Review Handout 1 – first draft

Math/Stat 491: Introduction to Stochastic Processes

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Part 1: Terminology and Definitions

1. sigma field
2. probability measure
3. probability space
4. random variable
5. independent events $A_1, \ldots, A_n$; independent random variables $X_1, \ldots, X_n$.
6. stochastic process (sample path of a stochastic process)
7. Borel function
8. characteristic function of a random variable; joint characteristic function of random variables $X_1, \ldots, X_n$.
9. conditional expectation (in terms of fundamental identity)
10. martingale, sub-martingale, super martingale
11. stopping time
12. predictable process
13. random walk

Part 2: Results and theorems

1. Properties of conditional expectation
2. Variance decomposition: $Var(Y) = E\{Var(Y|X)\} + Var\{E(Y|X)\}$.
3. martingales connected with sums of independent random variables

4. martingales connected with products:
   (products of independent mean 1 random variables; exponential/mgf martingales; likelihood ratio martingales)

5. Doob's optional sampling theorem

6. Expected number of visits to origin for random walk.

7. Doob's decomposition of a submartingale.

8. Jensen's inequality for $Eg(X)$ with g convex.

9. Jensen's inequality for conditional expectation $E\{g(Y)|X\}$ with g convex.

10. Stirling's formula

Part 3: Facts and calculations to know:

1. Binomial and Bernoulli process facts.

2. Poisson process facts.

3. Gaussian $N(\mu, \sigma^2)$ distribution.

4. bivariate Gaussian distribution $N_2(\mu, \Sigma)$ distribution with

$$\Sigma = \begin{pmatrix} \sigma^2 & \rho\sigma\tau \\ \rho\sigma\tau & \tau^2 \end{pmatrix}.$$

   Marginal and conditional distributions of bivariate Gaussian distribution.

5. Stirling's formula

6. Newton's binomial formula and Newton's series.