CORRECTIONS AND CHANGES FOR:

EMPIRICAL PROCESSES
WITH APPLICATIONS TO STATISTICS
(Wiley, 1986)

by

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1. Introduction

Since publication of our book

Empirical Processes with Applications to Statistics

in 1986, we have become aware of several mathematical errors and a number of typographical and other minor errors. Our purpose here is to give corrections of the errors of which we are currently aware. Although we would now do many things differently, we have not made any of that type of revisions here.

We encourage readers finding further errors to let us know of them.

We owe thanks to the following friends, colleagues, reviewers, and users of the book for telling us about errors, difficulties, and shortcomings: N. H. Bingham, M. Csorgo, S. Csorgo, Kjell Doksum, Peter Gaenssler, Richard Gill, Paul Janssen, Keith Knight, Ivan Mizera, D. W. Muller, David Pollard, Peter Sasieni, and Ben Winter.

We owe special thanks to Peter Gaenssler for providing us with a long list of typographical errors which provided the starting point for section 3 here.

The corrections of chapters 7, 21, and 23 given in section 2 were aided by discussions and correspondence with Richard Gill and Ben Winter (in the case of chapter 7), I. Bomze and E. Reschenhofer, and W.D. Kaigh (in the case of chapter 21), and Keith Knight (in the case of section 23.3).

A list of reviews is given in section 6.

2. Major changes and revisions

Here we give substantial corrections and revisions of section 7.3 (pages 304 - 306) and section 23.3 (pages 767 - 771).

2.1. Revision and correction of section 7.3.

The last two lines (pp. 305, -7 and -6) of the proof of (1) of theorem 1, page 304 are false. Hence there are also difficulties in the cases (i) - (v) on pages 305-306. The follow revision of section 7.3 should replace that entire section. As indicated in the following text, these results are due to Peterson (1977), Gill (1981), and Wang (1987).

We owe thanks to Richard Gill and Ben Winter for pointing out these difficulties and for correspondence concerning their solution.

Section 7.3, pages 304 - 306, should be replaced by the following:

3. CONSISTENCY OF $\hat{\Lambda}_n$ AND $I\!\hat{F}_n$

In this section we use the representations of Theorem 7.2.1 and continuity of the product integral map E which takes Λ to F (see section B.6 and especially example B.6.1, page 898) to establish weak and strong consistency of $\hat{\Lambda}_n$ and $I\!\hat{F}_n$. Our first result gives strong consistency of both $\hat{\mathbb{A}}_n$ and $I\!\hat{F}_n$ on any interval $[0, \theta]$ with $\theta < \tau \equiv \tau_H \equiv H^{-1}(1)$.

Theorem 1. Suppose that F and G are arbitrary df's on $[0, \infty)$. Recall $\tau \equiv \tau_H \equiv H^{-1}(1)$ where $1 - H \equiv (1-F)(1-G)$. Then for any fixed $\theta < \tau$

$$\text{(1)} \qquad ||I\!\hat{F}_n - F||_0^\theta \rightarrow_{a.s.} 0 \qquad \text{as} \quad n \rightarrow \infty$$

and

$$\text{(2)} \qquad ||\hat{\mathbb{A}}_n - \Lambda||_0^\theta \rightarrow_{a.s.} 0 \qquad \text{as} \quad n \rightarrow \infty .$$

The following theorems strengthen (1) of theorem 1 in different directions.

Theorem 2. Suppose F and G are df's on $[0, \infty)$ with $\tau \equiv \tau_H \equiv H^{-1}(1)$ satisfying either $H(\tau-) < 1$ or $F(\tau-) = 1$. Then

$$\text{(3)} \qquad \sup_{0 \le t \le \tau} |I\!\hat{F}_n(t) - F(t)| = ||I\!\hat{F}_n - F||_0^\tau \rightarrow_{a.s.} 0 \qquad \text{as} \quad n \rightarrow \infty ,$$

and, with $T \equiv Z_{n:n}$,

$$\text{(4)} \qquad \sup_{0 \le t \le T} |I\!\hat{F}_n(t) - F(t)| = ||I\!\hat{F}_n - F||_0^T \rightarrow_{a.s.} 0 \qquad \text{as} \quad n \rightarrow \infty .$$

The following theorem is more satisfactory since F and G are completely arbitrary; the price is that the consistency is in probability (and the supremum in (5) is just over the interval $[0, \tau)$).

