Description of the S-programs for calculating the LSE of a k-monotone density

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- *LSEComputeSpline*: finds the unique spline of degree $2k-1$ whose k-derivative is the minimizer of the LS criterion function over the cone generated by the current set of support points.
- *LSEInitialCond*: if $\underline{\theta} = (\theta_1, \cdots, \theta_m)$ is the set of support points of the current iterate f, then the minimizer $\tilde{f}$ of the LS criterion function over the cone generated by $\underline{\theta}$ is a spline of degree $k-1$ that have to satisfy some boundary conditions. This function puts these conditions in a numerical vector B which is one important input of the function *LSEComputeSpline*.
- *IndexFuncMLE*: finds the index of the support point to be eliminated from the vector of all support points of the current iterate (reduction step).
- *Integr.Fn*: calculates the $(k-1)$-fold integral of the empirical distribution $\mathbb{F}_n$.
- *J.Func*: calculates the $(k-1)$-fold integral of

$$g_\theta(x) = \frac{k(\theta - x)_+^{k-1}}{\theta^k}.$$

- *DirecDer*: calculates the directional derivative for the LS criterion function at some point θ.
- *FindMinFunc*: finds the minimum of the directional derivative over a chosen fine grid. The minimizer will be added to the current set of support points.
- *LSESupReducAlgo*: this is the main function that calls all the other sub-programs, and it calculates after a finite number of iterations an approximation of the LSE of a k-monotone density based on n independent observations. The function takes the following arguments:

  1. The integer k (≥ 2): the smoothness parameter.
  2. X: the vector of observations.
  3. *prec*: a parameter that controls how much fine the user wants to choose the grid over which the minimization problem is solved.

4. *eps*: the tolerance chosen by the user (should be very small). If the next iteration results in a directional derivative that is bigger or equal than $-eps$, then the algorithm stops.

The algorithm returns two vectors called S and C: S is the vector of support points and C is the corresponding vector of weights. The user might use any other software to plot the LSE in the direct and inverse problems.