Back to the Future

Do recent developments tell us anything about the future of statistics?

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- Talk at Joint Statistical Meetings
- Washington, D.C.
- August 3, 2009
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Outline

- Background

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- What recent developments?

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- Possible future directions?

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- What recent developments?
- Possible future directions?
- A few specific problems.

1. Background

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  - Longer on-line version of above: *Statistics: Challenges and Opportunities for the Twenty-First Century*.
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- Personal, subjective, and biased view, with lots ignored mostly due to personal ignorance.

Problems for the future (of Statistical Inference) mentioned by Kiefer, Savage, and Le Cam, 1967;
Conference at University of Wisconsin on "The Future of Statistics"

- Problems mentioned by Kiefer:
  - Theory of nonparametric inference: testing and estimation.
  - Theory of nonparametric Bayes procedures. (Theory developing over last 10+ years: e.g. Le Cam lecture by van der Vaart at this meeting.)
  - Rates of convergence. (Götze, Bickel, van Zwet, Peter Hall, ...)
  - Nonparametric regression / curve estimation. (Hints of model selection, penalized estimation.)

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- Problems mentioned by Savage:
  - Descriptive statistics (look at the data).
  - Multi-parameter problems and nonparametric problems
  - Weather forecasting?
  - Medical diagnosis?

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    - "... select some subset of the variables if we have more than five or six ... "
    - Change of perspective with computing power?

2. What recent developments?

The future ain't what it used to be. (Yogi Berra)

Data: Numbers of papers per year for several topics.

- Topics explicitly mentioned at the 1967 Madison meeting
  - Nonparametric Bayes methods (Kiefer)
  - Robustness (Kiefer)
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- Topics implicit or “hinted at” in discussion at the 1967 Madision meeting
  - Alternatives to Bayes methods (Le Cam) $\longrightarrow$ empirical Bayes estimation
  - Structure of stochastic models (Le Cam) $\longrightarrow$ graphical models
  - Multiple comparisons / multiple testing (Savage) $\longrightarrow$ false discovery rate
  - Multiparameter - nonparametric models (Savage) $\longrightarrow$ semiparametric models

