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Empirical processes with applications to statistics.
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This is a long-awaited book which fills a gap that was perhaps the widest in the whole literature on probability and statistics. Since many statistical procedures can be viewed as operations with functionals on the empirical process or the empirical distribution function, statistical problems have always motivated the development of new probabilistic theories that would cover the empirical process. The prime example is the development of weak convergence theory, where one of the two main motivating forces was the understanding of the asymptotic behaviour of the Kolmogorov-Smirnov and Cramér-von Mises statistics for testing goodness of fit. On the other hand, the new theories have led to new statistical procedures (for example, the Anderson-Darling statistic would not have been possible without the Doob-Donsker weak convergence theorem), which in their turn pose new theoretical problems. Because of this never-ending story, empirical processes, as a field of research, has always been and will be situated on and about the nonexistent boundary between probability and statistics.

The recent monographs by \n P. Gaenssler\en \ref[*Empirical processes*, Inst. Math. Statist., Hayward, Calif., 1983; MR0744668 (86f:60044)], \n R. M. Dudley\en \ref[*\'Ecole d'\'et\'e de probabilit\'es de Saint-Flour, XII---1982*, 1--142, Lecture Notes in Math., 1097, Springer Berlin, 1984] and \n D. Pollard\en \ref[*Convergence of stochastic processes*, Springer, New York, 1984; MR0762984 (86i:60074)] all deal with recent difficult convergence problems concerning empirical processes based on i.i.d. random elements in very general abstract spaces, where the greater part of the difficulties arises from the generality attempted. While certain aspects of the convergence and distribution theory of empirical processes based on i.i.d. real random variables have been nicely treated in the monographs by \n J. Durbin\en \ref[*Distribution theory for tests based on the sample distribution function*, SIAM, Philadelphia, Pa., 1973; MR0305507 (46 \#4637)], \n P. Billingsley\en \ref[*Convergence of probability measures*, Wiley, New York, 1968; MR0233396 (38

\#1718)], \n E. Csáki\en \ref[Magyar Tud. Akad. Fiz. Oszt. Közl. **23** (1977), no. 3-4, 239--327; MR0464485 (57 \#4415)], \n R. J. Serfling\en \ref[*Approximation theorems of mathematical statistics*, Wiley, New York, 1980; MR0595165 (82a:62003)], \n M. Csörgö\en and \n P. Révész\en \ref[*Strong approximations in probability and statistics*, Academic Press, New York, 1981; MR0666546 (84d:60050)] and \n M. Csörgö\en \ref[*Quantile processes with statistical applications*, SIAM, Philadelphia, Pa., 1983; MR0745130 (86g:60045)], and the very recent useful book edited by \n R. B. D'Agostino\en and \n M. A. Stephens\en \ref[*Goodness-of-fit techniques*, Dekker, New York, 1986; MR0874534 (88c:62075)] addressed to the practicing statistician reviews most of the tests based on the univariate empirical process (without any theory; the term "Brownian bridge" is not to be found in it), there has been no book that aims to grasp, arrange and narrate the whole richness of the field of empirical processes of i.i.d.\ real random variables in a well-digested fashion so that research workers could use it as a reference and graduate students could learn the subject from it. The encyclopædic masterpiece under review does exactly this job.

Most of the theory of empirical processes can be based upon the investigation of the uniform empirical and quantile processes $U_n(t)= \sqrt{n}(G_n(t)-t)$ and $V_n(t)=\sqrt{n}(G^{-1}_n(t)-t)$, $0\leq t\leq 1$, where G_n is the empirical distribution function of n independent random variables with the uniform distribution on $(0,1)$ and G^{-1}_n is the inverse to it. Indeed, the authors start the book with a nursery rhyme "The Uniform Song" to this effect. It is not going to supersede "Little Jack Horner" but many good kids from the kindergarten, like the reviewer, will like it. It is followed by a short and a long table of contents, the latter occupying 18 pages, and 26 chapters and 2 appendices.

