Nemirovski's inequality revisited

Jon A. Wellner

University of Washington

- joint work with Lutz Dümbgen and Sara van de Geer,
- Faculty talk at
  Graduate Student Orientation,
  Department of Statistics, University of Washington
  22 September, 2008
- *Email: jaw@stat.washington.edu*
  http: //www.stat.washington.edu/jaw/jaw.research.html

Outline

1. A problem from statistics: persistence
2. The theorem of Greenshtein and Ritov
3. First Proof – via Nemirovski's inequality
4. Second Proof – via bracketing entropy bounds
5. Proof of Nemirovski's inequality
6. Extensions and comparisons

1. A problem from statistics: persistence

Setting:

- Data: n i.i.d. copies $Z_1, \ldots, Z_n$ of $Z = (Y, X_1, \ldots, X_p) \equiv (Y, \underline{X})$; write $Z_i = (Y^i, X_1^i, \ldots, X_p^i)$, $i = 1, \ldots, n$.
- Dimension of $\underline{X}$, $p = p_n$ large, $p_n = n^\alpha$, $\alpha > 1$
- Goal: Predict Y on the basis of the covariates X_j, $j = 1, \ldots, p$
- Predictors $\widehat{Y}$ of Y of the form $\widehat{Y} = \sum_{j=1}^p \beta_j X_j = \underline{\beta}' \underline{X}$ with $\underline{\beta} \in B_n \subset \mathbb{R}^p$ for each n.
- Natural sets B_n to consider are

$$B_{n,k} \equiv \{\beta \in \mathbb{R}^p : \ \#\{j : \ \beta_j \neq 0\} = k\} = \{\beta \in \mathbb{R}^p : \ \|\beta\|_0 = k\},$$

$$B_{n,b} \equiv \{\beta \in \mathbb{R}^p : \ \|\underline{\beta}\|_1 \leq b\}.$$

where $k = k_n \to \infty$ and $b = b_n \to \infty$.

- For $Z = (Y, \underline{X}) \sim P$ on $(\mathbb{R}^{p+1}, \mathcal{B}_{p+1})$, define

$$L_P(\beta) = E_P \left(Y - \sum_{j=1}^{p} \beta_j X_j \right)^2 .$$

- For $Z = (Y, \underline{X}) \sim P$ on $(\mathbb{R}^{p+1}, \mathcal{B}_{p+1})$, define

$$L_P(\beta) = E_P\left(Y - \sum_{j=1}^{p} \beta_j X_j\right)^2 .$$

- For a given sequence of distributions $\{P_n\}$ of Z and sequence of sets $\{B_n\}$ with $B_n \subset \mathbb{R}^p$, define

$$\beta_n^*(P_n) \equiv \beta_n^* \equiv \text{argmin}_{\beta \in B_n} L_{P_n}(\beta).$$

  Thus β_n^* is a deterministic sequence in $\mathbb{R}^p$ determined by P_n and B_n.

- For $Z = (Y, \underline{X}) \sim P$ on $(\mathbb{R}^{p+1}, \mathcal{B}_{p+1})$, define

$$L_P(\beta) = E_P \left(Y - \sum_{j=1}^{p} \beta_j X_j \right)^2 .$$

- For a given sequence of distributions $\{P_n\}$ of Z and sequence of sets $\{B_n\}$ with $B_n \subset \mathbb{R}^p$, define

$$\beta_n^*(P_n) \equiv \beta_n^* \equiv \text{argmin}_{\beta \in B_n} L_{P_n}(\beta).$$

  Thus β_n^* is a deterministic sequence in $\mathbb{R}^p$ determined by P_n and B_n.

- This corresponds to the unknown "ideal predictor" $\widehat{Y}^* = \underline{\beta}_n^* \underline{X}$ which would be available to us if we knew P_n.

