Weak Convergence and Empirical Processes

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Changes and Corrections

December, 1996; updated June, 1997; April 2000; August 2000; October 2001; August 2002; February 2003

- Page vii, line 12: change "sets and functions" to "sets or functions".
- Page 20, line 8: Change: "for measurable X_α" to: "for measurable sequences X_α".
- Page 30, line 1: Change: "from" to: "form".
- Page 46, line 15: Change $f_m \uparrow f$ to $f_m \uparrow 1_G$.
- Page 61, line 5: Change: "that A_i^α and" to "A_i^α, and".
- Page 73, line -9: Change $L \in K$ to $L \in \mathcal{L}$.
- Page 80, line 3: change: "on a measurable" to "in a measurable".
- Page 80, line 7: Change "dirac" to "Dirac".
- Page 83, line 6: Change $(\mathcal{X}^\infty, \mathcal{B}^\infty, P^\infty)$ to $(\mathcal{X}^\infty, \mathcal{A}^\infty, P^\infty)$.
- Page 84, line 16: Change $(\int |f|^r)^{1/r}$ to $(\int |f|^r dQ)^{1/r}$.
- Page 88, line 7: Change ξ to ξ_1 on the right side.
- Page 95, line -2: Change $(\log 2)^{p/q}$ to $(\log 2)^{1/q-1/p}$.
- Page 100, line 18: Insert a constant K before the integral term on the right side.
- Page 100, line -4: Insert the word "independent" before "Rademacher" in this line.
- Page 112, line 13: Insert the word "independent" between the words "arbitrary" and "stochastic" in this line: it should read as "arbitrary independent stochastic processes" .
- Page 114, line 1: Change α to α^2 here.
- Page 120, line 2: Change: "$\mapsto$" to "$\rightarrow$".
- Page 120, line 3: Before the period add: "and $g_m \mapsto f$ in $L_2(P)$".
- Page 123, line 11: Add a full stop after "chapter".
- Page 132, line 13: Change $\leq 22^{-q}$ to $2 \cdot 2^{-2q}$.
- Page 143, line -11: Delete the "minus sign" before $(1/2)C_1 L n^{-W}$
- Page 145: In Corollary 2.6.12, add that F must be the envelope function of the underlying VC-subgraph class, also if $\mathcal{F}$ is a true subset of the symmetric convex hull of this VC-subgraph class.
- Page 150, line -4: Change "the previous corollary" to "Corollary 2.6.3".
- Page 152, problem 12: Change to: If $\mathcal{X}$ is the union of finitely many disjoint sets $\mathcal{X}_i$, and $\mathcal{C}_i$ is a VC-class of subsets of $\mathcal{X}_i$ for each i, $i = 1, \ldots, m$, then $\sqcup_{i=1}^m \mathcal{C}_i$ is a VC-class in $\cup_{i=1}^m \mathcal{X}_i$ of index $\sum_{i=1}^m V(\mathcal{C}_i) - m + 1$.
- Page 153, line 21: Change "$t_2 > t_1 \vee t_2$" to "$t_2 > t_1 \vee t_3$".
- Page 153, problem 2.6.17: Change the "Hint" to read as follows:

[**Hint:** Take $\mathcal{X} = \mathcal{Y} = \mathbb{N}$ with $\phi(2n-1) = \phi(2n) = n$. Let $\mathcal{C}$ be the collection consisting of the empty set, all singletons of odd integers, and all finite subsets of even integers with more than two elements: $\mathcal{C} =$

$\{\emptyset, \{1\}, \{3\}, \dots, \{2,4\}, \{2,6\}, \dots, \{2,4,6\}, \dots\}$. Show that no subset of two points in $\mathcal{X}$ is shattered, while $\phi(\mathcal{C})$ consists of all finite subsets of $\mathbb{N}$.]

- Page 156, line 12: Replace $2/\delta^{\alpha-k.} + 1$ by $2/\delta^{\alpha-k.} + 2$; and replace $2\delta^{-\alpha} + 1$ by $2\delta^{-\alpha} + 2$.
- Page 156, line -11: replace $|x_i - x_j|$ by $\|x_i - x_j\|$.
- Page 156, line -4: replace $2\delta^{-\alpha} + 1$ by $2\delta^{-\alpha} + 2$.
- Page 160, line -5: change $\sum_{j=1}^{i}$ to $\sum_{j=1}^{i-1}$.
- Page 161, line -12: change the left side of the displayed inequality to $(f(x_{j-1}) - f_i(x_{j-1}))(x_j - x_{j-1})^{1/r}$.
- Page 164, line 16: Replace $f(y)$ by $g(y)$ on the left side of the display.
- Page 184, line -10: Change "which is by" to "which are".
- Page 202, Example 2.10.25: Add a forward reference to Theorem 2.14.2, page 240, as follows: change the last line of the example to: "Suppose that $\alpha > d/2$ and $\sum_{j=1}^{\infty} M_j P(\mathcal{X}_j)^{1/2} < \infty$; then it follows from the bounds of Theorem 2.14.2 that the class $\mathcal{F}$ is P-Donsker."
- Page 202, Example 2.10.26: In the next to last line of the example, change $(1 + |xy|^{2+\delta})$ to $[(1+|x|)(1+|y|)]^{2+\delta}$.
- Page 215, -14: Change "nondecreasing" to "cadlag and nondecreasing".
- Page 291, line 17: Start a new paragraph (outside the corollary environment): "Thus the "continuity modulus ... ". (This should not be part of the Corollary.)
- Page 304, line 1: In Lemma 3.2.21 add the condition "$PM_\delta^2 \le \delta^2$".
- Page 304, line -7: Replace "second and third" by "two".
- Page 306, linew 4-5: Delete $\mathrm{sign}(Y - \theta_0' X)$.
- Page 306, line 16: Delete the $-$ sign in the display.
- Page 323, line 9: Change "porbability" to "probability".
- Page 324, footnote: Change 2.2 to 2.14.
- Page 325, line 4: Change "first" to "third".
- Page 328, line 6: The claimed inequality is false as stated, but it is true with the addition of the constant $(e^2-3)/(2(e^{-1}-1)^2) < 6$ multiplying the function $2(e^{x/2}-1)^2$ on the right side.
- Page 331, line -8: Change "regressions" to "regression".
- Page 332, line 16: Between "only" and "the" add "on".
- Page 334, line 13: Insert $\sum_{j=1}^{N_n}$ before the expectation on the right side.
- Page 337, lines -10, -13: Change $\le (1 + \frac{1}{2}\sqrt{2})$ to $\le (1+\sqrt{2})$.
- Page 347, line -14: Change "the multiplier" to "the conditional multiplier" here.
- Page 350, line 4: Change $m2_n\sqrt{n}$ to $2m_n\sqrt{n}$.
- Page 354, line 8: Change $\overset{\mathrm{P}}{\to} 0 \to 0$.
- Page 357, line -14: this paragraph should read "The following lemma can be called a "multiplier Glivenko-Cantelli theorem" with exchangeable multipliers". A special case of the lemma is used in the proof of Theorem 3.6.2
- Page 399, problem 9: In the hint, change $x \to z \circ \Psi^{-1}(x) - x$ to $x - x \to$