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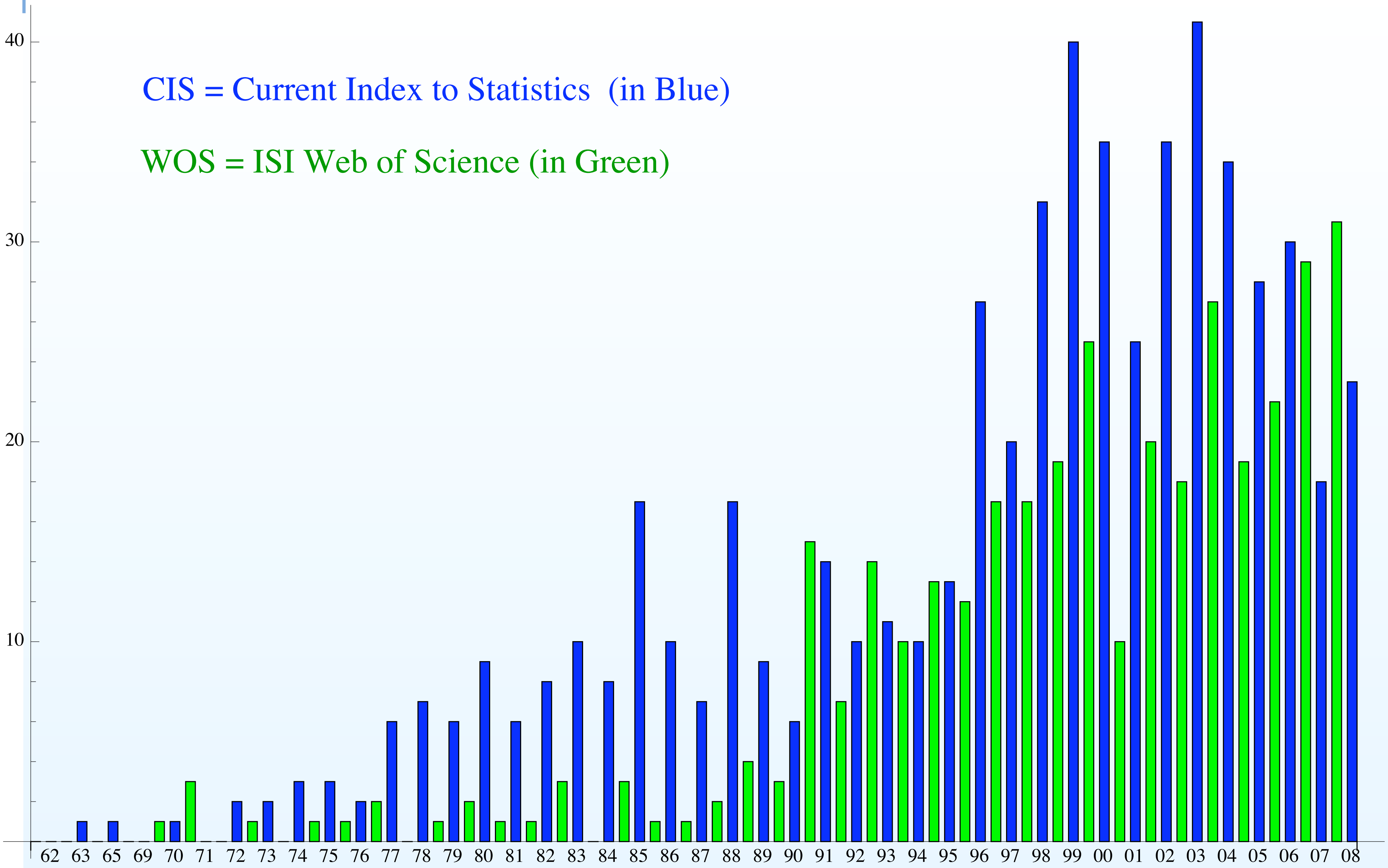
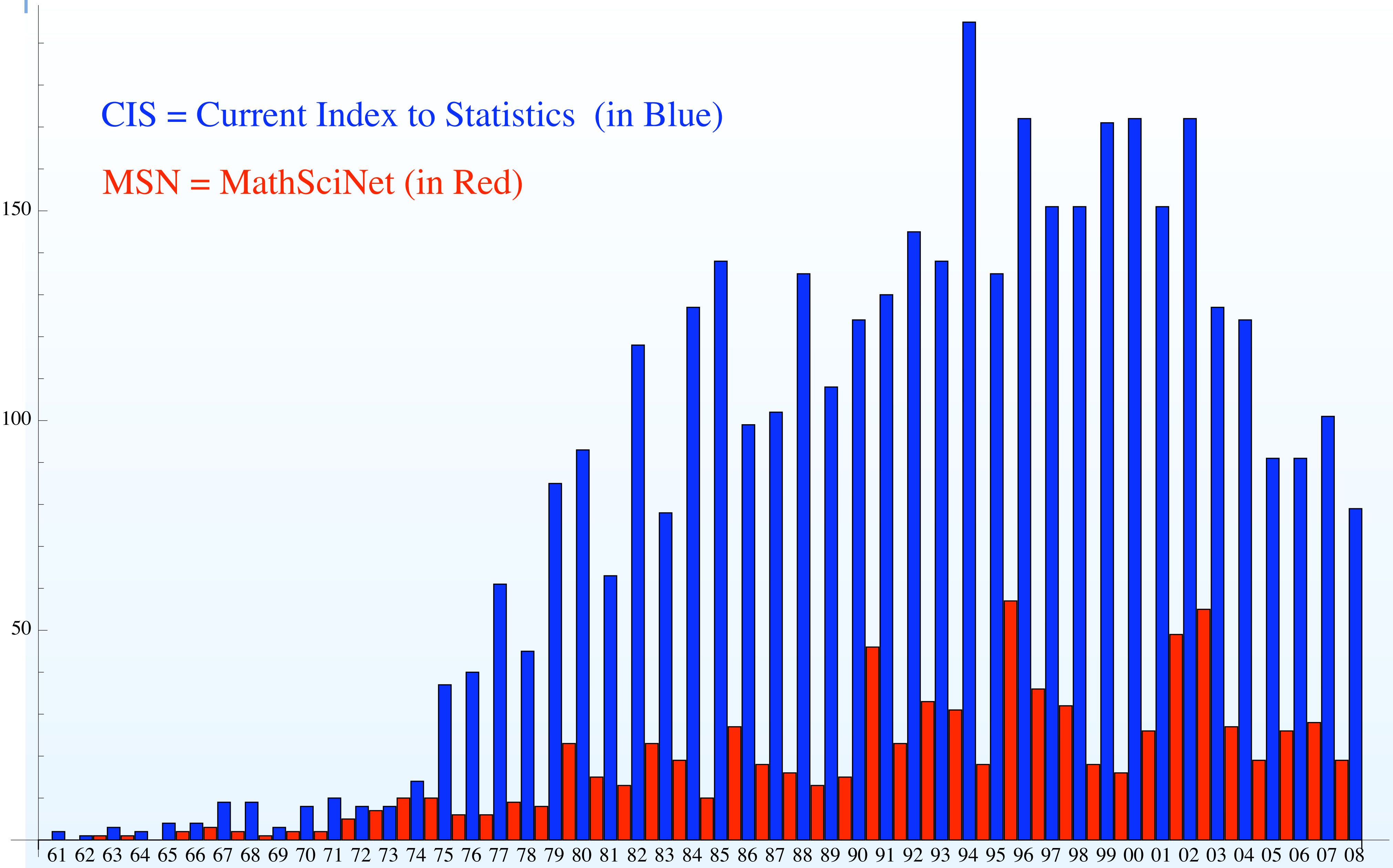
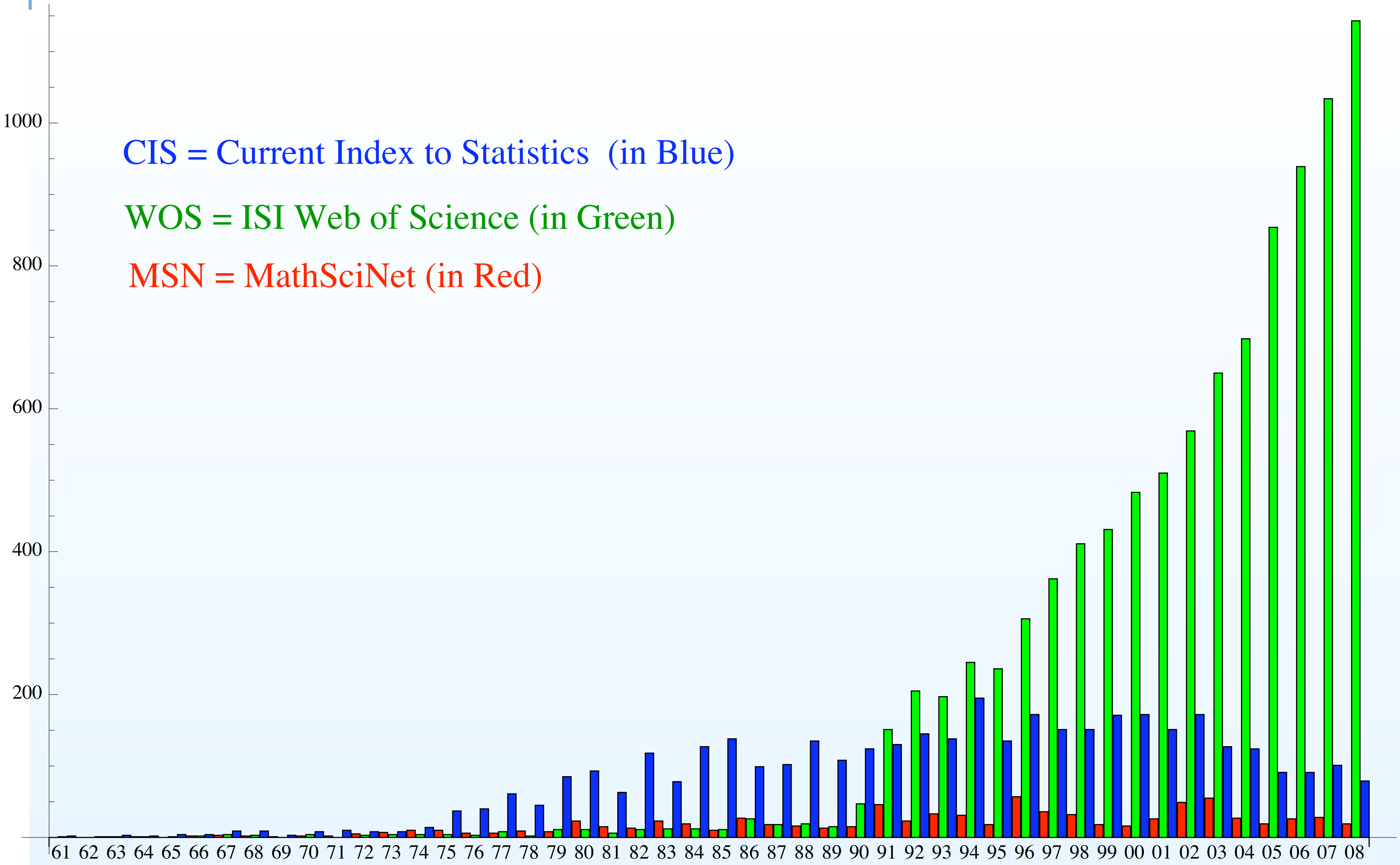
- Many other possible topics (not pursued here)
    - Hierarchical models
    - Metaanalysis
    - Nonparametric function estimation
    - Causal inference
    - Missing data



