Statistics 581, Plots for Weibull Example

Wellner; 11/20/2002, 11/03/2014

Example 1. Here $\mathcal{P} = \{P_\theta : \theta \in R^{+2}\}$ is the Weibull model and $\nu(P_\theta) = E_\theta(X) = \alpha\Gamma(1 + 1/\beta)$. The following plots show the information bounds $I^{-1}(P_\theta|\nu, \mathcal{P})$ and $I^{-1}(P_\theta|\nu, \mathcal{P}_0)$ for the submodel $\mathcal{P}_0$ when $\beta = \beta_0$ is known, together with the corresponding asymptotic variance $Var_\theta(X)$ of the non-parametric estimator of ν, namely $\overline{X}_n$.

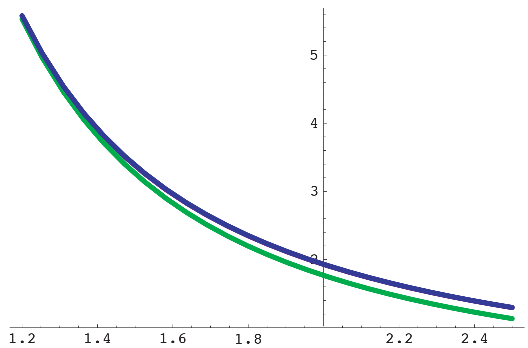

Figure 1: Information bounds, $\alpha = 3$, $1.2 \le \beta \le 2.5$; green $= I^{-1}(P|\nu, \mathcal{P}_0)$, purple $= I^{-1}(P|\nu, \mathcal{P})$, blue $= Var_\theta(X)$; (purple coincides with blue so not visible!)

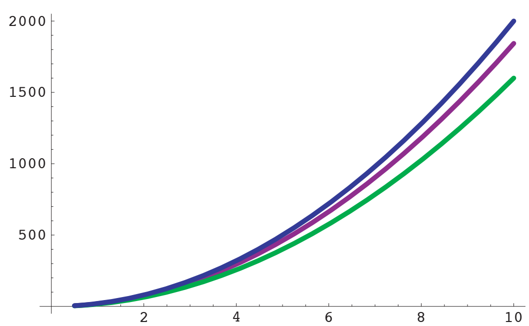

Figure 2: Information bounds, $.5 \le \alpha \le 10$, $\beta = .5$; green $= I^{-1}(P|\nu, \mathcal{P}_0)$, purple $= I^{-1}(P|\nu, \mathcal{P})$, blue $= Var_\theta(X)$

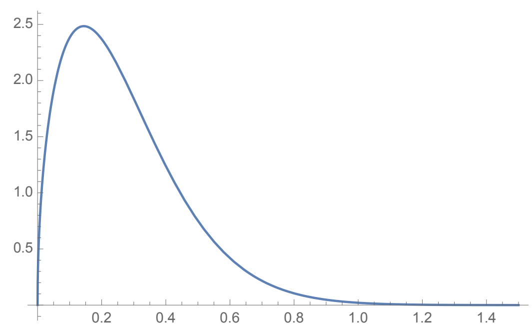

Figure 3: Weibull density $\alpha = .3$, $\beta = 1.5$

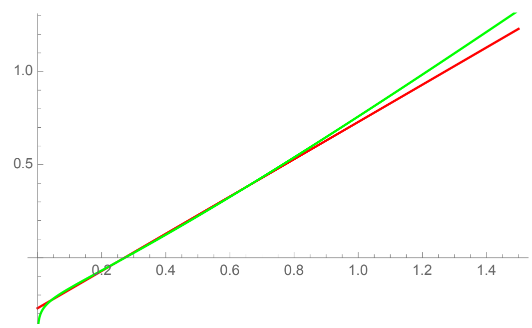

Figure 4: Two Influence functions: sample mean and efficient, Weibull model $\alpha = .3$, $\beta = 1.5$

