Haar function construction of Brownian bridge and Brownian motion

Wellner; 10/30/2008

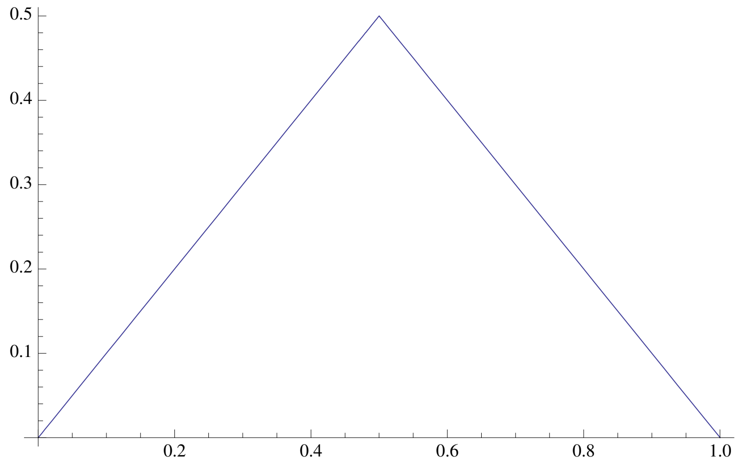

Figure 1: The Schauder function h_{00}.

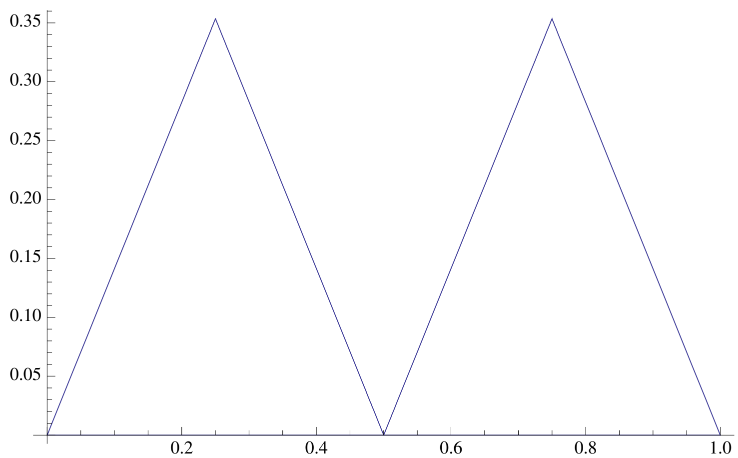

Figure 2: The Schauder functions h_{10} and h_{11}.

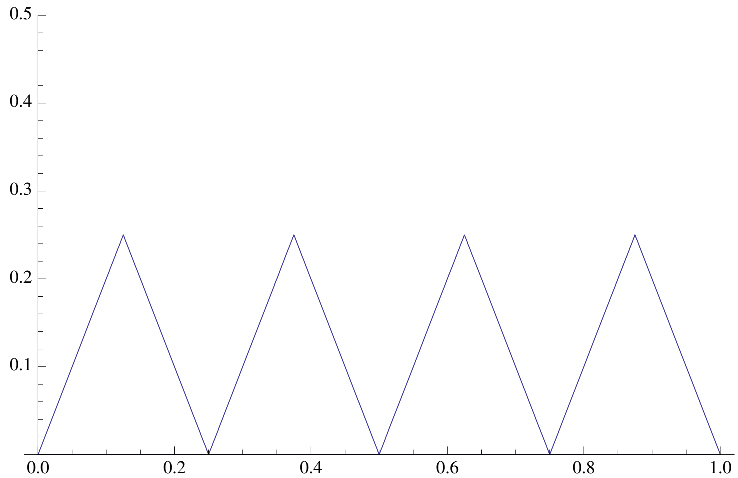

Figure 3: The Schauder functions h_{20}, h_{21}, h_{22}, h_{23}.