- **Definition.** (Greenshtein and Ritov, 2004).
  Given a set of possible predictors B_n, a sequence of procedures $\{\hat{\beta}_n\}$ is persistent (or *persistent relative to* $\{B_n\}$ and $\{\mathcal{P}_n\}$) if, for every sequence $P_n \in \mathcal{P}_n$

$$L_{P_n}(\widehat{\beta}_n) - L_{P_n}(\beta_n^*) \to_p 0.$$

2. The theorem of Greenshtein and Ritov

Theorem. If $p = p_n = n^\alpha$ and

$$F(Z_i) \equiv \max_{0 \le j,k \le p} |X^i_j X^i_k - E_{P_n}(X^i_j X^i_j)|$$

satisfies $E_{P_n} F^2(Z_1) \le M < \infty$ for all $n \ge 1$, then for $b_n = o((n/\log n)^{1/4})$ the procedures given by

$$\widehat{\beta}_n \equiv \text{argmin}_{\beta \in B_{n,b_n}} L_{\mathbb{P}_n}(\beta) \qquad (1)$$

are persistent with respect to

$$B_{n,b_n} \equiv \{\beta \in \mathbb{R}^p : \ \|\beta\|_1 \le b_n\}.$$

- **Comment 1.** The persistent procedures $\widehat{\beta}_n$ in (1) are equivalent to Lasso estimators with a particular range of the penalty parameters.

- **Comment 1.** The persistent procedures $\widehat{\beta}_n$ in (1) are equivalent to Lasso estimators with a particular range of the penalty parameters.
- **Comment 2.** Greenshtein and Ritov (2004) also prove related results for procedures based on the "model selection sets" $B_{n,k}$ under the assumption that $k = k_n = o((n/logn)^{1/2})$.

- **Comment 1.** The persistent procedures $\widehat{\beta}_n$ in (1) are equivalent to Lasso estimators with a particular range of the penalty parameters.
- **Comment 2.** Greenshtein and Ritov (2004) also prove related results for procedures based on the "model selection sets" $B_{n,k}$ under the assumption that $k = k_n = o((n/logn)^{1/2})$.
- **Proof, part 1:** Let $\gamma' = (-1, \beta_1, \ldots, \beta_p)' \equiv (\beta_0, \ldots, \beta_p)' \in \mathbb{R}^{p+1}$, and let $Y \equiv X_0$. Then

$$L_P(\beta) = E_P(Y - \beta' X)^2 = \gamma' \Sigma_P \gamma$$

where $\Sigma_P \equiv (\sigma_{ij}) = (E_P(X_i X_j))_{0 \le i,j \le p}$.

- **Proof, part 1:** With $\widehat{\beta}_n \equiv \text{argmin}_{\beta \in B_{n,b_n}} L_{\mathbb{P}_n}(\beta)$ it follows that

$$L_{P_n}(\widehat{\beta}_n) - L_{P_n}(\beta_n^*) \geq 0,$$
$$L_{\mathbb{P}_n}(\widehat{\beta}_n) - L_{\mathbb{P}_n}(\beta_n^*) \leq 0,$$

and hence

$$\begin{aligned} 0 &\leq L_{P_n}(\widehat{\beta}_n) - L_{P_n}(\beta_n^*) \\ &= L_{P_n}(\widehat{\beta}_n) - L_{\mathbb{P}_n}(\widehat{\beta}_n) + L_{\mathbb{P}_n}(\widehat{\beta}_n) - L_{\mathbb{P}_n}(\beta_n^*) \\ &\qquad + L_{\mathbb{P}_n}(\beta_n^*) - L_{P_n}(\beta_n^*) \\ &\leq 2 \sup_{\beta \in B_{n,b_n}} |L_{\mathbb{P}_n}(\beta) - L_{P_n}(\beta)|. \end{aligned}$$

- **Proof, part 1, continued:** Let $\mathbb{P}_n$ be the empirical measure of $Z_1, \ldots, Z_n$. Then

$$L_{\mathbb{P}_n}(\beta) = \gamma' \Sigma_{\mathbb{P}_n} \gamma \equiv \gamma'(\widehat{\sigma}_{ij})\gamma \equiv \gamma' \widehat{\Sigma} \gamma.$$

Define ϵ_{ij}^n and $E = (\epsilon_{ij}^n)$ by

$$\epsilon_{ij}^n \equiv \widehat{\sigma}_{ij} - \sigma_{ij}, \qquad E \equiv (\epsilon_{ij}^n) \equiv \widehat{\Sigma} - \Sigma_P.$$

Then

$$|L_{\mathbb{P}_n}(\beta) - L_{P_n}(\beta)| = |\gamma'(\Sigma_{\mathbb{P}_n} - \Sigma_{P_n})\gamma| \le \|\Sigma_{\mathbb{P}_n} - \Sigma_{P_n}\|_\infty \|\gamma\|_1^2.$$