$z \circ \Psi^{-1}(x)$.

- Page 412, line 8: Change to "parameters $\kappa_n(h)$ is "regular"...".
- Page 415, line 7: Add a minus sign in the definition of G_λ
- Page 415, line 12 (display): Change "G_λ" to "G".
- Page 417, line 1: change "$\tau(\mathbf{B}')$-measurable" to "$\mathbf{B}^*$-measurable".
- Page 417, line 2: change "$\tau(\mathbf{B}')$-measurable" to "$\mathbf{B}'$-measurable".
- Page 417, line 16 (display): Add a prime to b_{i,p_i}.
- Page 418, line 15: Change: "$\ell(y) > i/r$" to "$\ell(y) \leq i/r$".
- Page 419, line 10: Change: "$\tau(D[a,b]^*)$-measurable" to "$D[a,b]^*$-measurable".
- Page 425, notes, section 3.5, line -3: Before "Part of Theorem 3.5.5 ... ", insert the following: "Lemma 3.5.4 and its proof come from Giné and Zinn (1990), who only consider the symmetric case; this lemma also follows from the proof of a result of LeCam (1970b, Theoreme 3) on accompanying Poisson laws."
- Page 468, line -10: change "1494-1452" to "1494-1542".
- Page 472, line -20: Change "Probability Theory ... " to "Theory of Probability ... ".
- Page 473, line -6: Change "50-11" to "50-113".
- Page 483, line -5: Change (1994a) to (1994).
- Page 483, references: add the following reference:

Talagrand, M. (1987d). The Glivenko-Cantelli problem. *Annals of Probability* **15**, 837-870.

- Page 500, line 17: Change "M-estimators ... 270" to "M-estimators ... 284".
- Page 504, line -15: Change "uniform tightness ... 71" to "uniform tightness ... 73".
- Page 508, symbol list: add the symbols $\lfloor x \rfloor$ (page 88) and $\lceil x \rceil$ (page 227).

The following corrections were added to the list on August 1, 2000:

- Page 119, line -4: after $\in G$, insert $\wedge f \in U$.
- Page 130, line -13: insert $\sqrt{n}$ before $\|f\|_{P,2}/\sqrt{\log|\mathcal{F}|}$.
- Page 203, line 4: change $=$ to $\leq$.
- Page 319, line 3: change Ψ_{12} to $\dot{\Psi}_{12}$.
- Page 320, line -1: change $P_0(\cdots)^2$ to $1/P_0(\cdots)^2$.
- Page 329, line -15: insert log before the second $N_{[]}$. Thus the display should read as:

$$\log N_{[]}(\epsilon, \mathcal{P}, h) \leq \log N_{[]}(\epsilon, \mathcal{F}, L_2(\lambda)) \ldots .$$

- Page 347, line 9: change $\hat{Y}_n$ to $\hat{\mathbb{Y}}_n$
- Page 421, line -16: delete the $>$ before "F" (which results in the inverted question mark).

- Page 446, line 10: on the far right side of this display, replace $\sqrt{N(\eta, T, \rho)}$ by $\sqrt{\log N(\eta, T, \rho)}$.
- Page 487, line -18: change the date from (1990) to (1992).

The following correction was added to the list on February 17, 2003:

- Page 441, Proposition A.2.6: The probability inequality $\mathrm{P}(\sup_t X_t > \lambda) \leq \mathrm{P}(\sup_t Y_t > \lambda)$ requires the additional condition that $\mathrm{E}X_t^2 = \mathrm{E}Y_t^2$ for every $t \in T$. The inequality $\mathrm{E}\sup_t X_t \leq \mathrm{E}\sup_t Y_t$ is valid under the conditions as stated, and can be deduced from the probability inequality (with the strengthened hypothesis), but it is not an immediate consequence. For the inequality $\mathrm{E}\sup_t |X_t| \leq 2\mathrm{E}\sup_t |Y_t|$ it is required in addition either that $\mathrm{E}X_t^2 = \mathrm{E}Y_t^2$ for every $t \in T$ or that $X_t = 0$ almost surely for some $t \in T$. (In the second case we have $\mathrm{E}\sup_t |X_t| \leq \mathrm{E}\sup_{s,t}(X_s - X_t) = 2\mathrm{E}\sup_t X_t$, by symmetry of X.)

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