Theorem 3. (Wang). Suppose that F and G are completely arbitrary. Then

$$\sup_{0 \le t < \tau} |\hat{\mathbb{F}}_n(t) - F(t)| \to_p 0 \quad \text{as } n \to \infty , \tag{5}$$

and, with $T \equiv Z_{n:n}$,

$$\sup_{0 \le t \le T} |\hat{\mathbb{F}}_n(t) - F(t)| \to_p 0 \quad \text{as } n \to \infty . \tag{6}$$

Open Question 1. Does Wang's theorem 3 continue to hold with $\to_p$ replaced by $\to_{a.s.}$? (The hard case not covered by theorem 2 is $F(\tau-) < 1$, $G(\tau-) = 1$.)

Recall that for an arbitrary hazard function Λ (of a df F on R^+), the (product integral) or exponential map $(-\Lambda)$ recovers $1 - F$:

$$\begin{aligned} 1 - F(t) &= (-\Lambda)(t) \equiv \prod_{0 \le s \le t} (1 - d\Lambda) \\ &= exp(-\Lambda^c(t)) \prod_{0 \le s \le t} (1 - \Delta\Lambda(s)) ; \end{aligned}$$

see section B.6 and example B.6.1. Our proofs of theorems 1 - 3 will use the following basic lemma which is due to Peterson (1977), Gill (1981), and, in the present form, Wang (1987).

Lemma 1. (Continuity of the product integral map). Suppose that $\{g_n\}_{n \ge 0}$ is a sequence of nondecreasing functions on $A = [0, \tau]$ or $[0, \tau)$ satisfying $\Delta g_0 < 1$, and set $h_n \equiv (-g_n)$, $n = 0, 1, \cdots$. If

$$\sup_{t \in A} |g_n(t) - g_0(t)| \to 0 \quad \text{as } n \to \infty , \tag{7}$$

then

$$\sup_{t \in A} |h_n(t) - h_0(t)| \to 0 \quad \text{as } n \to \infty . \tag{8}$$

Proof of theorem 1. Now $||\mathbb{H}_n - H|| \to_{a.s.} 0$ by Glivenko-Cantelli, so that $||\mathbb{H}_{n-} - H_-|| \to_{a.s.} 0$ also. Thus for any fixed $t \le \theta$ we have a.s. that

$$\begin{aligned} |\hat{\Lambda}_n(t) - \Lambda(t)| \le & \int_0^t |(1 - \mathbb{H}_{n-})^{-1} - (1 - H_-)^{-1}| \, d\mathbb{H}_n^1 \\ & + \left| \int_0^t (1 - H_-)^{-1} \, d(\mathbb{H}_n^1 - H^1) \right| \end{aligned}$$

(a) $\qquad \to_{a.s.} \; 0 \; + \; 0 \; = \; 0$

by the Glivenko Cantelli theorem and $H(t-) \le H(\theta) < 1$ for the first term, and by the SLLN for the second term. Since $\hat{\mathbb{A}}_n$ and Λ are $\uparrow$, the standard argument of (3.1.83) improves (a) to (2).

But then (1) follows from (2) and continuity of the product integral map , lemma 1. □

Proof of theorem 2. First suppose $H(\tau-) < 1$. Then as in (a) of the proof of theorem 1,

$$|\hat{\mathbb{A}}_n(t) \; - \; \Lambda(t)| \; \le \; |\int_0^t \{(1 - \mathbb{H}_{n-})^{-1} \; - \; (1 - H_-)^{-1}\} \, d\mathbb{H}_n^1|$$

$$+ \; |\int_0^t (1 - H_-)^{-1} \, d(\mathbb{H}_n^1 \; - \; H^1)\,|$$

where the first term converges to zero uniformly on $[0\,,\tau]$ by the Glivenko - Cantelli theorem since $1 \; - \; H(\tau-) \; > \; 0$ and $H\#subn^1(\tau) \; \le \; 1$. Now the second term: for $0 \le t \le \tau$,

$$|\int_0^t \frac{1}{1 - H_-} \, d(\mathbb{H}_n^1 \; - \; H^1)\,|$$

$$\le \; |\frac{\mathbb{H}_n^1(t) \; - \; H^1(t)}{1 \; - \; H(t-)} \; - \; \int_0^t (\mathbb{H}_n^1(s) \; - \; H^1(s))\, d(\frac{1}{1 - H(s-)})\,|$$

