment

The Three R's (cont.)

- Blocking – Experimental units are divided into subsets (*blocks*) so that units *within* the same block are more similar than units from different subsets or blocks.

- If two units in the same block get different treatments, the treatments can be compared more precisely than if all the units in one block received one treatment, all in another received the second.

The Three R's (cont.)

- Why block?
  - ⇨ Partly because random assignment of treatments does not necessarily ensure a fair comparison when the number of experimental units is "small".
  - ⇨ If blocking variable is a good prognostic variable, you could effectively remove a source of variation in your response.
    - ✓ Basic principle of paired comparison t-test
  - ⇨ "Block what you can and randomize what you cannot." Box, Hunter and Hunter 1978

Examples

- Example 1 – An agricultural experimental station is going to test two varieties of wheat. Each variety will be tested with two types of fertilizers. Each combination will be applied to two plots of land. The yield will be measured for each plot.
- Treatment:
  - ✓ Varieties of wheat and fertilizer types
- Response:
  - ✓ yield
- Experimental unit:
  - ✓ plots
- Observational unit:
  - ✓ plots

Examples (cont.)

- Example 2 – Scientists want to study the effect of an anti-bacterial drug in fish lungs. The drug is administered at 3 dose-levels (0, 20, and 40 mg/L). Each dose is administered to a large controlled tank through the filtration system. Each tank has 100 fish. At the end of the experiment, the fish are sacraficed, and the amount of bacteria in each fish is measured.
- Treatment:
  - ✓ Dose levels of antibacterial drug
- Response:
  - ✓ Amount of bacteria
- Experimental unit:
  - ✓ Tanks
- Observational unit:
  - ✓ Fish

Examples (cont.)

- Example 3 – A study was conducted to examine the crop yield for 3 varieties of corn, *V*, under 5 different fertilizers, *F*. A 15 row field was available for the experiment. The experimenter first randomly assigned each of the 5 fertilizers to exactly 3 rows.

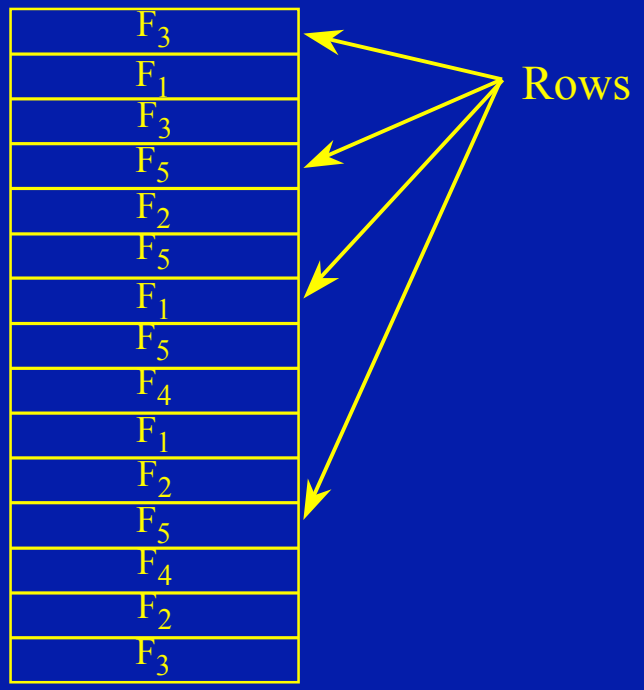
- Treatment:
  - ✓ Fertilizer
- Response:
  - ✓ Yield
- Experimental unit:
  - ✓ Row
- Observational unit:
  - ✓ Row

Examples (cont.)

- The treatment structure for *F* can be written as

  $Y_{ik} = \mu + F_i + \varepsilon_{ik}$, $i = 1,\ldots,5$; $k = 1,2,3$;

  where
  - ⇨ μ is the overall mean,
  - ⇨ F_i is the effect of fertilizer type i
  - ⇨ ε_{ik} is a mean zero random error term.