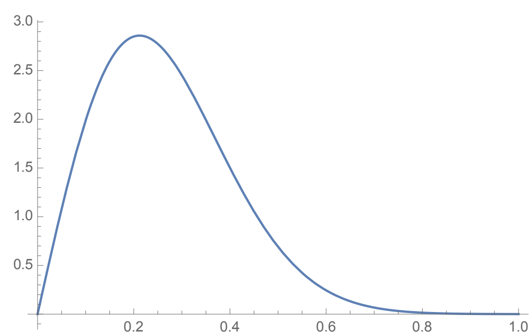

Figure 5: Weibull density $\alpha = .3$, $\beta = 2$

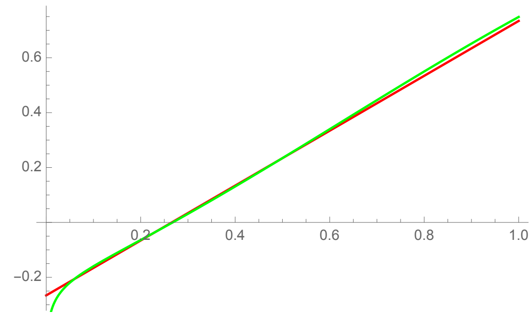

Figure 6: Two Influence functions: sample mean and efficient, Weibull model $\alpha = .3$, $\beta = 2$

Example 2. Here $\mathcal{P} = \{P_\theta : \theta \in R^{+2}\}$ is the Weibull model and $\nu(P_\theta) = P_\theta(X \ge x_0) = \exp(-(x_0/\alpha)^\beta)$. The following plots show the information bounds $I^{-1}(P_\theta|\nu, \mathcal{P})$ and $I^{-1}(P_\theta|\nu, \mathcal{P}_0)$ for the submodel $\mathcal{P}_0$ when $\beta = \beta_0$ is known, together with the corresponding asymptotic variance $Var_\theta(X)$ of the nonparametric estimator of ν, namely $\overline{X}_n$.

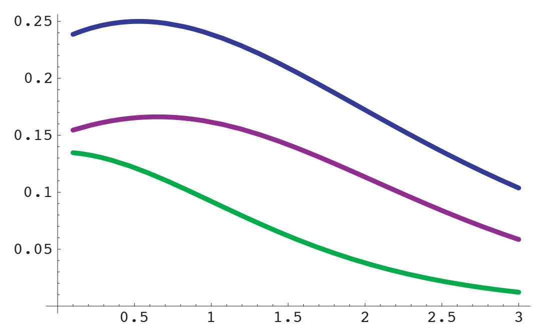

Figure 7: Information bounds for $\nu(P_\theta) = P_\theta(X \ge x_0)$, $\alpha = 1$, $x_0 = .5$, $.1 \le \beta \le 3$: green $= I^{-1}(P|\nu, \mathcal{P}_0)$, purple $= I^{-1}(P|\nu, \mathcal{P})$, blue $= nVar(\mathbb{F}_n(x_0) = P_\theta(X \ge x_0)(1 - P_\theta(X \ge x_0))$

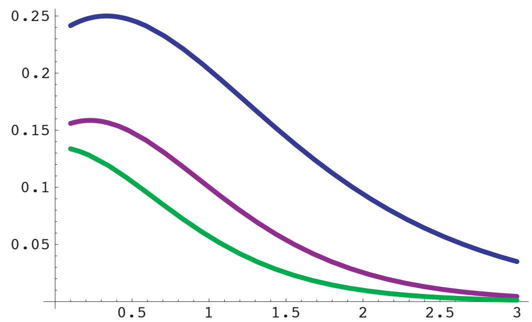

Figure 8: Information bounds for $\nu(P_\theta) = P_\theta(X \geq x_0)$, $\alpha = 3$, $x_0 = 1$, $.1 \leq \beta \leq 3.0$; green $= I^{-1}(P|\nu, \mathcal{P}_0)$, purple $= I^{-1}(P|\nu, \mathcal{P})$, blue $= nVar(\mathbb{F}_n(x_0)) = P_\theta(X \geq x_0)(1 - P_\theta(X \geq x_0))$

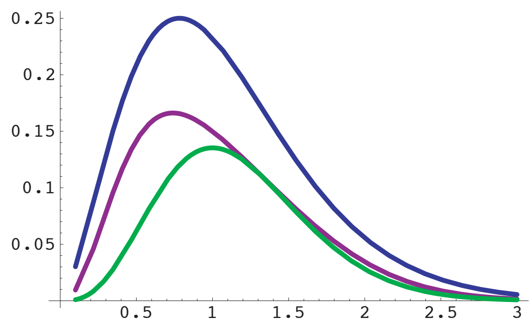

Figure 9: Information bounds for $\nu(P_\theta) = P_\theta(X \geq x_0)$, $\alpha = 1$, $\beta = 1.5$, $.1 \leq x_0 \leq 3.0$; green $= I^{-1}(P|\nu, \mathcal{P}_0)$, purple $= I^{-1}(P|\nu, \mathcal{P})$, blue $= nVar(\mathbb{F}_n(x_0)) = P_\theta(X \geq x_0)(1 - P_\theta(X \geq x_0))$