- **Proof, part 1, continued:** Thus for $B_{n,b_n} = \{\beta \in \mathbb{R}^p : \ \|\beta\|_1 \le b_n\}$,

$$
\begin{aligned}
& Pr\left(\sup_{\beta \in B_{n,b_n}} |L_{\mathbb{P}_n}(\beta) - L_{P_n}(\beta)| > \epsilon \right) && (2) \\
& \le \quad Pr(\|\Sigma_{\mathbb{P}_n} - \Sigma_{P_n}\|_\infty (1 + b_n)^2 > \epsilon) \\
& \le \quad \epsilon^{-1}(b_n + 1)^2 E\|\Sigma_{\mathbb{P}_n} - \Sigma_{P_n}\|_\infty . && (3)
\end{aligned}
$$

Thus if we can show that the expectation in the last display satisfies

$$E\|\Sigma_{\mathbb{P}_n} - \Sigma_{P_n}\|_\infty \le C\sqrt{\frac{\log n}{n}},$$

then the proof is complete:

3. First proof (part 2) – via Nemirovski's inequality

Lemma 1. (Nemirovski's inequality)
Let $X_1, \ldots, X_n$ be independent random vectors in $\mathbb{R}^d$, $d \geq 3$, with $EX_i = 0$ and $E\|X_i\|_2^2 < \infty$. Then for every $r \in [2, \infty]$

$$E\Big\| \sum_{i=1}^{n} X_i \Big\|_r^2 \leq \widetilde{C} \min\{r, \log d\} \sum_{i=1}^{n} E\|X_i\|_r^2$$

where $\|\cdot\|_r$ is the ℓ_r norm, $\|x\|_r \equiv \{\sum_1^d |x_j|^r\}^{1/r}$ and $\widetilde{C}$ is an absolute constant (i.e. not depending on r or d or n or the distribution of the X_i's).

- **First proof, part 2:** To apply Nemirovski's inequality to bound $E\|\Sigma_{\mathbb{P}_n} - \Sigma_{P_n}\|_\infty$, consider the matrix $\Sigma_{\mathbb{P}_n} - \Sigma_{P_n}$ as a $(p+1)^2-$dimensional vector, and write

$$\begin{aligned} \Sigma_{\mathbb{P}_n} - \Sigma_{P_n} &= \sum_{i=1}^n V_i \\ \equiv & \sum_{i=1}^n \frac{1}{n} \big(X_0^i X_0^i - E(X_0^i X_0^i), X_0^i X_1^i - E(X_0^i X_1^i), \ldots, \\ & \ldots, X_p^i X_p^i - E(X_p^i X_p^i) \big) . \end{aligned}$$

- **First proof, part 2:** To apply Nemirovski's inequality to bound $E\|\Sigma_{\mathbb{P}_n} - \Sigma_{P_n}\|_\infty$, consider the matrix $\Sigma_{\mathbb{P}_n} - \Sigma_{P_n}$ as a $(p+1)^2-$dimensional vector, and write

$$\begin{aligned}\Sigma_{\mathbb{P}_n} - \Sigma_{P_n} &= \sum_{i=1}^n V_i \\ &\equiv \sum_{i=1}^n \frac{1}{n}\Big(X_0^i X_0^i - E(X_0^i X_0^i), X_0^i X_1^i - E(X_0^i X_1^i), \ldots, \\ &\qquad \ldots, X_p^i X_p^i - E(X_p^i X_p^i)\Big)\,.\end{aligned}$$

- By our hypothesis

$$F(Z_i) \equiv \max_{0\le j,k\le p} |X_j^i X_k^i - E_{P_n}(X_j^i X_k^i)|$$

satisfies $E_{P_n} F(Z_i)^2 \le M < \infty$.

- **First proof, part 2, continued:** Then by Jensen's inequality followed by Nemirovski's inequality with $r = \infty$,

$$
\begin{aligned}
\{E_{P_n}\|\Sigma_{\mathbb{P}_n} - \Sigma_{P_n}\|_\infty\}^2 &= \left\{E_{P_n}\|\sum_{i=1}^n V_i\|_\infty\right\}^2 \le E_{P_n}\|\sum_{i=1}^n V_i\|_\infty^2 \\
&\le C\log((p_n+1)^2)\sum_{i=1}^n E_{P_n}\|V_i\|_\infty^2 \\
&\le C'\log(4n^{2\alpha})\frac{1}{n^2}\sum_{i=1}^n EF(Z_i)^2 \\
&\le C''\frac{\log n}{n},
\end{aligned}
$$

so that

$$
E_{P_n}\|\Sigma_{\mathbb{P}_n} - \Sigma_{P_n}\|_\infty \le C''\sqrt{\frac{\log n}{n}}. \qquad \square
$$

4. Proof (part 2) – via bracketing entropy bounds

- Let $\mathbb{G}_n \equiv \sqrt{n}(\mathbb{P}_n - P_n)$.