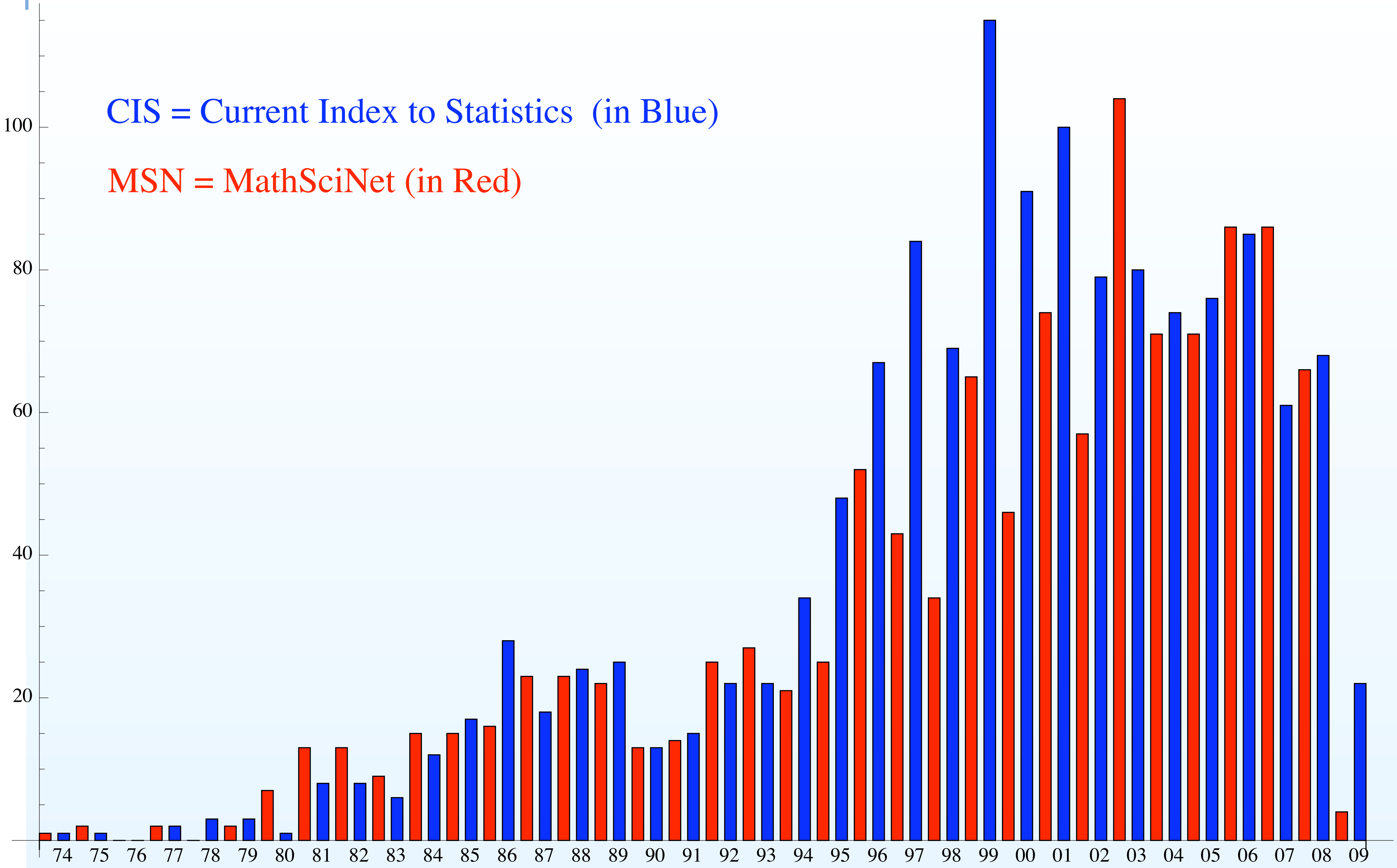
Numbers of papers by year, Nonparametric Bayes



Numbers of papers by year, Robustness



Numbers of papers by year, Robustness, all of MSN, CIS, WOS



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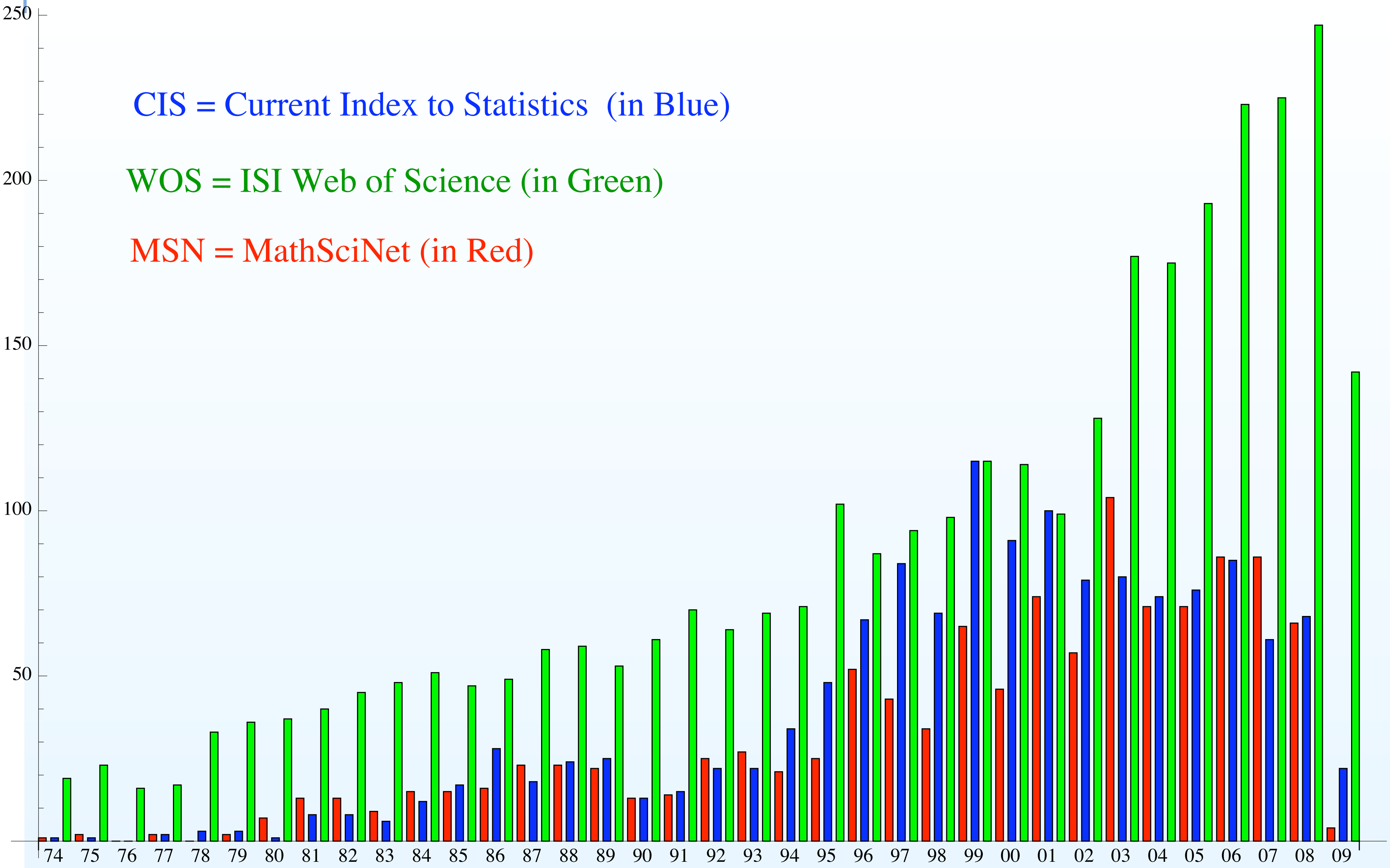