$$+ \; |\frac{\Delta\mathbb{H}_n^1(\tau) \; - \; \Delta H^1(\tau)}{1 - H(\tau-)}|$$

$$\le \; 2\,\frac{||\mathbb{H}_n^1 \; - \; H^1||_0^\tau}{1 \; - \; H(\tau-)} \; + \; |\frac{\Delta\mathbb{H}_n^1(\tau) \; - \; \Delta H^1(\tau)}{1 - H(\tau-)}|$$

$$\to_{a.s.} \; 0 \; + \; 0 \; = \; 0 \, ,$$

so the second term converges to zero a.s. uniformly in $t \in [0\,,\tau]$. Hence

(a) $$||\mathbb{A}_n \; - \; \Lambda||_0^\tau \; \equiv \; \sup_{0 \le t \le \tau} |\hat{\mathbb{A}}_n(t) \; - \; \Lambda(t)| \; \to_{a.s.} \; 0 \, .$$

If $\Delta\Lambda(\tau) < 1$, then (3) follows from lemma 1. If $\Delta\Lambda(\tau) = 1$ (so $F(\tau) = 1$), then lemma 1 and (a) imply that

$$\sup_{0 \le t < \tau} |\hat{I\!F}_n(t) \; - \; F(t)| \; \to_{a.s.} \; 0$$

and

$$0 \; \le \; 1 \; - \; \hat{I\!F}_n(\tau) \; \le \; 1 \; - \; \Delta\hat{\mathbb{A}}_n(\tau)$$

$$\to_{a.s.} \; 1 \; - \; \Delta\Lambda(\tau) \; = \; 0 \; = \; 1 \; - \; F(\tau) \, ,$$

so again (3) holds.

Now suppose that $F(\tau-) = 1$. Given $\epsilon > 0$, choose $\theta < \tau$ such that $F(\theta) > 1 - \epsilon$. For $\theta \leq t \leq \tau$ both

$$\hat{\mathbb{F}}_n(\theta) \leq \hat{\mathbb{F}}_n(t) \leq 1$$

and

$$1 - \epsilon \leq F(\tau) \leq 1 .$$

Hence

$$\begin{aligned} \|\hat{\mathbb{F}}_n - F\|_\theta^\tau &\leq \max\{\epsilon, 1 - \hat{\mathbb{F}}_n(\theta)\} \\ \text{(b)} \qquad &\rightarrow_{a.s.} \max\{\epsilon, 1 - F(\theta)\} = \epsilon \end{aligned}$$

by (1). Since ϵ is arbitrary, (1) and (b) imply (3) in this case ($F(\tau-) = 1$) too.

Since $T \equiv Z_{n:n} \leq \tau$ a.s., (4) follows from (3). □

Proof of theorem 3. We first suppose $\theta \leq \tau$ with $F(\theta-) < 1$, and show that

(a) $$\sup_{0 \leq t < \theta} |\hat{\boldsymbol{\Lambda}}_n(t) - \Lambda(t)| \rightarrow_p 0 \quad \text{as } n \rightarrow \infty$$

and

(b) $$\sup_{0 \leq t < \theta} |\hat{\mathbb{F}}_n(t) - F(t)| \rightarrow_p 0 \quad \text{as } n \rightarrow \infty .$$

Let $\mathbb{D}_n \equiv \hat{\boldsymbol{\Lambda}}_n - \Lambda$. Then, with $T \equiv Z_{n:n}$, $\mathbb{D}_n^T \equiv \{\mathbb{D}_n(t \wedge T) : t \geq 0\}$ is a square integrable martingale with predictable variation process

(c) $$< \mathbb{D}_n^T > (t) = \int_0^{t \wedge T} \frac{1 - \Delta\Lambda(s)}{n\,(1 - \mathbb{H}_n(s-))}\, d\Lambda(s) .$$