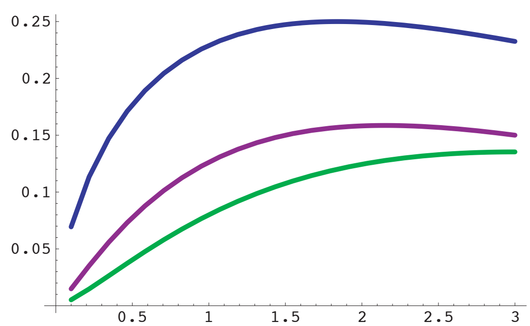

Figure 10: Information bounds for $\nu(P_\theta) = P_\theta(X \geq x_0)$, $\alpha = 3$, $\beta = .75$, $.1 \leq x_0 \leq 3.0$; green $= I^{-1}(P|\nu, \mathcal{P}_0)$, purple $= I^{-1}(P|\nu, \mathcal{P})$, blue $= nVar(\mathbb{F}_n(x_0)) = P_\theta(X \geq x_0)(1 - P_\theta(X \geq x_0))$

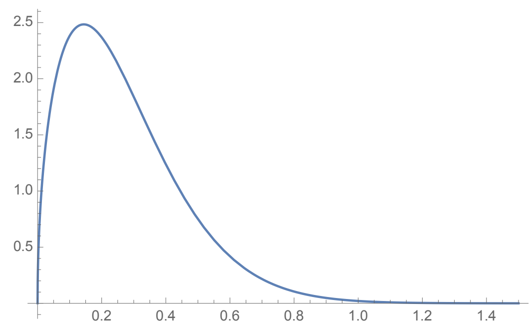

Figure 11: Weibull density $\alpha = .3$, $\beta = 1.5$

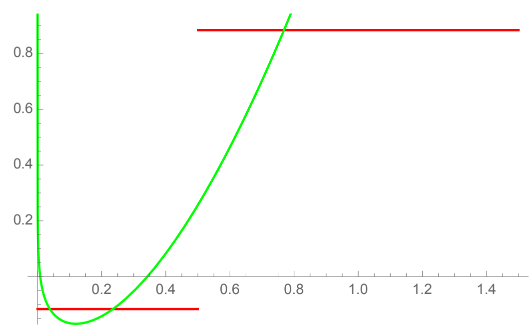

Figure 12: Two Influence functions: empirical and efficient, Weibull model $\alpha = .3$, $\beta = 1.5$

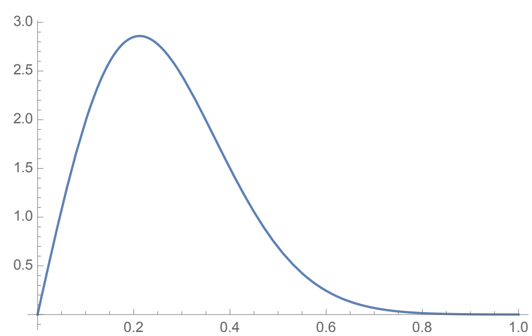

Figure 13: Weibull density $\alpha = .3$, $\beta = 2$

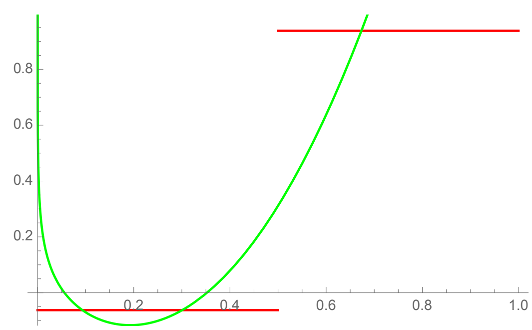

Figure 14: Two Influence functions: empirical and efficient, Weibull model $\alpha = .3$, $\beta = 2$