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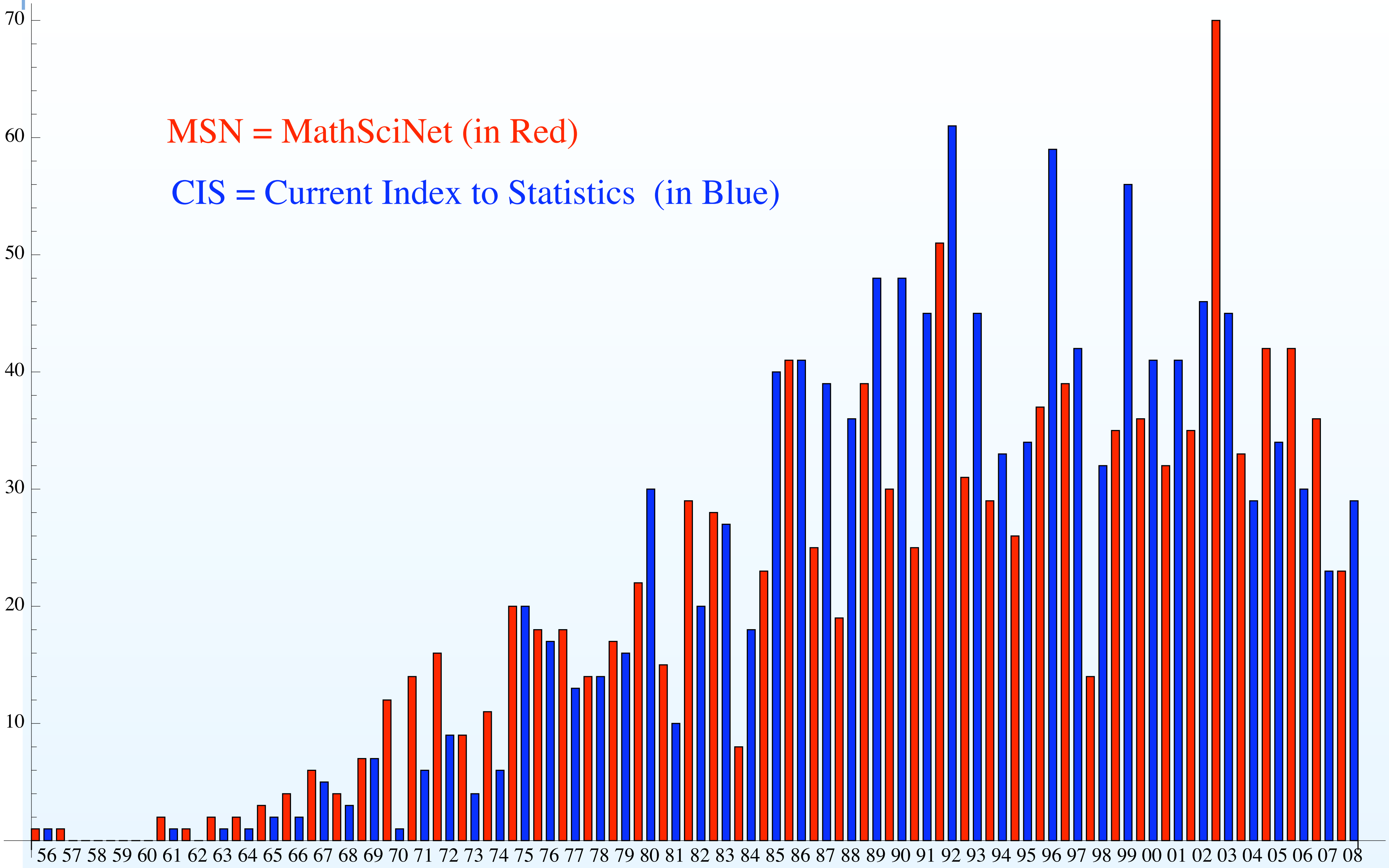
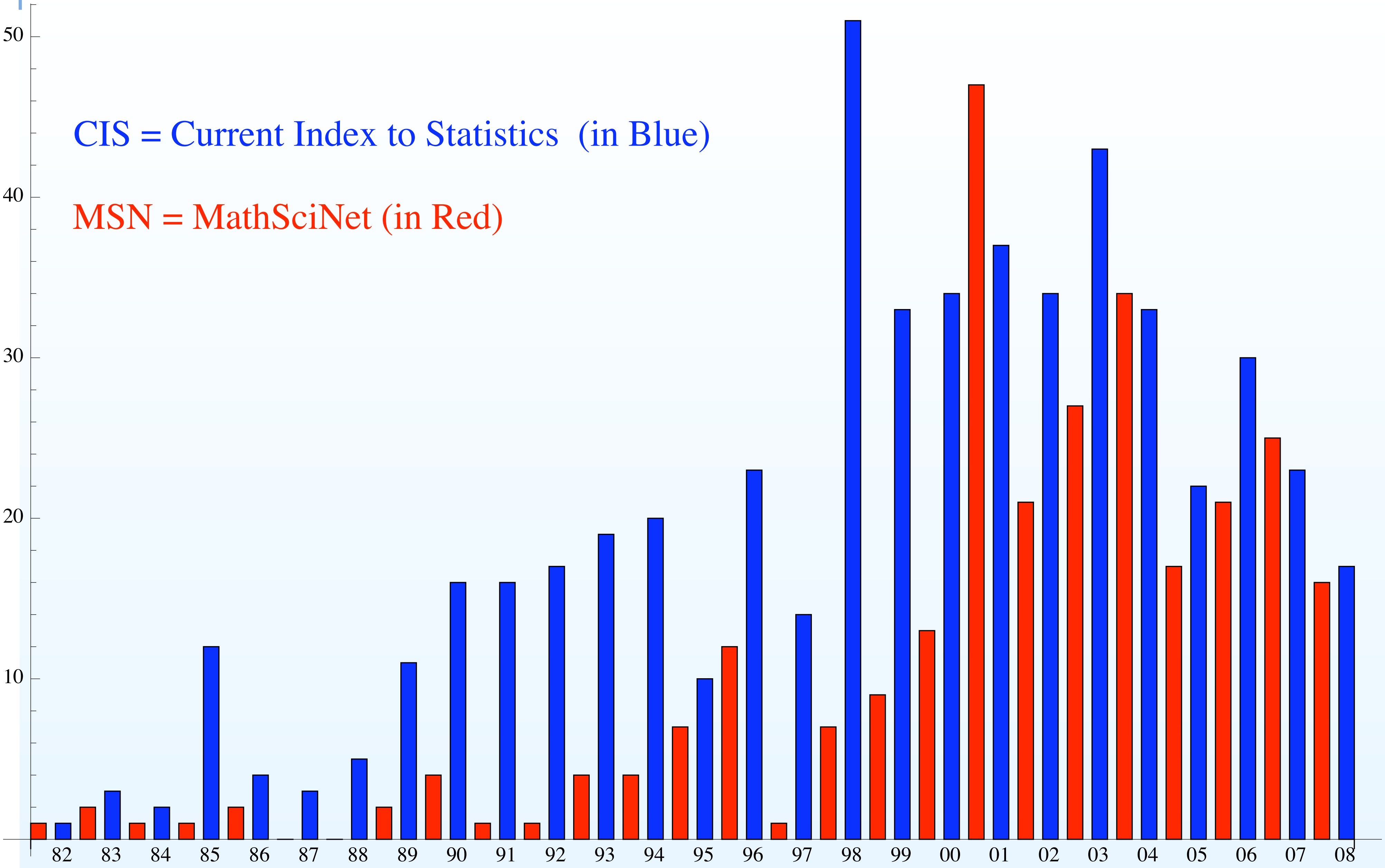
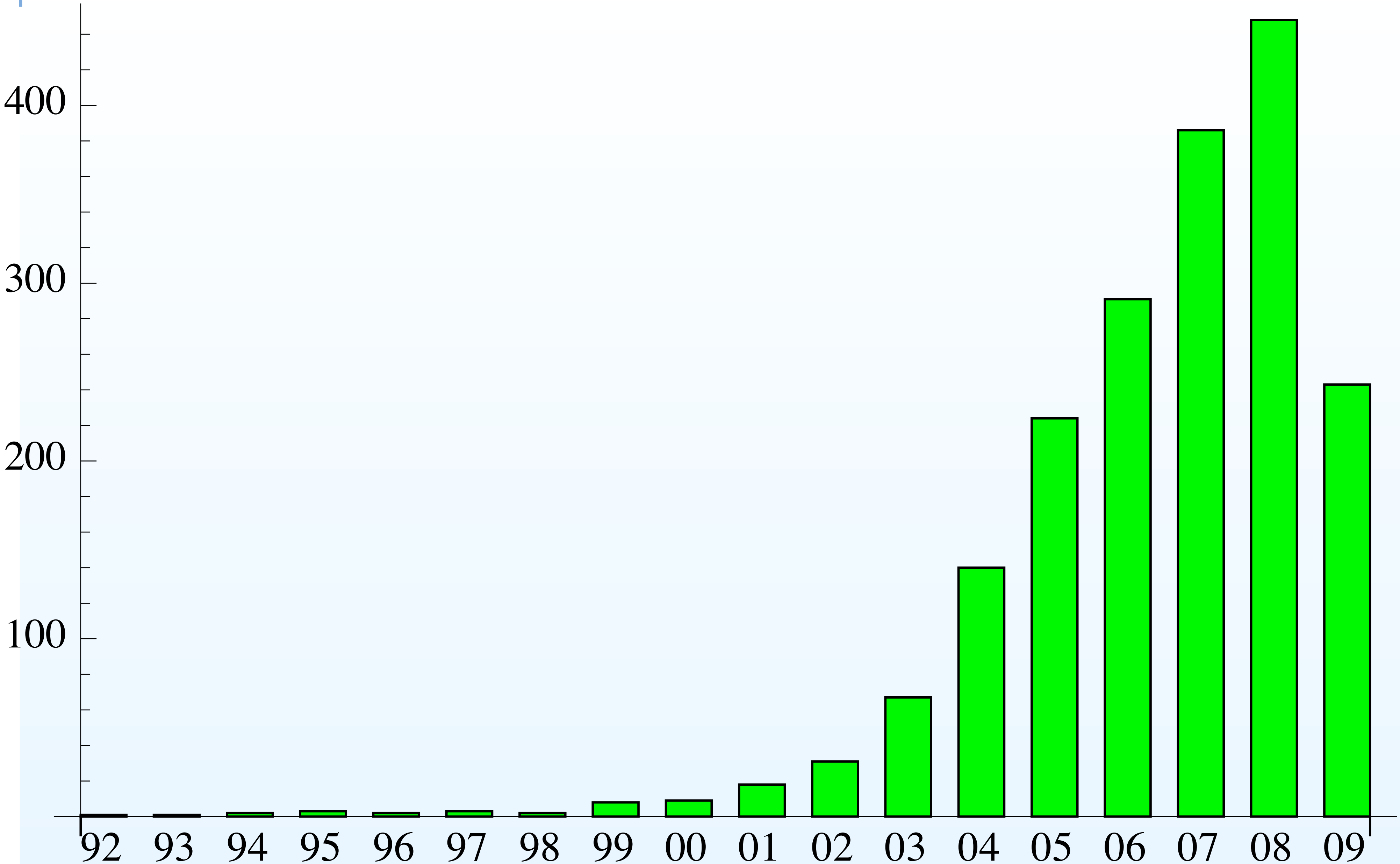