4. Proof (part 2) – via bracketing entropy bounds

- Let $\mathbb{G}_n \equiv \sqrt{n}(\mathbb{P}_n - P_n)$.
- For a class of functions $\mathcal{F} = \{f : \mathcal{Z} \to \mathbb{R}\}$ write $\|\mathbb{G}_n\|_{\mathcal{F}} = \sup_{f \in \mathcal{F}} |\mathbb{G}_n(f)|$. For $\mathcal{F}$ with $\#(\mathcal{F}) = d < \infty$, note that $\|\mathbb{G}_n\|_{\mathcal{F}} = \|\mathbb{G}_n(\underline{f})\|_\infty$ where $\mathbb{G}_n(\underline{f}) \equiv (\mathbb{G}_n(f_1), \ldots, \mathbb{G}_n(f_d))$.

4. Proof (part 2) – via bracketing entropy bounds

- Let $\mathbb{G}_n \equiv \sqrt{n}(\mathbb{P}_n - P_n)$.
- For a class of functions $\mathcal{F} = \{f : \mathcal{Z} \to \mathbb{R}\}$ write $\|\mathbb{G}_n\|_{\mathcal{F}} = \sup_{f \in \mathcal{F}} |\mathbb{G}_n(f)|$. For $\mathcal{F}$ with $\#(\mathcal{F}) = d < \infty$, note that $\|\mathbb{G}_n\|_{\mathcal{F}} = \|\mathbb{G}_n(\underline{f})\|_\infty$ where $\mathbb{G}_n(\underline{f}) \equiv (\mathbb{G}_n(f_1), \ldots, \mathbb{G}_n(f_d))$.
- For each $\epsilon > 0$ let the bracketing number $N_{[\,]}(\epsilon, \mathcal{F}, L_2(P))$ be the minimal number of brackets of $L_2(P)-$size ϵ needed to cover $\mathcal{F}$.

4. Proof (part 2) – via bracketing entropy bounds

- Let $\mathbb{G}_n \equiv \sqrt{n}(\mathbb{P}_n - P_n)$.
- For a class of functions $\mathcal{F} = \{f : \mathcal{Z} \to \mathbb{R}\}$ write $\|\mathbb{G}_n\|_{\mathcal{F}} = \sup_{f \in \mathcal{F}} |\mathbb{G}_n(f)|$. For $\mathcal{F}$ with $\#(\mathcal{F}) = d < \infty$, note that $\|\mathbb{G}_n\|_{\mathcal{F}} = \|\mathbb{G}_n(\underline{f})\|_\infty$ where $\mathbb{G}_n(\underline{f}) \equiv (\mathbb{G}_n(f_1), \ldots, \mathbb{G}_n(f_d))$.
- For each $\epsilon > 0$ let the bracketing number $N_{[\,]}(\epsilon, \mathcal{F}, L_2(P))$ be the minimal number of brackets of $L_2(P)-$size ϵ needed to cover $\mathcal{F}$.
- For $\delta > 0$, let

$$J_{[\,]}(\delta, \mathcal{F}, L_2(P)) \equiv \int_0^\delta \sqrt{\log(1 + N_{[\,]}(\epsilon, \mathcal{F}, L_2(P))} d\epsilon.$$

Lemma. (Empirical process theory bracketing entropy bound)

$$E^* \|\mathbb{G}_n\|_{\mathcal{F}} \lesssim J_{[\,]}(1, \mathcal{F}, L_2(P_n)) \|F\|_{P_n,2}.$$

Proof. Pollard (1989); see Theorem 2.14.2, van der Vaart and Wellner (1996), page 240. □

- In the current application take $\mathcal{F} = \{f_{j,k}(z) = x_j x_k, \ 0 \le j, k \le p\}$, a finite list of functions of cardinality $\#(\mathcal{F}) = (p_n + 1)^2$.

