

**STATISTICS 581:**  
**Advanced Theory of Statistical Inference**  
**Fall, 2010**

## **Tentative Outline**

**1. Chapter 0: Measures, Integration, Convergence.**

- Section 0.1: Measures.
- Section 0.2: Measurable functions and integration.
- Section 0.3: Absolute continuity, Radon-Nikodym theorem, Fubini's theorem.

**2. Chapter 1: Special Distributions.**

- Special distributions.
- Convolution and related formulas.
- The multivariate normal distribution.
- Integration by parts formulas.

**3. Chapter 2: Some basic large sample theory.**

- Modes of convergence.
- Classical limit theorems.
- Replacing  $\rightarrow_d$  by  $\rightarrow_{a.s.}$ : Skorokhod's theorem.
- Empirical distributions, empirical measures, empirical processes, and Brownian bridge.
- The partial sum process and Brownian motion.
- Sample quantiles and asymptotic linearity of estimators.

4. **Chapter 3: Lower bounds for estimation.**

- Introduction and examples.
- Cramér - Rao bounds for parametric models.
- Regular estimates; superefficiency; LAN and Le Cam's three lemmas.
- Hajek's convolution theorem and local asymptotic minimax (LAM) theorem.

5. **Chapter 4: Efficient Likelihood Estimation and Related Tests.**

- Maximum likelihood and efficient likelihood estimation.
- Likelihood ratio, Wald, and Rao (or score) tests.
- Examples.
- Consistency of maximum likelihood estimates.
- The EM algorithm and related methods.
- Nonparametric MLE.
- Limit theory for the statistical agnostic:  $P \notin \mathcal{P}$ .