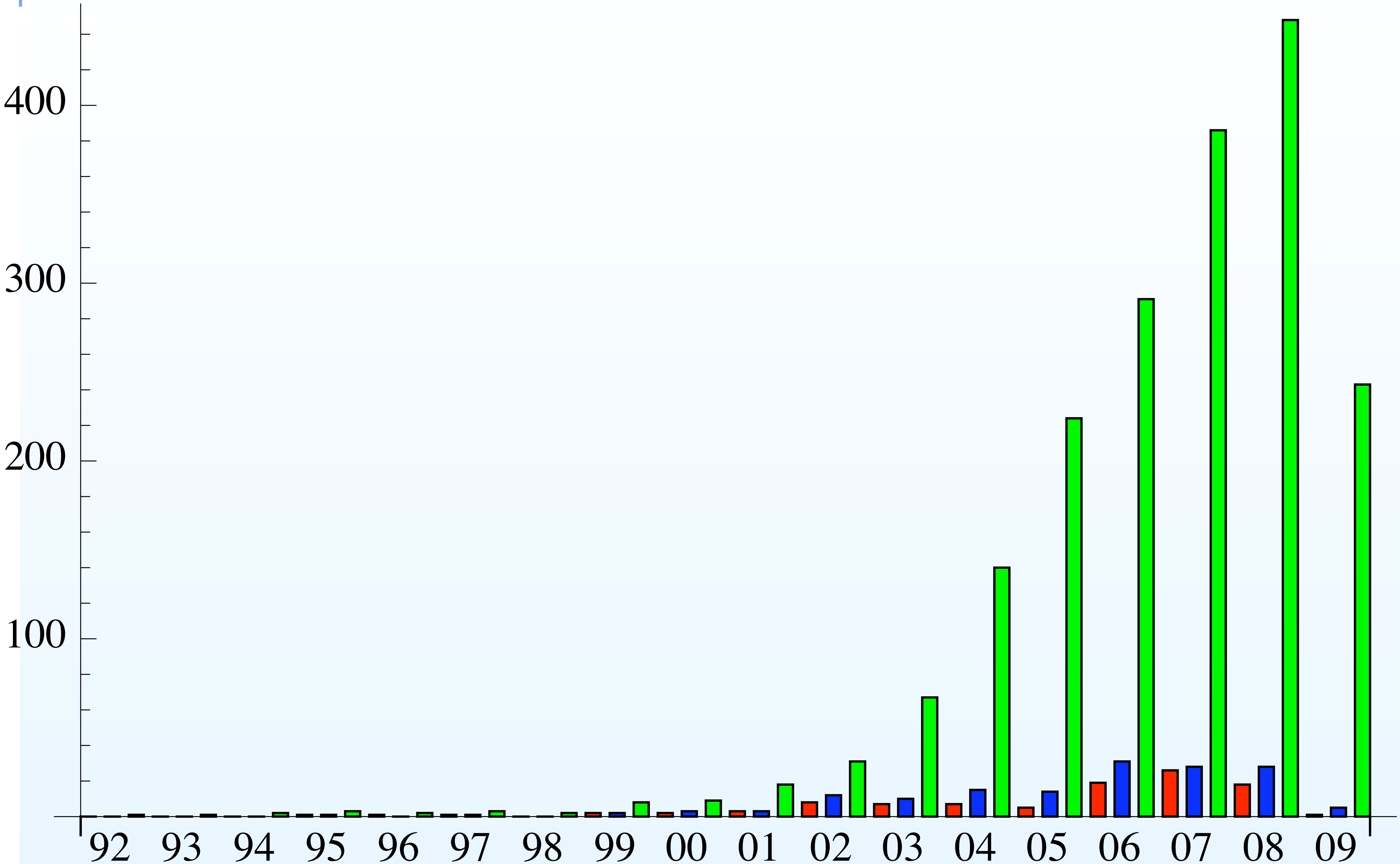
Numbers of papers by year, Empirical Bayes estimation