Lemma. (Empirical process theory bracketing entropy bound)

$$E^* \|\mathbb{G}_n\|_{\mathcal{F}} \lesssim J_{[\,]}(1, \mathcal{F}, L_2(P_n)) \|F\|_{P_n,2}.$$

Proof. Pollard (1989); see Theorem 2.14.2, van der Vaart and Wellner (1996), page 240. □

- In the current application take $\mathcal{F} = \{f_{j,k}(z) = x_j x_k,\ 0 \le j,k \le p\}$, a finite list of functions of cardinality $\#(\mathcal{F}) = (p_n + 1)^2$.
- Hence $N_{[\,]}(\epsilon, \mathcal{F}, L_2(P_n)) \le (p_n + 1)^2$ by choosing $\epsilon-$brackets $[l_{j,k}, u_{j,k}]$ given by $l_{j,k}(z) = f_{j,k}(z) - \epsilon/2$ and $u_{j,k}(z) = f_{j,k}(z) + \epsilon/2$.

Lemma. (Empirical process theory bracketing entropy bound)

$$E^*\|\mathbb{G}_n\|_{\mathcal{F}} \lesssim J_{[\,]}(1, \mathcal{F}, L_2(P_n))\|F\|_{P_n,2}.$$

Proof. Pollard (1989); see Theorem 2.14.2, van der Vaart and Wellner (1996), page 240. □

- In the current application take $\mathcal{F} = \{f_{j,k}(z) = x_j x_k,\ 0 \le j,k \le p\}$, a finite list of functions of cardinality $\#(\mathcal{F}) = (p_n + 1)^2$.
- Hence $N_{[\,]}(\epsilon, \mathcal{F}, L_2(P_n)) \le (p_n + 1)^2$ by choosing $\epsilon-$brackets $[l_{j,k}, u_{j,k}]$ given by $l_{j,k}(z) = f_{j,k}(z) - \epsilon/2$ and $u_{j,k}(z) = f_{j,k}(z) + \epsilon/2$.
- Thus the bound in the lemma becomes

$$E\|\mathbb{G}_n\|_{\mathcal{F}} \lesssim \sqrt{1 + \log\left[(p_n + 1)^2\right]}\|F\|_{P_n,2} \lesssim \sqrt{\log n},$$

- Or, equivalently

$$E\|\Sigma_{\mathbb{P}_n} - \Sigma_{P_n}\|_\infty = E\|\mathbb{P}_n - P_n\|_{\mathcal{F}} \lesssim \sqrt{n^{-1}\log n},$$

in agreement with the bound given by Nemirovski's inequality. □

5. Proof of Nemirovski's inequality

Proof: For given $r \in [2, \infty)$ consider the map V_r from $\mathbb{R}^d$ to $\mathbb{R}$ defined by

$$V_r(x) \equiv \|x\|_r^2.$$

Then V_r is continuously differentiable with Lipschitz continuous derivative ∇V_r. Furthermore

$$V_r(x+y) \leq V_r(x) + y' \nabla V_r(x) + CrV_r(y) \tag{4}$$

for an absolute constant C. Thus, writing $\sum_{i=1}^{n+1} X_i = \sum_{i=1}^{n} X_i + X_{n+1}$, it follows from (4) that

$$V_r(\sum_{i=1}^{n+1} X_i) \leq V_r(\sum_{i=1}^{n} X_i) + X'_{n+1} \nabla V_r(\sum_{i=1}^{n} X_i) + CrV_r(X_{n+1}).$$

Taking expectations across this inequality and using independence of X_{n+1} and $\sum_{i=1}^n X_i$ together with $E(X_{n+1}) = 0$ yields

$$\begin{aligned} EV_r\left(\sum_{i=1}^{n+1} X_i\right) &\leq E\left\{V_r\left(\sum_{i=1}^{n} X_i\right) + X'_{n+1}\nabla V_r\left(\sum_{i=1}^{n} X_i\right)\right\} \\ &\quad + CrEV_r(X_{n+1}) \\ &= EV_r\left(\sum_{i=1}^{n} X_i\right) + CrE\|X_{n+1}\|_r^2. \end{aligned}$$

By recursion this yields

$$EV_r\left(\sum_{i=1}^{n+1} X_i\right) \leq Cr\sum_{i=1}^{n+1} EV_r(X_i) \tag{5}$$

and hence the desired result with r rather than $\min\{r, \log d\}$.

