Statistics 492, Problem Set 5

Wellner; 2/18/14

Reading: Klebaner, chapter 5, pages 123-149,
Klebaner, chapter 6, pages 151-167.

Due: Tuesday, 25 February 2014.

1. Let X denote an Itô process. Obtain the formula

$$[X, X](t) = X^2(t) - X^2(0) - 2\int_0^t X(s)dX(s)$$

for the quadratic variation of X by applying Itô's formula to $X^2(t)$.

2. Klebaner, Exercise 4.7, page 121. Suppose that $X(t)$ has a stochastic differential with $\mu(x) = bx + c$ and $\sigma^2(x) = 4x$. Assuming $X(t) \geq 0$, find the stochastic differential for the process $Y(t) = \sqrt{X(t)}$.

3. Klebaner, Exercise 4.9, page 121. Suppose that $X(t)$ has a stochastic differential with $\mu(x) = cx$ and $\sigma^2(x) = x^a$, $c > 0$. Let $Y(t) = X(t)^b$. What choice of b will give a constant diffusion coefficient for Y?

4. **Optional bonus problem:** Klebaner, Exercise 4.8, page 121. A process $X(t)$ on $(0, 1)$ has a stochastic differential with $\sigma(x) = x(1-x)$. Assuming $0 < X(t) < 1$, show that the process defined by $Y(t) = \log(X(t)/(1 - X(t))$ has a constant diffusion coefficient.

5. **Optional bonus problem:** Klebaner, Exercise 4.10, page 121. Let $X(t) = tB(t)$ and $Y(t) = e^{B(t)}$. Find $d(X(t)/Y(t))$.