Statistics 581, Handout

Local Power Approximations

Pearson Chi-square; Wellner and Ferguson compared

Wellner; 10/17/08

Ferguson (1996), pages 64-65, gives a treatment of the limiting distribution of the Pearson chi-square statistic under alternatives which is, rigorous, but somewhat unnatural. Here is a summary of the differences between my treatment of this material in class and Ferguson's treatment, with some effort to understand why Ferguson made some particular choices.

Table 1:

	Wellner	Ferguson
null hypothesis	$\underline{p}_0$, fixed	$\underline{p}_{0,n}$ depends on n
alternative hypothesis	$\underline{p}_n = \underline{p}_0 + \underline{c} n^{-1/2}$	$\underline{p}$, fixed
shift vector	$\underline{c} = \sqrt{n}(\underline{p}_n - \underline{p}_0)$	$\underline{\delta} = \sqrt{n}(\underline{p} - \underline{p}_{0,n})$
noncentrality parameter	$\delta = \sum_{j=1}^k \frac{c_j^2}{p_{0,j}}$	$\lambda = \sum_{j=1}^k \frac{\delta_j^2}{p_j}$
CLT	Cramér - Wold device + Liapunov CLT in $\mathbb{R}$	multivariate CLT at fixed $\underline{p}$
Goal	natural treatment that carries over to other problems	use multivariate CLT avoid Cramér - Wold device and Liapunov CLT

In summary, it seems to me that Ferguson made his treatment fit within the scope of the multivariate central limit theorem (for i.i.d. multivariate summands data), and therefore was forced into the somewhat unnatural position of letting the null hypothesis depend on the sample size n. He did this in part to avoid the Cramér - Wold device (which is not included in his book) together with an application of either the Liapunov CLT or Lindeberg-Feller CLT.