To show that we can replace r by $\min\{r, \log d\}$ up to an absolute constant, first note that this follows immediately for $r \leq r(d) \equiv 2\log d$ with C replaced by $2C$. Now suppose $r > r(d) = 2\log d$. Recall that for $1 \leq r' \leq r$ we have

$$\|x\|_r \leq \|x\|_{r'} \leq d^{(1/r')-(1/r)}\|x\|_r$$

for all $x \in \mathbb{R}^d$ (by Hölder's inequality).

Thus with $r' = r(d) < r$

$$
\begin{aligned}
E\|\sum_{i=1}^n X_i\|_r^2 &\le E\|\sum_{i=1}^n X_i\|_{r(d)}^2 \\
&\le Cr(d)\sum_{i=1}^n E\|X_i\|_{r(d)}^2 \qquad \text{by (5)} \\
&\le Cr(d)\sum_{i=1}^n E\left\{d^{\frac{2}{r(d)}-\frac{2}{r}}\|X_i\|_r^2\right\} \\
&\le Cr(d)d^{2/r(d)}\sum_{i=1}^n E\|X_i\|_r^2 \\
&= 2Ce\log d\sum_{i=1}^n E\|X_i\|_r^2.
\end{aligned}
$$

Thus Nemirovski's inequality is proved for $r \in [2, \infty)$ with constant $\widetilde{C}$ given by $2eC$ and C the constant of (4).

6. Extensions and comparisons

Nemirovski's inequality yields bounds of order comparable to those achieved by bracketing methods from empirical process theory. Since the proofs are very different, it may be worthwhile to explore the exact constants achieved by the two methods in more detail.

- In fact Nemirovski's basic deterministic inequality

$$V_r(x+y) \le V_r(x) + y'\nabla V_r(x) + CrV_r(y) \tag{6}$$

holds in the following more precise form:

$$V_r(x+y) \le V_r(x) + y'\nabla V_r(x) + (r-1)V_r(y) \tag{7}$$

where $V_r(x) = \|x\|_r^2$.

- Thus Nemirovski's inequality for sums of independent X_i's holds in the form

$$E\|\sum_{i=1}^{n} X_i\|_r^2 \leq C(r,d) \sum_{i=1}^{n} E\|X_i\|_r^2 \qquad (8)$$

where $C(r,d) = \min\{r-1, e(2\log(d)-1)\}$. In particular, when $r = \infty$,

$$E\|\sum_{i=1}^{n} X_i\|_\infty^2 \leq e(2\log(d)-1) \sum_{i=1}^{n} E\|X_i\|_\infty^2.$$

Two alternative methods for deriving similar bounds:

- By "type and co-type" theory together with symmetrization, (8) holds with

$$C(r,d) = \begin{cases} 8\left(\frac{\Gamma((r+1)/2)}{\pi}\right), & 2 \le r < \infty \\ 2\pi c_d^2, & r = \infty. \end{cases}$$

where $c_d^2 = E \max_{1 \le j \le d} Z_j^2 \le 2\log(d)$ for $d \ge 3$.

Two alternative methods for deriving similar bounds:

- By “type and co-type” theory together with symmetrization, (8) holds with

$$C(r,d) = \begin{cases} 8\left(\frac{\Gamma((r+1)/2)}{\pi}\right), & 2 \le r < \infty \\ 2\pi c_d^2, & r = \infty. \end{cases}$$

  where $c_d^2 = E\max_{1\le j\le d} Z_j^2 \le 2\log(d)$ for $d \ge 3$.