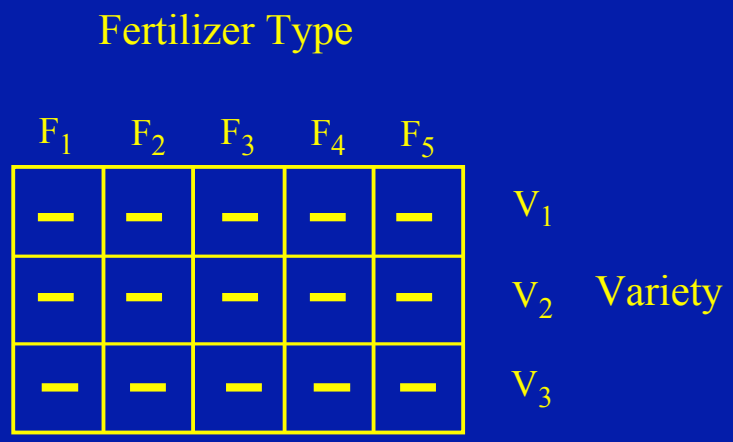
Examples (cont.)

- The experimenter also wants to study the 3 varieties of corn. Suppose the experimenter randomly assigns the 3 varieties to exactly 5 rows.

- Treatments:
  - ✓ Corn varieties and fertilizers
- Response:
  - ✓ Yield
- Experimental unit:
  - ✓ Row
- Observational unit:
  - ✓ Row

Examples (cont.)

- The model for *F* and *V* is
  $Y_{ijk} = \mu + F_i + V_j + (FV)_{ij} + \varepsilon_{ijk}$
  where
  - μ is the overall mean,
  - F_i is the effect of fertilizer type *i*,
  - V_j is the effect of variety *j*,
  - $(FV)_{ij}$ is the fertilizer by variety interaction,
  - ε_{ijk} is a mean zero random error term.



Examples (cont.)

- The experimenter *does* wish to investigate a fertilizer by variety interaction. S/he decides to divide each of the 15 rows, *r*, into 3 *subplots*, then randomly assigns one of the 3 corn varieties, *V*, to each of the subplots.

- Treatment:
  - ✓ Corn varieties
- Response:
  - ✓ Yield
- Experimental unit:
  - ✓ Subplot
- Observational unit:
  - ✓ Subplot

Examples (cont.)

- The model for the subplot EU is

$$Y_{ij} = \mu + r_i + V_j + (rV)_{ij} + \varepsilon_{ij},\ i = 1,\ldots,15;\ j = 1,2,3$$

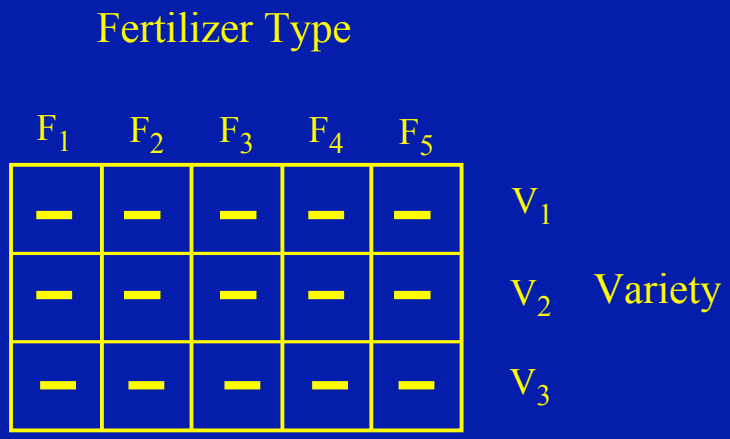
where

- r_i is the i-th block effect,
- V_j is the effect for variety j,
- $(rV)_{ij}$ is the block by variety interaction,
- ε_{ij} is a random error term.