Here is the Mathematica code that produced the plots for the case $m = 8$.

```
Needs["Histograms`"]
ndist = NormalDistribution[0, 1]
m = 8
Z = Table[RandomReal[ndist, 2^n], {n, 0, m}]
Histogram[Flatten[Z]]

h[t_] := t /; 0 <= t <= 1/2
h[t_] := 1 - t /; 1/2 < t <= 1
h[t_] := 0 /; t < 0 || t > 1
h1[t_, n_, j_] := 2^(-n/2)*h[2^n *t - j]
V0[t_] := Z[[1, 1]]*h[t]
V[t_, n_] := Sum[Z[[n + 1, j + 1]]*h1[t, n, j], {j, 0, 2^n - 1}]
U[t_, m_] := Sum[V[t, n], {n, 1, m}] + V0[t]
P1 = Plot[h[t], {t, 0, 1}]
P2 = Plot[h1[t, 1, 0], {t, 0, 1}]
P3 = Plot[h1[t, 1, 1], {t, 0, 1}]
Show[P2, P3]
P4 = Plot[h1[t, 2, 0], {t, 0, 1}, PlotRange -> {0, .5}]
P5 = Plot[h1[t, 2, 1], {t, 0, 1}, PlotRange -> {0, .5}]
```

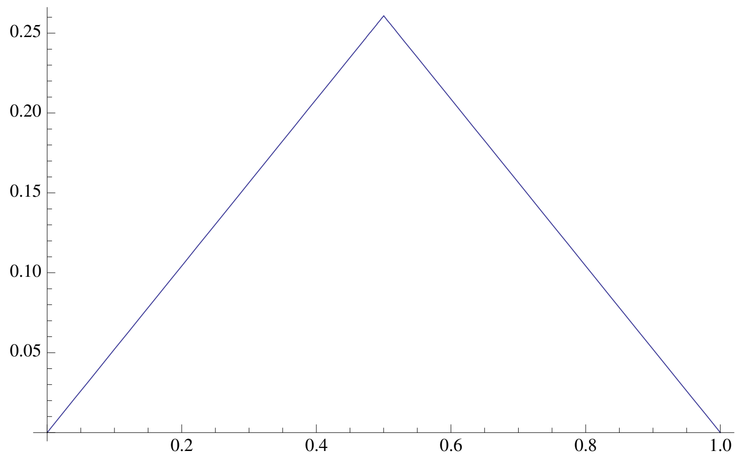

Figure 4: A sample path of the random function $V_0(t)$.

```
P6 = Plot[h1[t, 2, 2], {t, 0, 1}, PlotRange -> {0, .5}]
P7 = Plot[h1[t, 2, 3], {t, 0, 1}, PlotRange -> {0, .5}]
Show[P4, P5, P6, P7]
Plot[V0[t], {t, 0, 1}]
Plot[V[t, 1], {t, 0, 1}, PlotStyle -> {Thickness[1/200], RGBColor[0.0, 0.0, 1.00],
Plot[V[t, 2], {t, 0, 1}, PlotStyle -> {Thickness[1/200], RGBColor[0.1, 0.00, .90],
Plot[V[t, 3], {t, 0, 1}, PlotStyle -> {Thickness[1/200], RGBColor[0.2, 0.00, .80],
Plot[V[t, 4], {t, 0, 1}, PlotStyle -> {Thickness[1/200], RGBColor[0.3, 0.00, .70],
Plot[V[t, 5], {t, 0, 1}, PlotStyle -> {Thickness[1/200], RGBColor[0.4, 0.00, .60],
Plot[V[t, 6], {t, 0, 1}, PlotStyle -> {Thickness[1/200], RGBColor[0.5, 0.00, .50],
Plot[V[t, 7], {t, 0, 1}, PlotStyle -> {Thickness[1/200], RGBColor[0.6, 0.00, .40],
Plot[V[t, 8], {t, 0, 1}, PlotStyle -> {Thickness[1/200], RGBColor[0.7, 0.00, .30],

Plot[U[t, 8], {t, 0, 1}, PlotStyle -> {Thickness[1/200], RGBColor[0.0, 0.50, .50],
```