- By truncation and Bernstein’s inequality (8) holds with

$$C(\infty,d) = \{1 + 3.46\sqrt{\log(2d)}\}^2.$$

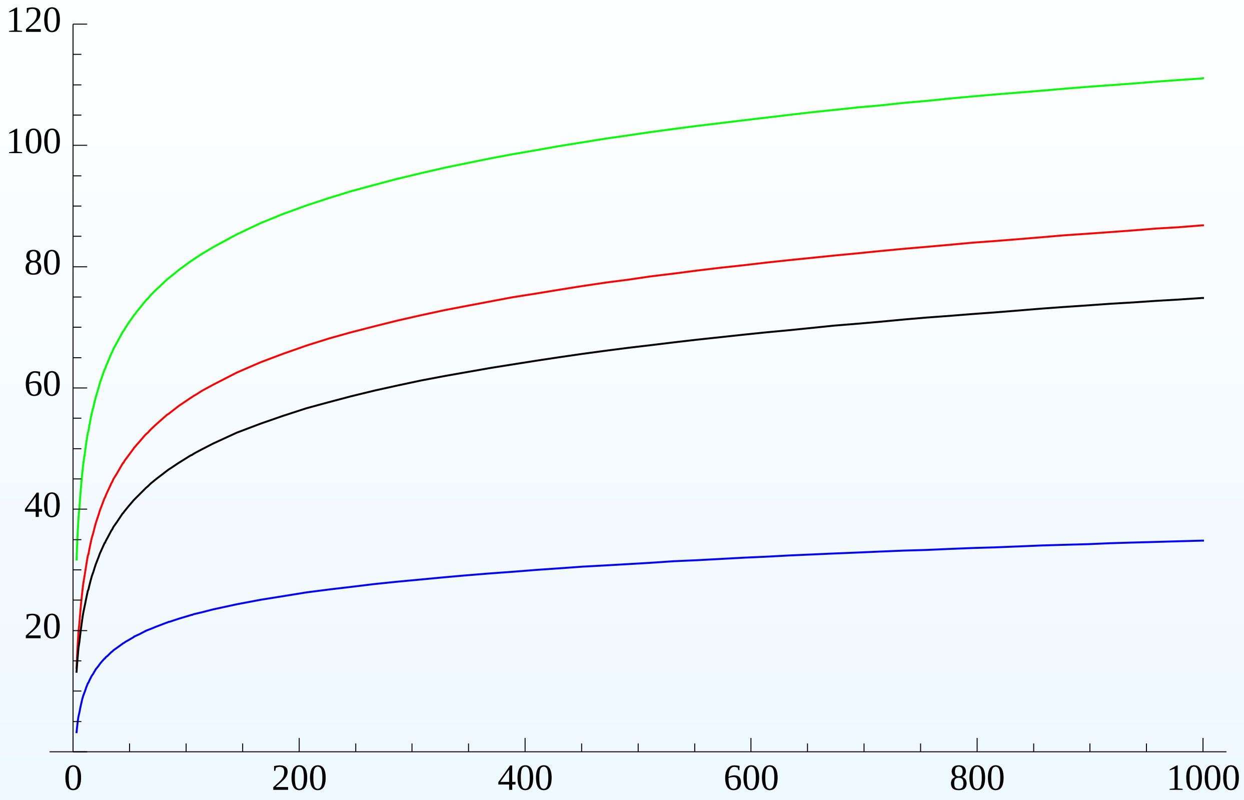

Figure 1: Comparison of $C(\infty, d)$ obtained via the three proof methods: Blue (bottom) $=$ Nemirovski; Red and Black (middle) $=$ type-inequalities / probability in Banach spaces; Green (top) $=$ truncation and Bernstein inequality

- - What sort of skill set(s) does a successful researcher in that area usually have?
  strong math background; interest in statistical theory

- - What sort of skill set(s) does a successful researcher in that area usually have?
  strong math background; interest in statistical theory
- - Why/How did you get involved in this subfield/area?
  Became interested in statistical theory as a grad student; wrote a book or two on empirical process theory.

- - What sort of skill set(s) does a successful researcher in that area usually have?
  strong math background; interest in statistical theory
- - Why/How did you get involved in this subfield/area?
  Became interested in statistical theory as a grad student; wrote a book or two on empirical process theory.
- - Who else in the department works on similar things?
  Galen Shorack and I have worked together on empirical process theory. Most other colleagues work on some aspects of theory / methodology.

- - What sort of skill set(s) does a successful researcher in that area usually have?
  strong math background; interest in statistical theory
- - Why/How did you get involved in this subfield/area?
  Became interested in statistical theory as a grad student; wrote a book or two on empirical process theory.
- - Who else in the department works on similar things?
  Galen Shorack and I have worked together on empirical process theory. Most other colleagues work on some aspects of theory / methodology.
- - If your project is interdisciplinary, how did you connect with people from other departments? Interdisciplinary in the sense of interactions between math (probability theory) and statistics. Connections made by attending national and international meetings + (random ?) contacts via internet.