Now

(d) $$< \mathbb{D}_n^T > (\theta-) \rightarrow_{a.s.} 0 .$$

To see this, let $\epsilon > 0$, and choose $\sigma < \theta$ so that $\Lambda(\theta-) - \Lambda(\sigma) < \epsilon$, and hence $H(\sigma) < 1$ also. Then

$$\begin{aligned} < \mathbb{D}_n^T > (\theta-) - < \mathbb{D}_n^T > (\sigma) &= \int_{(\sigma, \theta)} 1_{[T \geq s]} \frac{1 - \Delta\Lambda(s)}{n\,(1 - \mathbb{H}_n(s-))}\, d\Lambda(s) \\ &\leq \Lambda(\theta-) - \Lambda(\sigma) < \epsilon , \end{aligned}$$

and, by the Glivenko - Cantelli theorem

$$\begin{aligned} n < \mathbb{D}_n^T > (\sigma) &= \int_0^\sigma \frac{1 - \Delta\Lambda(s)}{1 - \mathbb{H}_n(s-)}\, d\Lambda(s) \\ &\rightarrow_{a.s.} \int_0^\sigma \frac{1 - \Delta\Lambda(s)}{1 - H(s-)}\, d\Lambda(s) < \infty . \end{aligned}$$

Therefore

$$< \mathbb{D}_n^T > (\sigma) \ \to_{a.s.} \ 0$$

and

$$\limsup_{n \to \infty} < \mathbb{D}_n^T > (\theta-) \ \le \ \epsilon \qquad a.\,s.$$

Since $\epsilon > 0$ is arbitrary, (d) holds.

By Lenglart's inequality B.4.1,

(e) $$\sup_{0 \le t < \theta} |\mathbb{D}_n^T(t)| \ \to_p \ 0 \qquad \text{as} \quad n \to \infty\,.$$

Since we also have (recall $T \equiv Z_{n:n}$)

$$\{\Lambda(\theta-) \ - \ \Lambda(T)\} 1_{[T < \theta]} \ \to_{a.s.} \ 0 \qquad \text{as} \quad n \to \infty$$

in view of $F(\theta-) < 1$, (a) holds.

Now (a) implies that for every subsequence $\{n'\}$ there is a further subsequence $\{n''\} \subset \{n'\}$ so that

(f) $$\sup_{t < \theta} |\hat{\mathbb{\Lambda}}_{n''}(t) \ - \ \Lambda(t)| \ \to_{a.s.} \ 0 \qquad \text{as} \quad n \to \infty\,.$$

But by continuity of given by lemma 1, it follows from (f) that

(g) $$\sup_{0 \le t < \theta} |\hat{\mathbb{F}}_{n''}(t) \ - \ F(t)| \ \to_{a.s.} \ 0\,,$$

and hence (b) holds when $F(\theta-) < 1$.

To complete the proof of (5), it remains only to consider the case $F(\tau-) = 1$. But then (5) follows from (3).

To prove (6), consider the two cases $H(\tau-) = 1$ and $H(\tau-) < 1$: If $H(\tau-) = 1$, then $T \equiv Z_{n:n} < \tau$ a.s. and hence (6) follows from (5). If $H(\tau-) < 1$, then (6) follows from (4). □

Proof of lemma 1. By (7) and $\Delta g_0 < 1$ we can assume that

(a) $$\Delta g_n \ < \ 1 \qquad \text{for} \quad n = 1, 2, \cdots.$$

Since g_n are nondecreasing, finite, and (a) holds, it is easy to verify that $h_n > 0$, $n = 0, 1, \cdots$. For $t \in A$ and $\epsilon > 0$, define (note (B.5.3))

(b) $$\underline{g}_n^\epsilon(t) \ \equiv \ g_n^c(t) \ - \ \sum_{s \le t} \log(1 \ - \ \Delta g_n(s))\, 1_{[|\Delta g_n(s)| \le \epsilon]}$$

and

(c) $$\overline{g}_n^\epsilon(t) \ \equiv \ - \sum_{s \le t} \log(1 \ - \ \Delta g_n(s))\, 1_{[|\Delta g_n(s)| > \epsilon]}$$

so that

(d) $$\underline{g}_n^{\epsilon}(t) \;+\; \overline{g}_n^{\epsilon}(t) \;=\; -\log h_n(t)\,.$$

Now $\overline{g}_n^{\epsilon}(t)$ is the sum of at most a finite number of terms. Thus by (7) for every $\epsilon > 0$ with