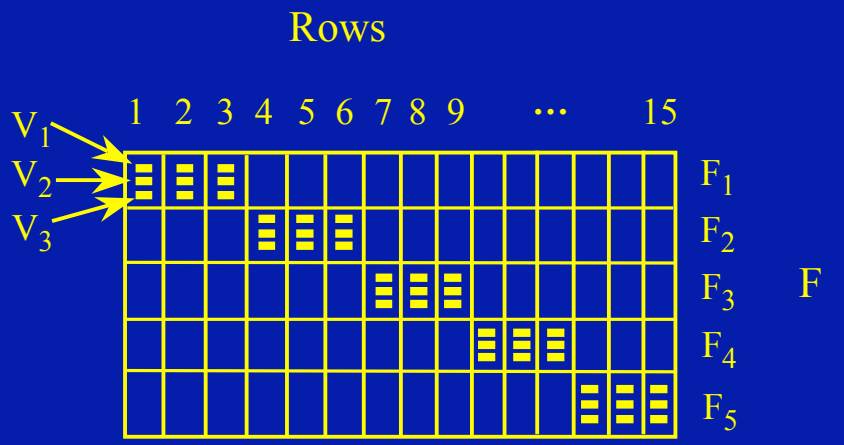
- Recall: rows, *r,* are nested within fertilizers, *F*.

Examples – Split Plot Model

- In the first design, rows were the EUs; the factors *F* and *V* were completely crossed.



- In the split plot design, subplots form one level of the EU. The row is a (blocking) factor. Rows are nested within fertilizers and crossed with varieties.



Examples – Split Plot Model

- Experimental Units – 2 levels
  - ⇨ 1. The EUs (rows) are (randomly) assigned one level of the whole plot factor (e.g., fertilizer type F_4).
  - ⇨ 2. EUs are then split into smaller EUs (subplots) and receive all levels of the subplot factor (e.g., varieties V_1, V_2, V_3).

Model: $Y_{ijk} = \mu + F_i + r_{k(i)}$ } *whole plot part*

$+ V_j + (FV)_{ij} + \varepsilon_{ijk}$ } *sub plot part*

Examples – Split Plot Model

- ANOVA Table

Source	df	E[MS]
Between plot	14	
Fertilizer, F	4	$\sigma_\varepsilon^2 + 3\sigma_r^2 + 3*3\ \Sigma_i\ F_i^2/(5-1)$
Row(F)	10	$\sigma_\varepsilon^2 + 3\sigma_r^2$
Within plot	30	
Variety, V	2	$\sigma_\varepsilon^2 + 3\sigma_{vr}^2 + 3*5\ \Sigma_j\ V_j^2/(3-1)$
V x F	8	$\sigma_\varepsilon^2 + 3\sigma_{vr}^2 + 3\ \Sigma_{ij}\ (FV)_{ij}^2/\{(5-1)(3-1)\}$
V x Row(F)	20	$\sigma_\varepsilon^2 + 3\sigma_{vr}^2$

Split Plot Model

- What's so special about the split plot model?
  - ⇨ Allows one to model correlated data in a univariate model.
  - ⇨ Relatively easy to fit.
  - ⇨ Model assumes
    - ✓ Balance design,
    - ✓ exchangeable correlation for the whole plot EU.

Repeated Measures Model

- Example 4 – An experimenter is interested in vigilance performance. S/he desires to evaluate the relative effectiveness of two modes of signal presentation: an auditory signal and a visual signal . The second treatment corresponding to four successive two-hour monitoring periods. Eight graduate students were randomly assigned to one of the two modes of presentation. Response latency scores were recorded on each participant at each successive 2-hour period.

- Whole plot factor:
  - ⇨ Signal presentation

- Subplot factor:
  - ⇨ Time (in 2-hour increments)

- Problems?

Repeated Measures Model

- Subjects are randomized to signal presentation, *S*, and then receive four levels of the *time* treatment, *T*.

Model: $Y_{ijk} = \mu + S_i + bl_{k(i)}$ } *whole plot part*
$+ T_j + (ST)_{ij} + \varepsilon_{ijk}$ } *sub plot part*

- What if the within subject correlation is not exchangeable?
  - ⇨ Corrected F-statistics (Huyhn and Feldt, 1976).
  - ⇨ Mixed model estimation (Harville, 1977).
  - ⇨ Generalized Estimating Equations (Zeger & Liang, 1986)