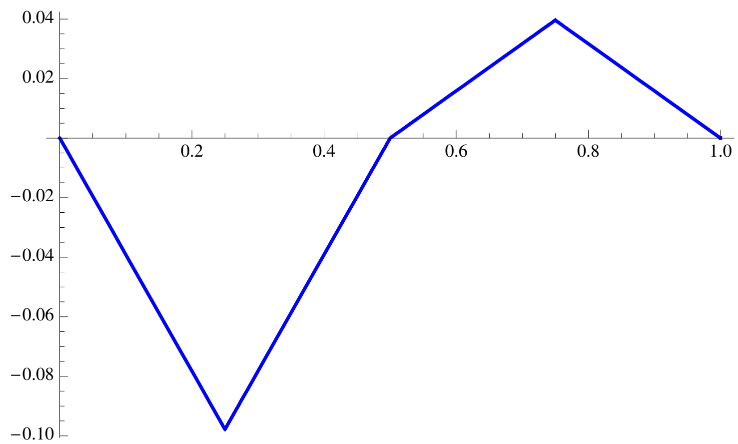

Figure 5: A sample path of the random function $V_1(t)$.

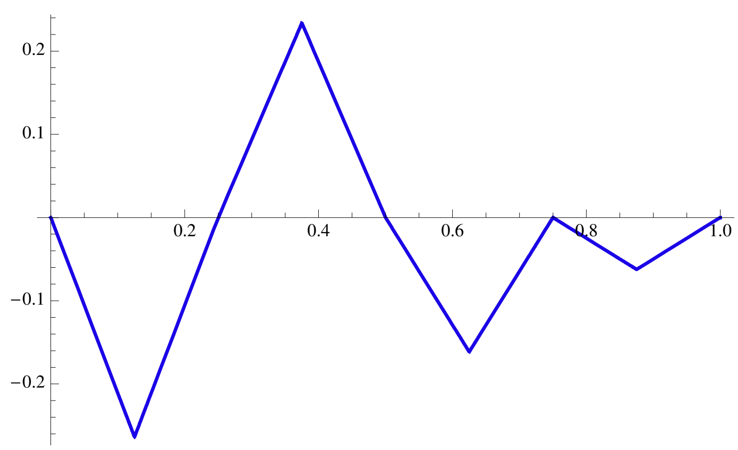

Figure 6: A sample path of the random function $V_2(t)$.

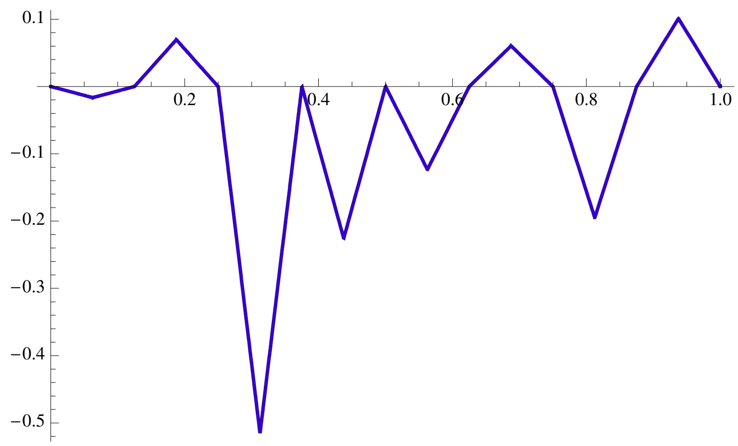

Figure 7: A sample path of the random function $V_3(t)$.

Figure 8: A sample path of the random function $V_4(t)$.

Figure 9: A sample path of the random function $V_5(t)$.