(e) $$\epsilon \in \{ a < 1/2 :\; \Delta g_0(t) \neq a \text{ for all } t \in A \}$$

it follows that

(f) $$\sup_{t \in A} \Big| \sum_{s \le t} \Delta g_n(s))\, 1_{[|\Delta g_n(s)| > \epsilon]} \;-\; \sum_{s \le t} \Delta g_0(s))\, 1_{[|\Delta g_0(s)| > \epsilon]} \Big| \;\to\; 0$$

as $n \to \infty$ and

(g) $$\sup_{t \in A} |\overline{g}_n^{\epsilon}(t) \;-\; \overline{g}_0^{\epsilon}(t)| \;\to\; 0 \qquad \text{as} \quad n \to \infty\,.$$

But note that

$$\begin{aligned}
&|\underline{g}_n^{\epsilon}(t) \;-\; \underline{g}_0^{\epsilon}(t)| \\
&\le \Big|\, \underline{g}_n^{\epsilon}(t) \;-\; g_n^{c}(t) \;-\; \sum_{s \le t} \Delta g_n(s))\, 1_{[|\Delta g_n(s)| \le \epsilon]} \Big| \\
&\quad + \Big|\, g_n^{c}(t) \;+\; \sum_{s \le t} \Delta g_n(s))\, 1_{[|\Delta g_n(s)| \le \epsilon]} \;-\; g_0^{c}(t) \;-\; \sum_{s \le t} \Delta g_0(s))\, 1_{[|\Delta g_0(s)| \le \epsilon]} \Big| \\
&\quad + \Big|\, \underline{g}_0^{\epsilon}(t) \;-\; g_0^{c}(t) \;-\; \sum_{s \le t} \Delta g_0(s))\, 1_{[|\Delta g_0(s)| \le \epsilon]} \Big| \\
&\le \Big| \sum_{s \le t} \{ \log(1 - \Delta g_n(s)) \;+\; \Delta g_n(s) \}\, 1_{[|\Delta g_n(s)| \le \epsilon]} \Big| \\
&\quad + \Big| \sum_{s \le t} \Delta g_n(s))\, 1_{[|\Delta g_n(s)| > \epsilon]} \;-\; \sum_{s \le t} \Delta g_0(s))\, 1_{[|\Delta g_0(s)| > \epsilon]} \Big| \\
&\quad + |g_n(t) \;-\; g_0(t)| \\
&\quad + \Big| \sum_{s \le t} \{ \log(1 - \Delta g_0(s)) \;+\; \Delta g_0(s) \}\, 1_{[|\Delta g_0(s)| \le \epsilon]} \Big| \\
&\le \epsilon\,(|g_n(t) \;+\; |g_0(t)|\,) \\
&\quad + \Big| \sum_{s \le t} \Delta g_n(s))\, 1_{[|\Delta g_n(s)| > \epsilon]} \;-\; \sum_{s \le t} \Delta g_0(s))\, 1_{[|\Delta g_0(s)| > \epsilon]} \Big|
\end{aligned}$$

(h) $$\quad + |g_n(t) \;-\; g_0(t)|\,.$$

Therefore, for every ϵ satisfying (e), (f) yields

$$\limsup_{n \to \infty} \sup_{t \in A} |\underline{g}_n^{\epsilon}(t) \;-\; \underline{g}_0^{\epsilon}(t)| \;\le\; 2\,\epsilon\, g_0(\tau)$$

and hence, by (d) and (g),

(i) $$\limsup_{n \to \infty} \sup_{t \in A} |\log h_n(t) - \log h_0(t)| \le 2\epsilon\, g_0(\tau)\,.$$

Since ϵ is arbitrary, (i) implies (8). $\square$

2.2 Revision and correction for section 7.7 Weak convergence $\Longrightarrow$ of $\mathbb{B}_n$ and $\mathbb{X}_n$ in $\|\cdot/q\|_0^T$ – metrics

On page 325, exercise 3: The displayed equation should read as

$$(1 - K)/(1 - F) = \left\{1 + \int_0^{\cdot} \tilde{C}\, dF\right\}^{-1}$$

And then: "Hence $(1 - K)/(1 - F)$ is $\downarrow$."

2.3 Revision and correction of section 23.3. The Shorth.

There is an error here in the grouping of the $n^{1/6}$ factor leading to (i) on page 768; and exercise 1 on page 771) is not correct. The following correction is perhaps the simplest. A different, somewhat longer correction was suggested to us by Keith Knight. (Knight's alternative correction changes the "centering" in the definition of M_n in (6) from $2F^{-1}(1-a)$ to $\mathbb{F}_n^{-1}(1-a) - \mathbb{F}_n^{-1}(a)$.)