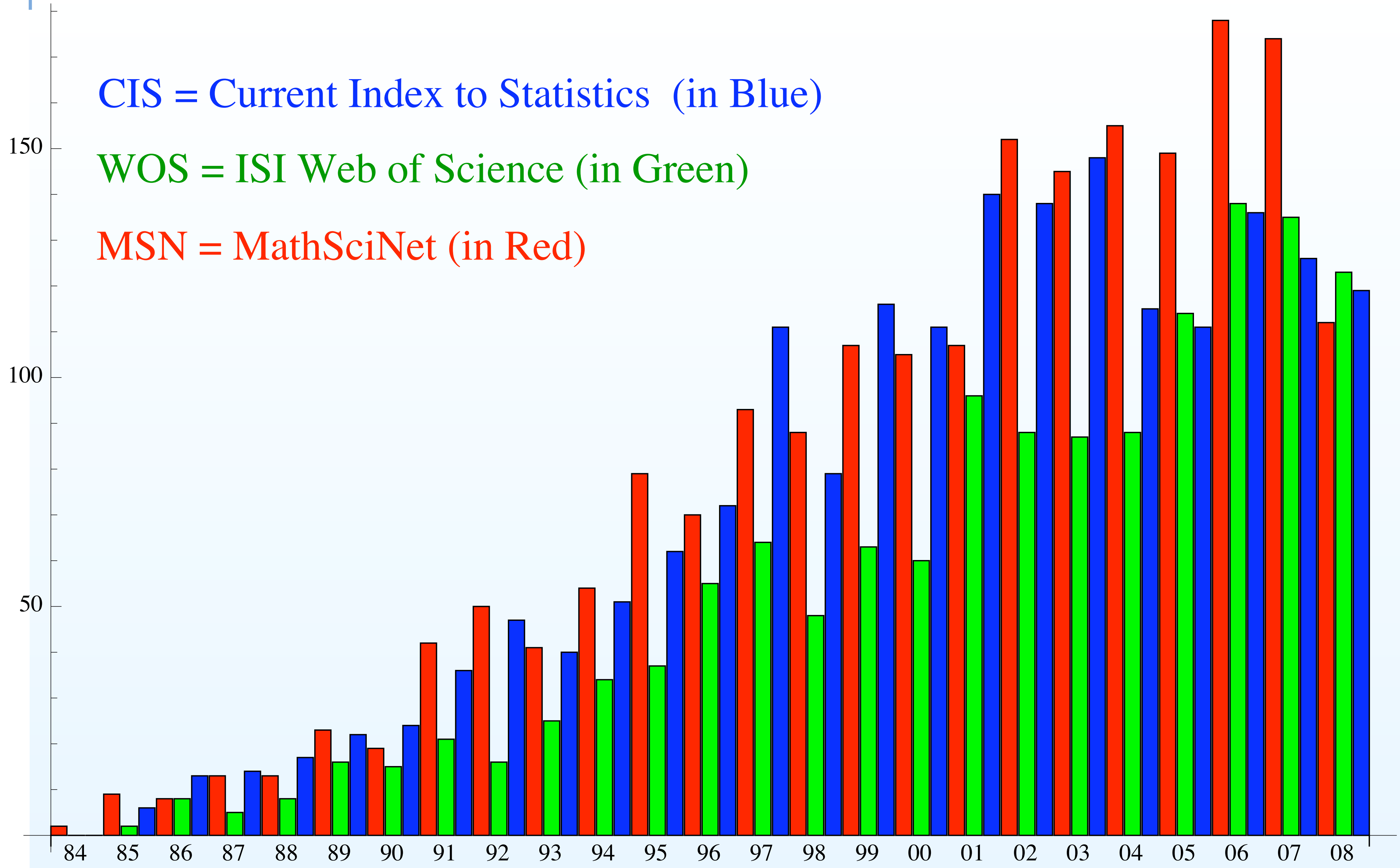
Numbers of papers by year, Graphical Model



