Statistics 491, Problem Set 2

Wellner; 10/2/13

Reading: Ross; Chapter 3, pages 97-189
Durrett; Appendix A, pages 241-257

Due: Wednesday, October 9, 2013.

1. Ross, Chapter 2, problem 17, page 88.
2. Ross, Chapter 2, problem 74, page 94.
3. Ross, Chapter 2, problem 77, page 95.
4. Ross, Chapter 3, problem 56, page 182.
5. Ross, Chapter 3, problem 81, page 186.
6. Suppose that $N \sim \text{Poisson}(\lambda)$ and $(Y|N = n) \sim \text{Binomial}(n, p)$. Show that $Y \sim \text{Poisson}(\lambda p)$.
7. Suppose that $(X|N = n) \sim \text{Binomial}(n, p)$ and that N is itself Binomially distributed with parameters q and m.
(a) Show analytically that the marginal distribution of X is $\text{Binomial}(m, pq)$.
(b) Give a probabilistic argument for this result.
8. Let $X(t) = At + B$ where A and B are independent $N(0, \sigma^2)$. Find the expected value of the following random variables.
(i) $Y_1 = \max_{0 \le t \le 1} X(t)$; (ii) $Y_2 = \max_{0 \le t \le 1} |X(t)|$; (iii) $Y_3 = \int_0^1 X(t)dt$; (iv) $Y_4 = \int_0^1 X^2(t)dt$.
Hint: For all four parts, and especially for (i) and (ii), draw sample paths of the process $\{X(t) : 0 \le t \le 1\}$ and express Y_j carefully in terms of A and B.
9. The number of accidents occurring in a factory in a week is a random variable with mean μ and variance σ^2. The number of individuals injured in different accidents are independently distributed each with mean ν and variance τ^2. Determine the mean and variance of the number of individuals injured in a week.
10. Let $X_1, X_2, \ldots, X_n, X_{n+1}, \ldots, X_{m+n}$ be independent identically distributed random variables. Let $V = \max\{X_1, \ldots, X_n\}$. Find: (i) $P(X_{n+1} \ge V)$. (ii) $P(\min\{X_{n+1}, \ldots, X_{n+m}\} \ge V)$.

11. **Optional bonus problem:** Let X be a non-negative random variable with (cumulative) distribution function $F(x) = F_X(x) = P(X \leq x)$.
(a) Show that
$$E(X) = \int_0^\infty (1 - F(x))dx.$$
(b) Extend the formula you derived in (a) to $E(X^r)$ for any $r > 0$.
(c) Use the result of (a) to compute the mean of $X \sim \text{Exponential}(\lambda)$; that is, $1 - F_X(x) = \exp(-\lambda x)$, $x > 0$.