Begin on page 768 just after (g):

Moreover, since g' exists and is continuous,

$$\sup_{|t| \le K} |g(1 - a + \frac{At}{n^{1/3}})\, n^{1/6}\, \mathbb{B}_n(1 - a + \frac{At}{n^{1/3}}) - g(1-a)\, n^{1/6}\, \mathbb{B}_n(1 - a + \frac{At}{n^{1/3}})|$$

$$\le \left\{n^{1/6} \sup_{|t| \le K} |g(1 - a + \frac{At}{n^{1/3}}) - g(1-a)|\right\} \times \left\{\sup_{|t| \le K} |\mathbb{B}_n(1 - a + \frac{At}{n^{1/3}})|\right\}$$

$$\le \left\{n^{1/4} \sup_{|t| \le K} |g(1 - a + \frac{At}{n^{1/3}}) - g(1-a)|\right\} \times \left\{n^{-1/12} \sup_{|t| \le K} |\mathbb{B}_n(1 - a + \frac{At}{n^{1/3}})|\right\}$$

(h) $$= o(1)\, O(1) \quad \text{a.s.}$$

(i) $$= o(1) \quad \text{a.s.}$$

continue on page 769, line 1.

Correction of Exercise 23.3.1, page 771. Replace the present exercise 1 by the following:

Exercise 1. Show that for any $0 \le K < \infty$, $0 \le A < \infty$, and $0 \le a < 1$ we have

$$n^{-1/12} \sup_{|t| \le K} |\mathbb{B}_n(a + K/n^{1/3})| = O(1) \quad \text{a.s.}\,.$$

(Knight's alternative correction for this section involves the following alternative exercise:

Exercise 1′. Show that for any $0 \le K < \infty$, $0 \le A < \infty$, and $0 \le a < 1$ we have

$$\sup_{|t| \le K} n^{1/12} |\mathbb{B}_n(a + K/n^{1/3}) - \mathbb{B}_n(a)| = O(1) \quad \text{a.s.}\,.$$

3. Typographical and spelling errors, and minor changes

page	line or equation	error or change
12	(10)	factor of $(-1)^{k+1}$ is missing
14	(7)	factor of $(-1)^{k+1}$ is missing
15	(14)+1	$\sum_{j=1}^{\infty} \chi_i^2 \rightarrow \sum_{j=1}^{\infty} \chi_j^2$
16	-1	(2.2.11) $\rightarrow$ (2.2.13)
25	exercise 4	replace G on the LHS by g (lower case)
28	(15)	$X \rightarrow x$ 5 times (not consistent)
29	(18)+1	$x \rightarrow X$ (not consistent)
29	(18)+5	the set of continuous
30	(5)+1	$(s_1 \quad s_2)(t_1 \quad t_2 - t_1 t_2)$
37	(j)+1	5.9 $\rightarrow$ 9.9
37	(15)-4	5.6 $\rightarrow$ 9.6
47	-13	replace "to then" by "then to"
47	theorem 4	referred to on pg 113 as "Skorokhod's theorem"
47	-3	$\mathbf{M}_\delta^B - \mathbf{A}^* \rightarrow \mathbf{A}^* - \mathbf{M}_\delta^B$
47	(16)+1	$M_S \quad \delta -$ separable implies M_S
		is $\mathbf{M}_\delta^B -$ measurable (cf lemma in Gaenssler)
49	(24)-1	$\|\|Z - A_m Z\|\| \rightarrow \|\|Z - A_m \circ Z\|\|$
59	4	change to: ... independent of $\mathbb{B} \cdots$
59	5	$dF(a) \rightarrow dF(-a)$
61	exercise 8	Kiefer
61	-2	(23) $\rightarrow$ (30)
69	(2) - (3)	$b \rightarrow b_n$
70	middle	Brieman $\rightarrow$ Breiman
88	(21)	$\xi_{ni} \rightarrow \xi_i$
90	(33)+1	$X \rightarrow \xi$ twice
90	(35)	$= \rightarrow \equiv$
90	(35)+1	identify $\rightarrow$ identity
92	(54)	Σ
126	7	$\nu^n \rightarrow \nu_n$
138	-5	martingale $\rightarrow$ submartingale
140	(37)	$x_{ni} \rightarrow X_{ni}$
150	-8, -9	replace: with $m = n$ by with $m = m' = n$
151	(2)	$\sum_{i=1} \rightarrow \sum_{i=1}^{n}$
153	-2	$X_{ni} = F_n^{-1}(\xi_{ni}) \rightarrow X_{ni} = F_{ni}^{-1}(\xi_{ni})$