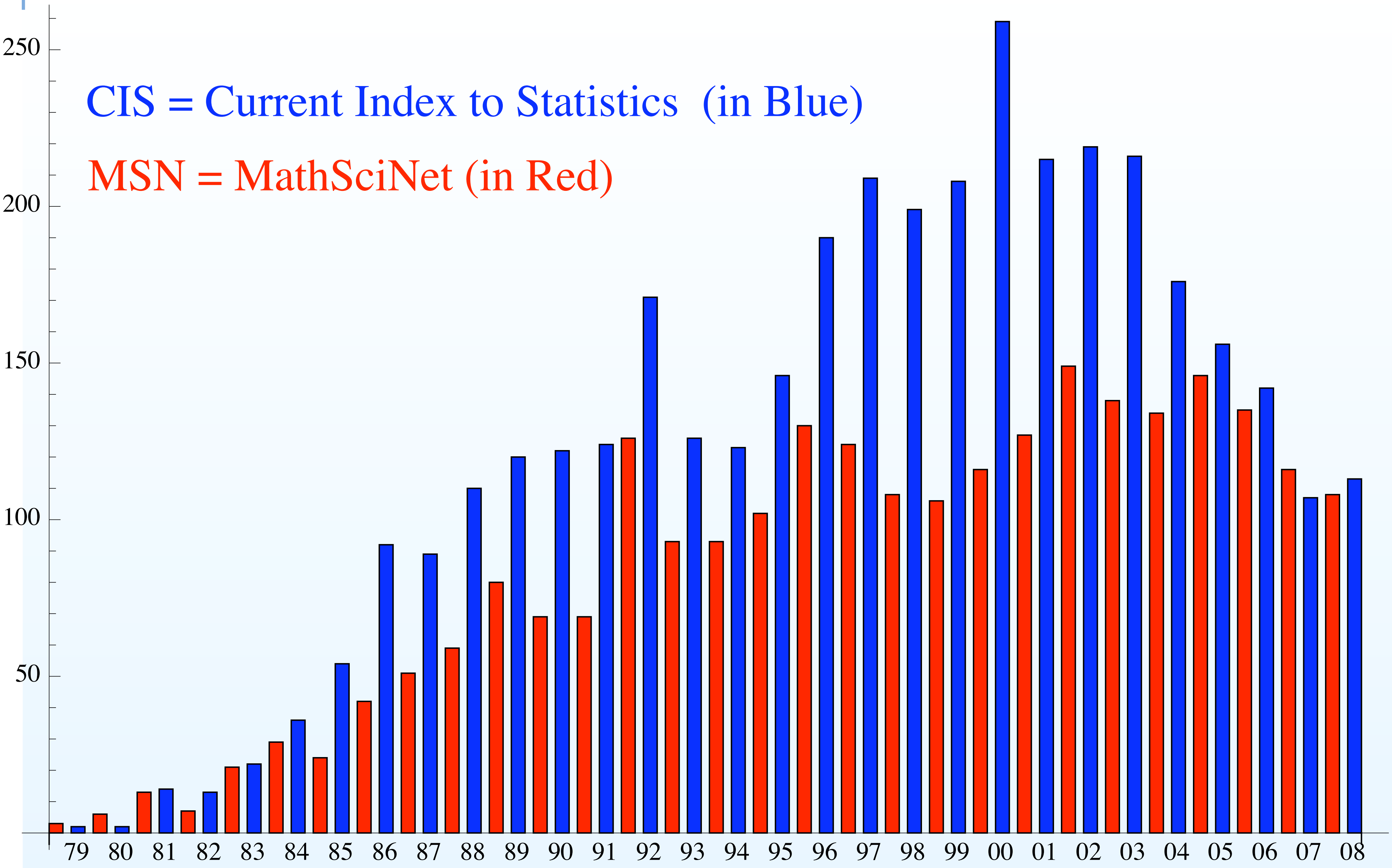
Numbers of papers by year, False Discovery Rate: green=WOS



Numbers of papers by year, False Discovery Rate, combined MSN, CIS, WOS



Numbers of papers by year, Semiparametric Models



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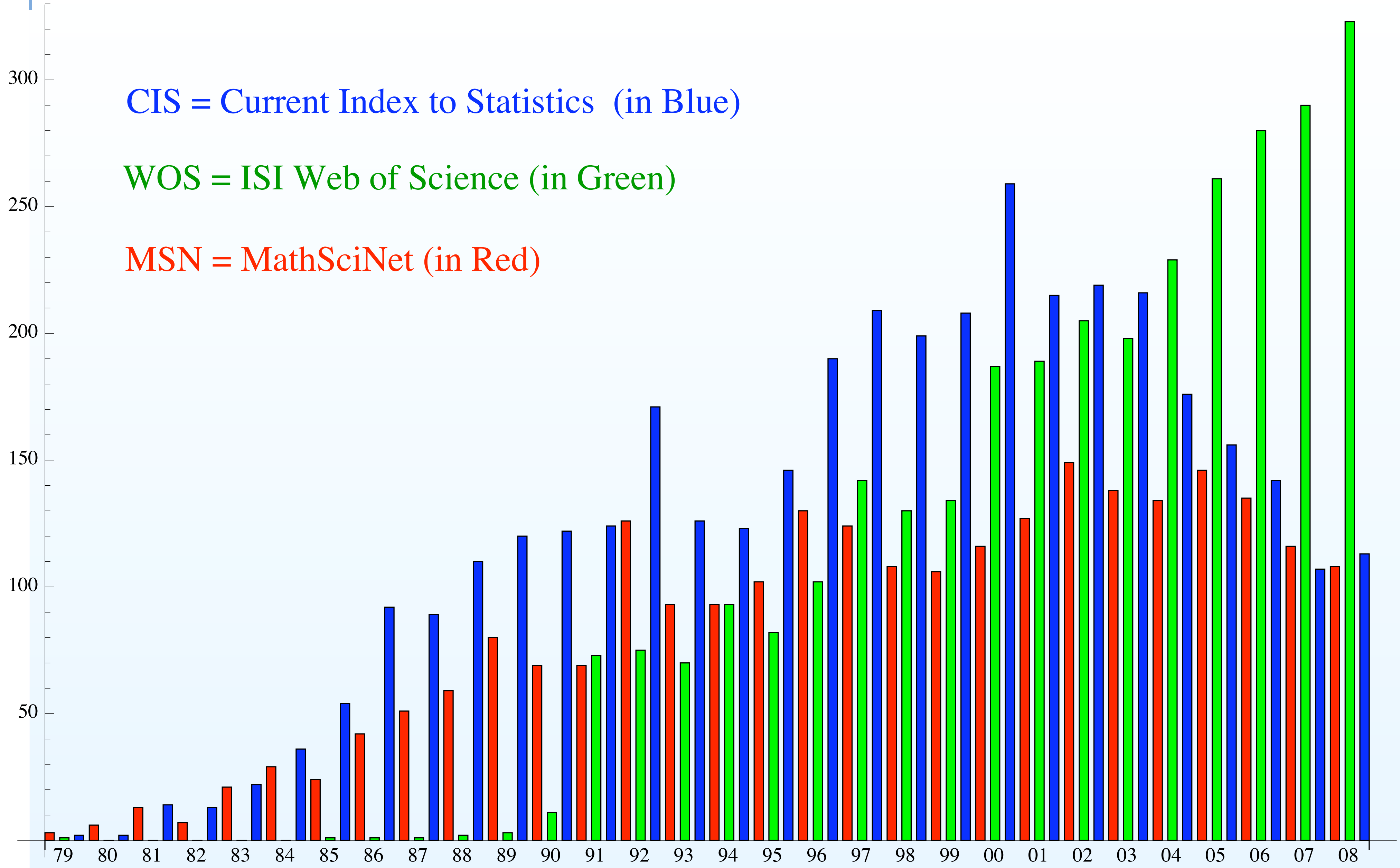