Chapter 1 (pp. 1--22) introduces with great care the probability integral transformation and its inverse which allow one to sing the Uniform Song and gives a condensed survey of some results and techniques from some later chapters indicating the kind and scope of the whole work. Chapter 2 (pp.\ 23--84) lays the foundation. It describes the basic notions of weak convergence and almost sure relative compactness and the basic spaces such as the various C and D spaces. Very rightly so, the emphasis is on the constructional approach to weak convergence by means of the Skorokhod-Dudley-Wichura (SDW) theorem which says that weak convergence in a metric space to a limiting random element with a separable range is equivalent to almost sure convergence in the metric of the space of distributionally equivalent copies of the original sequence. Sidestepping certain topological and measurability problems, this approach is conceptually simpler and technically stronger for the vast majority of problems encountered in empirical process theory than the more widely known classical approach based on various characterizations of tightness, i.e., compactness of sets. Any introduction to the theories of weak convergence and almost sure relative compactness is first illustrated on the partial-sum process. This special process allows one to further highlight the constructional approach by bringing in the Skorokhod embedding, which in turn leads to Strassen's relative compactness results, and the "best" constructions of Komlós, Major and Tusnády (KMT) which are simply called here the "Hungarian Construction". All these are explained here. Although many basic results are not proved here, the reader can leave this chapter, which also contains a section on Vasersste\u\i n distance and a very nice one on the distribution of various functionals of the Wiener and Brownian bridge processes, with a strong and active picture of convergence. Chapter 3 (pp. 85--150), at the heart of the book, is on "Convergence and distributions of empirical processes". It gives a beautiful unified treatment of (joint constructional) weak convergence of general weighted empirical processes based on a triangular array of row-wise independent, generally not identically distributed random variables and associated rank processes, together with the corresponding reduced quantile processes and stochastic integrals obtained by integrating a deterministic function with respect to these processes. The reduction of the general processes (a source of many annoying errors in the literature) is very carefully treated. Martingale inequalities are deduced from those of Birnbaum and Marshall for processes weighted by a function, and the limiting distributions of classical statistics for testing simple goodness of fit are derived under the null hypotheses and the corresponding tabulations of these distributions are reproduced from the original sources. The next two chapters represent the first large harvest in statistical theory that can be based on Chapter 3. Chapter 4 (pp. 151--200) provides the basic convergence theory of empirical and rank processes and supremum and other test statistics under contiguous and fixed alternatives and of empirical processes of residuals in a linear model. Some power results and Beran's optimality theorem for the empirical distribution function are also derived. Chapter 5 (pp. 201--257) is an elegantly compiled L^2-theory of the empirical process motivated by the Cramér-von Mises statistic and its weighted analogues both for the simple fit problem and for the case when various parameters are also estimated from the sample. Although the book is not divided into larger units, here is the end of an imaginary Part 1 (and we still have a good 700 pages ahead of us).

The recent martingale theory-counting process approach to some convergence problems of empirical processes are expounded neatly in Chapter 6 (pp.\ 258--292) and used to prove (constructional) weak convergence theorems for certain integral processes associated with empirical processes. This chapter is of special interest to those who want to learn this method because most of the results are proved directly rather than by appealing to the complicated general theory. On the other hand, the companion chapter, Chapter 7 (pp.\ 293--333), dealing with the uniform consistency and weak convergence of the Kaplan-Meier product-limit estimator and the Aalen-Nelson cumulative hazard estimator based on randomly right-censored data, is already based on this general theory, sketched in Appendix B (pp. 884--900).

The short Chapter 8 (pp. 334--342) is on the classical results of exponential and Poisson distributional representation of the V_n and U_n processes. It is not easy to see clearly in the jungle of results on exact distributions associated with the uniform empirical distribution function. Chapter 9 (pp. 343--403) solves this problem intelligently by concentrating on the essence of three different methods. The Dvoretzky-Kiefer-Wolfowitz inequality is derived here.

Chapter 10 (pp. 404--437) is on in-probability linear and almost sure nearly linear bounds on G_n and G^{-1}_n and the corresponding strong ratio and other theorems. These results, due mainly to the authors and \n D. M. Mason\en, are important technical tools in a great variety of problems. The next chapter (pp. 438--490) presents sharp exponential bounds on binomial, Poisson, gamma and beta probabilities, for uniform order statistics and for moments of functions of these statistics, and for weighted supremum norms of the processes U_n and V_n. The latter are used to give a proof for the Chibisov-O'Reilly weak convergence theorems. Convergence in L^p norms is also considered.

Chapter 12 (pp. 491--503) lists the results concerning the "Hungarian" constructions of U_n and V_n. No proofs are given here.

Three strong chapters (pp. 504--530, pp. 531--583 and pp. 584--596) follow on strong behaviour. These are on the laws of the iterated logarithm and relative compactness of U_n and V_n, on the moduli continuity of U_n and V_n, and the Bahadur-Kiefer theory of U_n+V_n, respectively.

Chapter 16 (pp. 597--620) is on the asymptotic distributional and strong behaviour of the standardized processes $U_n(t)(t(1-t))^{-1/2}$ and $V_n(t)(t(1-t))^{-1/2}$, while Chapter 18 (pp. 637--659) does the same for the general quantile processes together with some results on Q-Q plots and the quantile process of the product-limit process of Chapter 7. The intermediate Chapter 17 (pp. 621--636) is a glimpse into the weak convergence theory of U_n indexed by functions.

