In order to contrast the mollification method with others, a few comments are given. This method is uniquely simple in concept and can be used with virtually all other methods. (When using with other methods, the optimal choice of parameters may no longer be clear, however.) Another positive aspect of the method is that it can be used for a variety of problems. Some negative aspects (from my perspective but maybe not that of mathematicians) is that there is little usage of statistics, that multiple sensors cannot be used in one-dimensional problems, and that many sensors which are equally-spaced along a single line are needed for two-dimensional problems.

There is no question regarding the importance of this book for applied mathematicians working in inverse problems and for engineers, physicists, geologists, and others who need to recover functions from data. This book should be on the bookshelf of all practitioners of the solution of inverse problems.

JAMES V. BECK
Michigan State University

Information Bounds and Nonparametric Maximum Likelihood Estimation. *By Piet Groeneboom and Jon A. Wellner.* Birkhäuser-Verlag, Basel, 1992. viii+126pp. $29.50, paper. ISBN 3-7643-2794-4.

This is a monograph based on two courses of DMV (Deutsche Mathematiker Vereinigung) lectures given by Piet Groeneboom and Jon Wellner, respectively.

The topics covered, information bounds (by Wellner) and nonparametric maximum likelihood estimation (by Groeneboom) complement each other nicely in several ways.

Wellner deals with the question of how well can one (asymptotically) estimate Euclidean and "curve valued" parameters, such as the distribution function. The framework is very general, essentially all models for independent and identically distributed observations. Wellner gives, in an elegant and succinct fashion, a version of the modern nonparametric functional analytic approach to these bounds which does not, in principle, give a prescription for how they can be achieved. To a considerable extent his treatment is a synopsis of the high points of a forthcoming lengthy treatment of these bounds (and methods for achieving them) by himself, jointly with Klaassen, Ritov, and myself.

His treatment is marked by a number of illuminating examples of the uses of the bounds ranging from very classical ones like estimation of the mean of a normal distribution to very novel ones such as estimation of the mean for observations subject to interval censoring, one of the main topics covered by Groeneboom. We draw the reader's attention particularly to the treatment of score operators, an essential tool for calculating information bounds in many semiparametric models and van der Vaart's differentiability theorem, an important tool in determining what parameters can be estimated at the $n^{-1/2}$ rate.

Groeneboom takes a different tack than Wellner and studies in great detail the behaviour of nonparametric maximum likelihood estimates (NPMLE) in three important and difficult types of semiparametric models, interval censoring I and II and convolution models.

Interval censoring arises naturally in cross sectional data. Groeneboom gives some interesting examples, and Gaussian convolution models have long been of interest. Groeneboom's treatment is more self-contained than Wellner's. He begins by showing how NPMLE's in the interval censoring models can be characterized both as solutions of self-consistency equations and isotone regression problems more or less from scratch rather than relying as he could in part on general convex analysis and the classical book of Barlow, Bartholomew, Bremner, and Brunk. He then goes on to show how these characterizations can be extended to convolution models in which the known "error" component has a decreasing density and then goes on to analyze, to the extent he can, the structure of the NPMLE when the "error" is symmetric. This discussion is laced with interesting tidbits on the size and structure of the support of NPMLE's and a novel and nearly explicit form for the NPMLE in the exponential convolution model recently studied by Vardi and Zhang.

He then gives a very insightful discussion of the behaviour of the EM algorithm and competitors based on the isotonic regression formulations he has introduced. Not surprisingly, the competitors, which use the structure of the particular models he considers, perform considerably better. The concluding sections deal first with established asymptotic theory, consistency of the NPMLE of the distribution function in these models, and the limiting distribution of the estimate at a point for interval censoring, case I. He ends with a partly heuristic treatment of the limiting

distribution for interval censoring II and convolution models and finally a link with Wellner. He shows that the mean of the NPMLE in interval censoring I is efficient via the information bound introduced by Wellner.

As is customary in the DMV course, a detailed set of problems testing the reader's mastery of the concepts and, more importantly, providing a number of complements, alternative proofs and approaches to the topics discussed forms an integral part of the book. The book also has numerous references to the literature, some of it as yet unpublished, on which the lectures are based as well as references to the needed supporting results in functional analysis, empirical process theory, etc.

The only critique I have of these excellent lectures is that they do not present enough of what underlies their rigorous treatment. Given its generality, Wellner's presentation is necessarily abstract. But a little bit of discussion of the heuristics underlying the formulation and relating it to the examples would not have been amiss. Groeneboom's proofs are excellent but usually rather technical and the characterization of the procedures and proofs of their properties are often rather unmotivated.

The book on the whole seems relatively free of typos. But here are a couple: p6,1.2: Chang and Yang (1987) not (1976), p.43 1.11-: Exercise 6 not 5.

PETER J. BICKEL
University of California – Berkeley

Rational Iteration: Complex Analytic Dynamical Systems. By *Norbert Steinmetz*. de Gruyter Studies in Mathematics. Vol. 16, 1993. 189 pp. $49.95, hardcover.
Complex Dynamics. By *Lennart Carleson and Theodore W. Gamelin*. Springer-Verlag, New York, 1993. 174 pp. $29.00. ISBN 0-387-97942-5.

The field of complex analytic dynamical systems has undergone dramatic growth since the first computer graphic images of Julia sets and the Mandelbrot set appeared in 1980. But the field has a rich history, dating back to the work of Leau, Koenigs, Schröder, and Böttcher in the late nineteenth century.

The basic problem in complex dynamics is the following: Given a complex analytic function F :

$C \to C$, what can be said about the successive iterates $F^{n+1} = F^n \circ F$ of F?

In the early days, the primary focus of complex dynamicists was on the local theory, primarily near a fixed point of F. The basic question was, roughly speaking: Is F dynamically equivalent to a simple mapping (i.e., a linear map of $z \to z^n$) in a neighborhood of the fixed point? By the end of the nineteenth century, this question had satisfactory answers for fixed points z_0 which were attracting ($|F'(z_0)| < 1$) and repelling ($|F'(z_0)| > 1$), but the intermediate neutral case remained essentially open.

The years 1918–20 brought a dramatic change to the subject. Thanks to the pioneering work of Julia, Fatou, and Lattés, the study of complex dynamical systems proceeded from a more global point of view. Of central interest was what we now call the Julia set, the set of points at which the family of iterates behaves chaotically, and the complementary set, the Fatou set, where iterates behave more tamely. The behavior of a complex dynamical system on its Fatou set was almost completely characterized during this period, the exceptions being the aforementioned neutral fixed points and the possibility of "wandering domains."

After the 1920s, the study of complex dynamics languished somewhat. A notable exception occurred in 1942 when Siegel proved that certain neutral fixed points admitted Siegel disks. These are domains about the fixed point in which the map is dynamically equivalent to a rotation.

The field reawakened in 1980 when the first computer graphics images of the Mandelbrot set and Julia sets appeared. At the same time, Sullivan proved the "No wandering domains" theorem, effectively culminating the work begun in 1918–20. In the ensuing 15 years, a number of major results have appeared, including Douady and Hubbard's analysis of the Mandelbrot set and theory of polynomial like maps, Thurston's classification theorem for rational maps, Shiskikura's bound on the number of attracting and neutral periodic points, and many others. Thus the field is ready for a collection of new texts that attempt to present both the classical and modern ideas in complex dynamics. This is the goal of the two books under review.

Steinmetz' book most closely resembles a textbook. According to the author, "the book may serve as a textbook for a course following a one-year introduction to analytic function theory." Such a course had better include Carathéodory's