154	(16)+1	$X_{ni} \equiv F_{ni}^{-1}(\xi_{ni})$ again
156	6	theorem 1 $\to$ theorem 3
168	corollary 1	$F_0 \to F$
168	(5)-1	3.6 $\to$ 3.8
169	1	14.1.4 $\to$ 4.1.1
169	-12	4.1.2 $\to$ 4.1.5
169	-1	$\tilde{P} \to \bar{P}$; 4.1.2->4.1.5
195	1	vector $\to$ matrix; constant $\to$ constants
224	(32)	$G_n^2 \to G^2$
224	(32)+1	change $G_n \to_d G$ to $G_n^2 \to_d G^2$
224	(33)	change $P(G > \lambda)$ to $P(G^2 > \lambda)$
262	(25)+3	$d\Lambda(x) = \quad \to \quad d\Lambda(x) \equiv$
264	(6)	$1_{[X_i \le y]} \to 1_{[X_{ni} \le y]}$
264	(6)	$1 \le i \le n. \to 1 \le i \le n_j$
265	(14)	$X_i \to X_{ni}$ twice
266	7 - 8	$X_i \to X_{ni}$ throughout
270	(32) -1	change (A.9.6) to (A.9.16)
272	(40)	$X_i \to X_{ni}$ twice
273	(1)	$X_i \to X_{ni}$ twice
274	-3	$\psi(x) = x^2 \to \psi(x) = x$
275	(9)	$\|\cdot\|_0^1, \ \|\cdot\| \quad \to \quad \|\ \|_0^1, \ \|\ \|_0^1$
276	(1)	$X_i \to X_{ni}$
279	(9)	$I\!N. \to I\!K$ on RHS
282	(21)	delete garbage before =
294	(4)+3	$\tau \equiv \tau_H = \tau_F \max \tau_G \to \tau \equiv \tau_H = \tau_F \min \tau_G$
304-6		see section 2
323	2	change "proof of (10)" to "proof of (9)"
325	Exercise 3	See section 2
425	(15)+5	Mason (1981) $\to$ Mason (1981b)
425	-1	Mason (1981) $\to$ Mason (1981b)
478	Exercise 4. +1	Anderson's $\to$ Anderson's
492	-1	Esseen $\to$ Esséen
545	2 (18)	$\# I\!U_n \to I\!U_n^{\#}$
558	section title	$I\!K_n \to I\!K$
584	(3)-1	$((log_2 n)^{1/4}\sqrt{\log n}/\sqrt{n} \to ((log_2 n)(log\, n)^2/n)^{1/4}$
604	(2)+10	$n \to t$
604	(2)+11	$t \to n$
661	(4)+1	$\psi \to \psi_n$ twice
661	(9)+1	in the next section $\to$ in section 4