The next three chapters give applications to the theories of linear combinations of functions of order statistics (pp. 660--694), rank statistics (pp. 695--719) and spacing processes (pp. 720--742). Both the probabilistic theory and the statistical applications are finely delineated. As further applications, special estimation and testing problems are treated in Chapter 22 (pp. 743--762) when the underlying distribution is symmetric about a known or unknown centre, while even further applications concerning the bootstrap, kernel estimates of a distribution function, the so-called Shorth and U-statistic, mean residual life, Lorenz curve and total-time-on-test empirical processes are found in Chapter 23 (pp. 763--780).

Chapter 24 (pp. 781--795) is a short overview of the large deviation theory of the empirical process with the motivating notion of Bahadur efficiency.

Chapter 25 (pp. 796--825) explains how and why empirical processes based on independent but not identically distributed variables are "smaller" than i.i.d. processes from various viewpoints, with applications concerning the extension of some results for L-statistics from Chapter 19. Finally, Chapter 26 (pp. 826--841) provides a very brief glance into the theory of empirical processes on general state spaces, given in much more detail in the books by Dudley, Gaenssler, and Pollard referred to above.

Last, but indeed not least, Appendix A (pp. 842--883) is an extremely nicely compiled chapter on moment, maximal, exponential, large deviation, martingale and various other inequalities and selected miscellaneous probabilistic and deterministic facts, with many proofs also given, which in itself will be a good reason for frequent reference to the book. Following Appendix B noted above, the references occupy pages 901--921, and an author index (pp. 922--925) and a subject index (pp. 926--938) complete the work.

It is impossible to avoid small misprints and errors in such a voluminous book. There are inconsistencies in the usage of Czech, French, Hungarian and Swedish accent marks. There are misstatements such as the attribution of the empirical symmetry process to Butler (1969) on p. 746 when it was considered 22 years before by \n N. V. Smirnov\en \ref[C. R. (Dokl.) Acad. Sci. URSS (N.S.) **56** (1947), 11--14; MR0021260 (9,46a)], stating the "other Hungarian construction" of KMT for the bridge approximation of U_n is in a triangular array form on p. 495 when it is true for a single sequence, and calling the proof of the very same result appearing in the book of \n M. Csörgö\en and Révész referred to above as "readable" when it is not complete and is in fact in error, and the attribution of a form of the Borel-Cantelli lemma to Rényi alone on p. 859 when it is due to Erdös and Rényi. There are a number of items in the reference list which are not referred to in the text, such as Földes and Rejtö (1981 a,b), while Wang (1978), referenced on p. 778, is not in the reference list (and the author index gives p. 768 for \n G. L. Wang\en where this name is not to be found). A number of workers will find that the reference list of 519 items could have been extended by one or two of their important papers. A factor $(-1)^{k+1}$ is missing from displays (10) on p. 12 and (7) on p. 14 for the Kolmogorov distribution function, while the same formula is correct on pp. 34, 37 (where, however, the references should be to Sections 9.9 and 9.6 instead of 5.9 and 5.6), 142 and 401. Any worker in the field will find places where he/she would have said it otherwise or would have emphasized other things. The reviewer, for one, would have included a proof for the SDW theorem which is so crucial for the whole book, or would have included an exercise (indeed in the style of the book) to the effect that Theorem 17.2.1 is almost trivial via the "other Hungarian construction". And the reviewer would have referred to the nice booklet by \n G. V. Martynov\en \ref[*Omega-square tests* (Russian), "Nauka", Moscow, 1978; MR0527912 (80g:62028)] and would have taken many of the tables for the integral tests from this booklet. (For example, Table 3.8.5 for the Anderson-Darling statistic is almost useless compared to the detailed table of Martynov.) However, these little insufficiencies that the reviewer found are so small in number that they are in fact amazingly negligible in a book of this size. The reviewer's main criticism is addressed to the publisher: in a book of this size the page headings should contain the chapter and section numbers to facilitate easier orientation.

The book is very well balanced. Whenever possible the authors try to show not only one but a few approaches to the same problem. A number of results are new, and a good number of other results are polished and perfected versions of ones appearing in the periodical literature. Heuristic considerations form an integral part of the authors' writing style. They concentrate on the essence of the matter and do not hesitate to sidestep unnecessary technical details whenever this is feasible and advisable. Many of these details are listed as exercises, while other exercises are themes not pursued by the authors. At the same time, there is a sufficient amount of detail to learn the subject. Many nice figures help the reader, and some open problems are also formulated. In spite of the enormous wealth and diversity of results, methods, problems and topics covered, there is a strong unifying effect caused by the systematic use of the constructional approach to weak convergence and of the strong exponential probability inequalities, many of which are due to the authors.

It is not difficult to predict that this book will be a standard reference of research workers for many years to come, and generations of graduate students will learn empirical processes from it. A triumph of true scholarship!

Reviewed by Sándor Csörgö