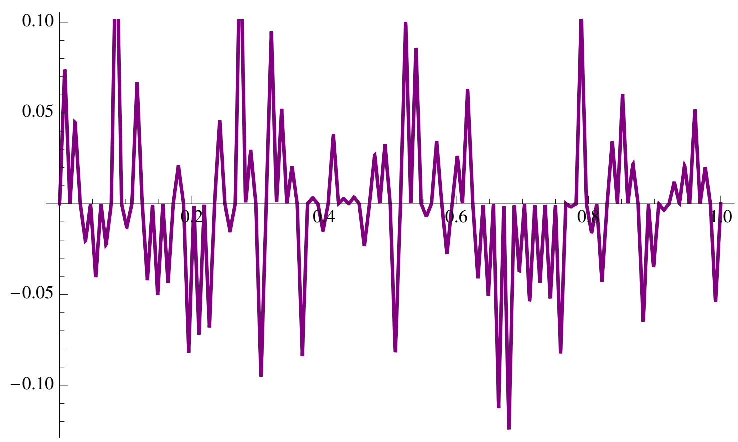

Figure 10: A sample path of the random function $V_6(t)$.

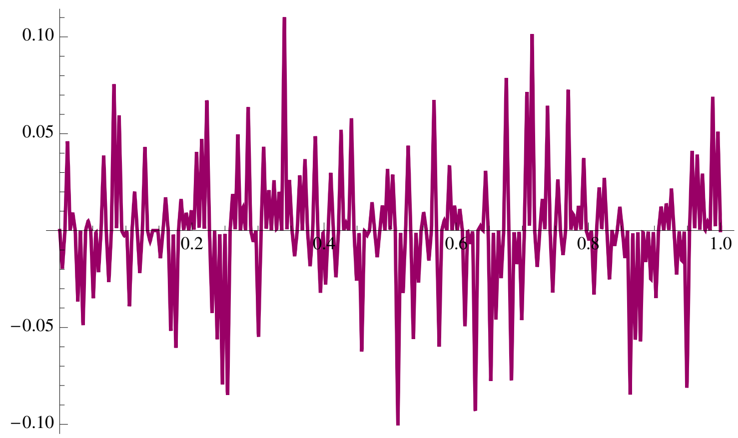

Figure 11: A sample path of the random function $V_7(t)$.

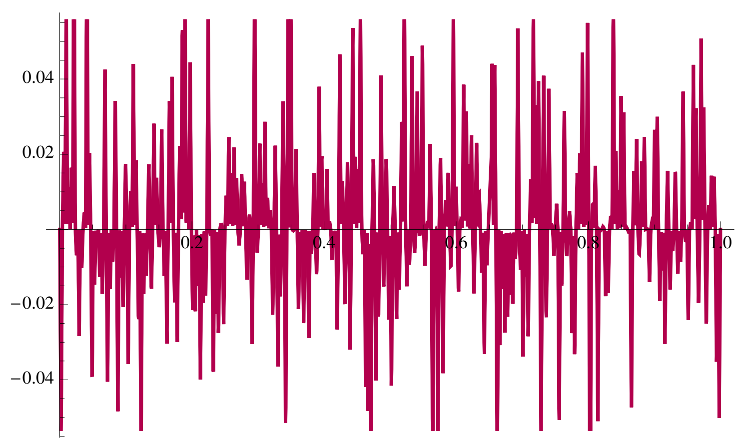

Figure 12: A sample path of the random function $V_8(t)$.

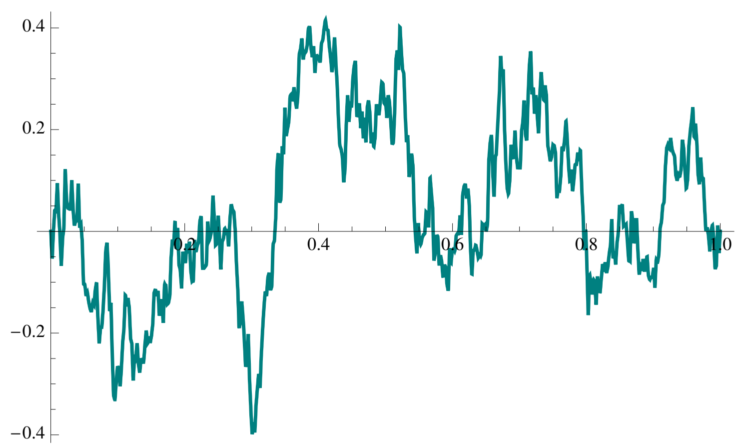

Figure 13: A sample path of the random function $U_8(t)$.