662	(12)	$=\ \rightarrow\ \doteq$
688	(1)-1	since the ... $\rightarrow$ since for the ...
695	(3)+2	$\frac{k}{n} \rightarrow \frac{k}{n+1}$
696	(3)+1	(3.7.4) $\rightarrow$ (3.6.4)
697	(15)	$0 \leq t \leq 1$
698	(21)	$\int_0^{P_{n,i+1}} \rightarrow \int_0^{p_{n,i+1}}$
699	(7)	$\int_0^1 \rightarrow \int_0^t$
746	+5	change to: The definition of $\mathbb{B}_n$ is found first in Smirnov (1947); see also Butler (1969).
747	(11)+2	change to: Smirnov (1947) and Butler (1969) give an expression for the exact distribution.
771	2	t missing just before K
778	(16)+4	Wang $\rightarrow$ Yang
790	(4)	$\pi \rightarrow \Pi$
802	(d)	$2.\,1_{[T_2,\infty)} \rightarrow 2\,1_{[T_2,\infty)}$
804	(j)-1	$\underline{F}$ should go with S_1 and S_m as a subscript
819	-3	$(e^x - 1 - x^2) \rightarrow (e^x - 1 - x)$
821	-2	nonidentically $\rightarrow$ not identically
821	-3	combinations of ... $\rightarrow$ combinations of a function of ...
844	(6)	$\sqrt{2s_n} \rightarrow \sqrt{2}\, s_n$
851	(5)	$exp(-\frac{\lambda}{2\sigma^2}\psi(\frac{\lambda b}{\sigma^2\sqrt{n}})) \rightarrow exp(-\frac{\lambda^2}{2\sigma^2}\psi(\frac{\lambda b}{\sigma^2\sqrt{n}}))$
852	(a)	$E\exp(\sum_1^n X_i) \rightarrow E\exp(r\sum_1^n X_i)$
853	7	$0 < \lambda < 1-\mu \rightarrow 0 < \lambda/\sqrt{n} < 1-\mu$
855	(12)	$exp(-2\lambda^2/\sum_{i=1}^n(b_i - a_i)^2) \rightarrow exp(-2n\lambda^2/\sum_{i=1}^n(b_i - a_i)^2)$
856	(21)+2	Steinback $\rightarrow$ Steinebach
859	-6	Renyi $\rightarrow$ Erdős and Rényi
890	(8)+2	replace $A^c(t) - \sum_{s \leq t} \Delta A(s)$ by $A^c(t) \equiv A(t) - \sum_{s \leq t} \Delta A(s)$
896	(2)	$dX \rightarrow dX^i$
896	(3)	$dX \rightarrow dX^i$
897	(6)	$\int_{(0,t]} \rightarrow \int_{[0,t]}$
898	+3 and +4	$(0,t] \rightarrow [0,t]$

903	Bretagnolle	enchantillon → echantillon
904		Burk→ Burke
910	Hu	tal → tail
915	Steinbach	Steinbach → Steinebach
916	Rényi	theroy → theory
925		Wang → Yang
925		Steinbach → Steinebach
929	-7, right	877 → 878
936	9, left	Rebelledo → Rebolledo
938	17, left	676 → 677

4. Accent mark revisions

page	line or equation	error or change
xxxiii	3.8.3	Renyi
xvii	-3	Renyi
16	+8	Tusnady -> Tusnady
19	-4	Levy → Levy
223	-6, -3	Csorgo -> Csorgo
559	+4	Csorgo -> Csorgo
274	Horvath	Horvath -> Horvath
492	Horvath	Horvath -> Horvath
903	Bretagnolle	nonequireparti -> nonequireparti
904	Horvath	Horvath -> Horvath
905	Horvath	Horvath -> Horvath
906	Horvath	Horvath -> Horvath (3 times)
923	Horvath	Horvath -> Horvath
843		Loeve -> Loeve
844		Loeve -> Loeve
846		Loeve -> Loeve
855		Loeve -> Loeve
861		Loeve -> Loeve
913		Loeve -> Loeve
924		Loeve -> Loeve
924		Komlos -> Komlos
905	Csaki and Tusnady	Csaki -> Csaki
905	Csaki and Tusnady	Tusnady -> Tusnady
908	Foldes and Rejto	Rejto -> Rejto

913	Lenglart	Poincare -> Poincare
270	9	Doleans-Dade -> Doleans-Dade
897	7	Doleans-Dade -> Doleans-Dade
907	1	Doleans-Dade -> Doleans-Dade

5. Solutions of "Open Questions"

Problem	reference for solution
OQ 9.2.1, p. 353	Götze (1985)
OQ 9.2.2, p. 356	Massart (1988)
OQ 9.8.1, p. 400	D. Khoshnevisan, Berkeley Ph.D. (1989)
OQ 9.8.2, p. 400	Khoshnevisan (1992)
OQ 9.8.3, p. 400	Bass and Khoshnevisan (1995)
OQ 10.6.1, p. 428	
OQ 10.7.1, p. 431	
OQ 12.1.1, p. 495	
OQ 12.1.3, p. 495	
OQ 13.4.1, p. 526	
OQ 13.5.1, p. 530	
OQ 13.6.1, p. 530	
OQ 14.2.1, p. 544	
OQ 14.2.2, p. 545	Einmahl and Ruymgaart (1987)
OQ 15.2.1, p. 596	
OQ 16.2.1, p. 605	
OQ 16.4.1, p. 616	Einmahl and Mason (1988)
OQ 17.2.1, p. 628	
OQ 25.3.1, p. 809	

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