Disclaimer: *These notes have not been subjected to the usual scrutiny reserved for formal publications. They may be distributed outside this class only with the permission of the Lecturer.*

Prelude

"Everything transitory is but an image."

— Faust: First Part

"PLOTINUS (A.D. 204-270), the founder of Neoplatonism, is the last of the great philosophers of antiquity. His life is almost coextensive with one of the most disastrous periods in Roman history. ... Of all this there is no mention in the works of Plotinus. He turned aside from the spectacle of ruin and misery in the actual world, to contemplate an eternal world of goodness and beauty. In this he was in harmony with all the most serious men of his age. To all of them, Christians and pagans alike, the world of practical affairs seemed to offer no hope, and only the Other World seemed worthy of allegiance. To the Christian, the Other World was the Kingdom of Heaven, to be enjoyed after death; to the Platonist, it was the eternal world of ideas, the real world as opposed to that of illusory appearance."

— "The History of Western Philosophy", by Bertrand Russel

"In the temple of science are many mansions, and various indeed are they that dwell therein and the motives that have led them thither. ... Now let us have another look at those who have found favor with the angel. Most of them are somewhat odd, uncommunicative, solitary fellows, really less like each other, in spite of these common characteristics, than the hosts of the rejected. What has brought them to the temple? ... to escape from personal life into the world of objective perception and thought; ... to escape from his noisy, cramped surroundings into the silence of high mountains; ... to make for himself in the fashion that suits him best a simplified and intelligible picture of the world."

— "Principles of Research", by Albert Einstein

(recently suggested by Professor Song-Chun Zhu)

What will the course be about?

This is a 10-week course focused on introducing advanced mathematical tools for statisticians, and more generally, researchers who work on stochastic models and are interested in understanding the driving forces beneath them.

The course centers around three concepts: (1) consistency and rate of convergence; (2) limiting distribution; and (3) optimality, some core insights that were formalized in 1950's-1960's using the language of large sample theory, a.k.a., asymptotics.

For this, we will largely follow the history, understand what statisticians were facing back to 1970's and 80's (Glory to robust statistics, semiparameric regression, and bootstrap!), appreciate step by step the development of the so-called empirical processes (EP) theory, and lastly, use them to solve some new problems.

Overview

This course is separated to three parts. The first part introduces

(1) the universality theory (VC classes, BUEI classes, VC stability, VC-hull, VC-major, uniform entropy, and finally, the celebrated Dudley-Pollard-Koltchinskii universality theorem).

The second part introduces

(2) Glivenko-Cantelli and Donsker classes and their preservations, M- and Z-estimation theory, functional delta method, and bootstrap theory (in both Peter Hall and Donsker senses).

The third part (if time permits) introduces

(3) rate and constant optimality in statistical inference, where the general tools for deriving minimax and semiparametric efficiency lower bounds will be detailed.

There shall be, accordingly, four chapters:

Chapter 1: Big Picture, where we shall introduce the basic notion of P-GC and P-Donsker, and several motivating examples.

Chapter 2: Uniform Entropy, where we shall provide the elegant universality approaches to proving P-GC and P-Donsker using VC dimension and preservation results.

Chapter 3: Statistical Application, where M- and Z-estimation theory, functional delta method, and bootstrap theory will be covered.

Chapter 4: Optimality, where we shall give the general framework for proving a statistical procedure's optimality.

References

The course is not built on any specific textbook, but an extraction and combination of the following ones:

[GN2015]. Evarist Giné and Richard Nickl (2015), Mathematical Foundations of Infinite-Dimensional Statistical Models;

[**K2008**]. Michael Kosorok (2008), Introduction to Empirical Processes and Semiparametric Inference;

[**P1990**]. David Pollard (1990), Empirical Processes: Theory and Applications;

[**T2008**]. Alexandre Tsybakov (2008), Introduction to Nonparametric Estimation;

[**V2000**]. Aad van der Vaart (2000), Asymptotic Statistics;

[**VW1996**]. Aad van der Vaart and Jon Wellner (1996), Weak Convergence and Empirical Processes: With Applications to Statistics;

[**W2016**]. Jon Wellner (2016), Lecture Notes on STAT581-583.

Prerequisites

This course is appropriate for a graduate student of a probability/statistics/econometrics/machine learning background, and requires a certain level of mathematical maturity. Please do not hesitate to approach the instructor if you have any concern.

Evaluation

3 homework assignments (50%), a midterm exam (20%), and a final project (30%).

Miscellanea

Instructor: Fang Han (fanghan@uw.edu)

Lectures: 10:30AM-11:20AM (M&W&F)

Office hour: Check the website