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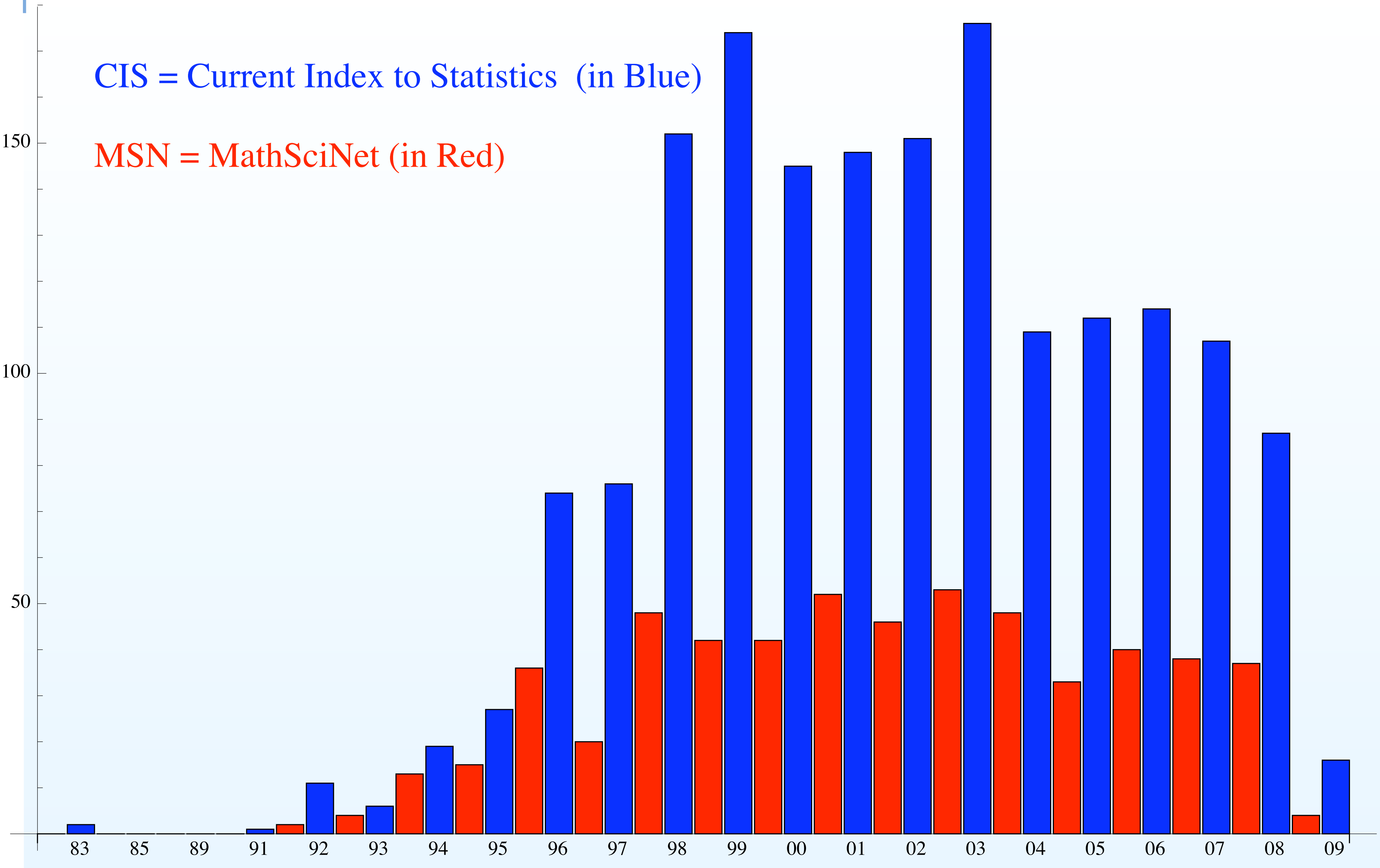

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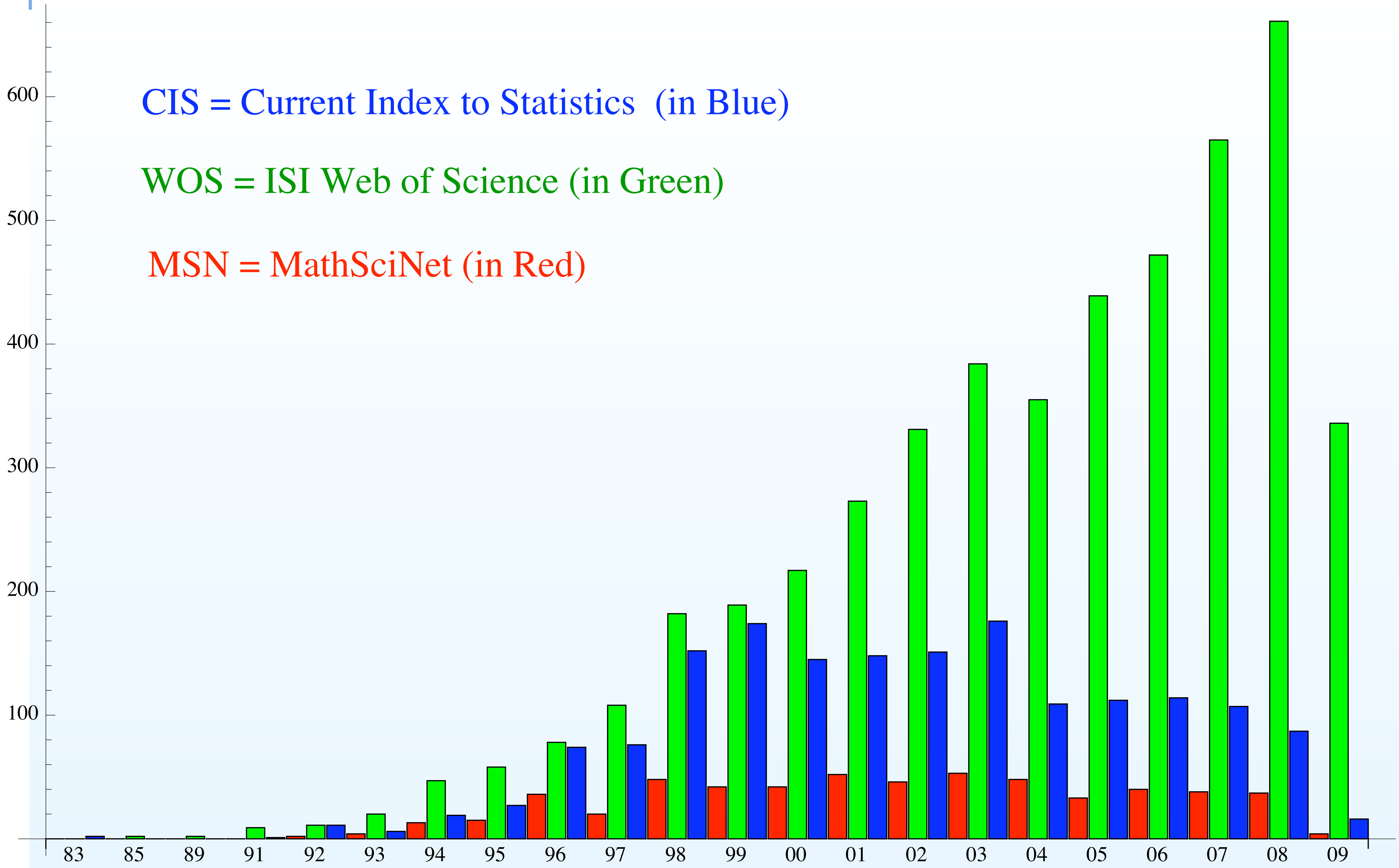

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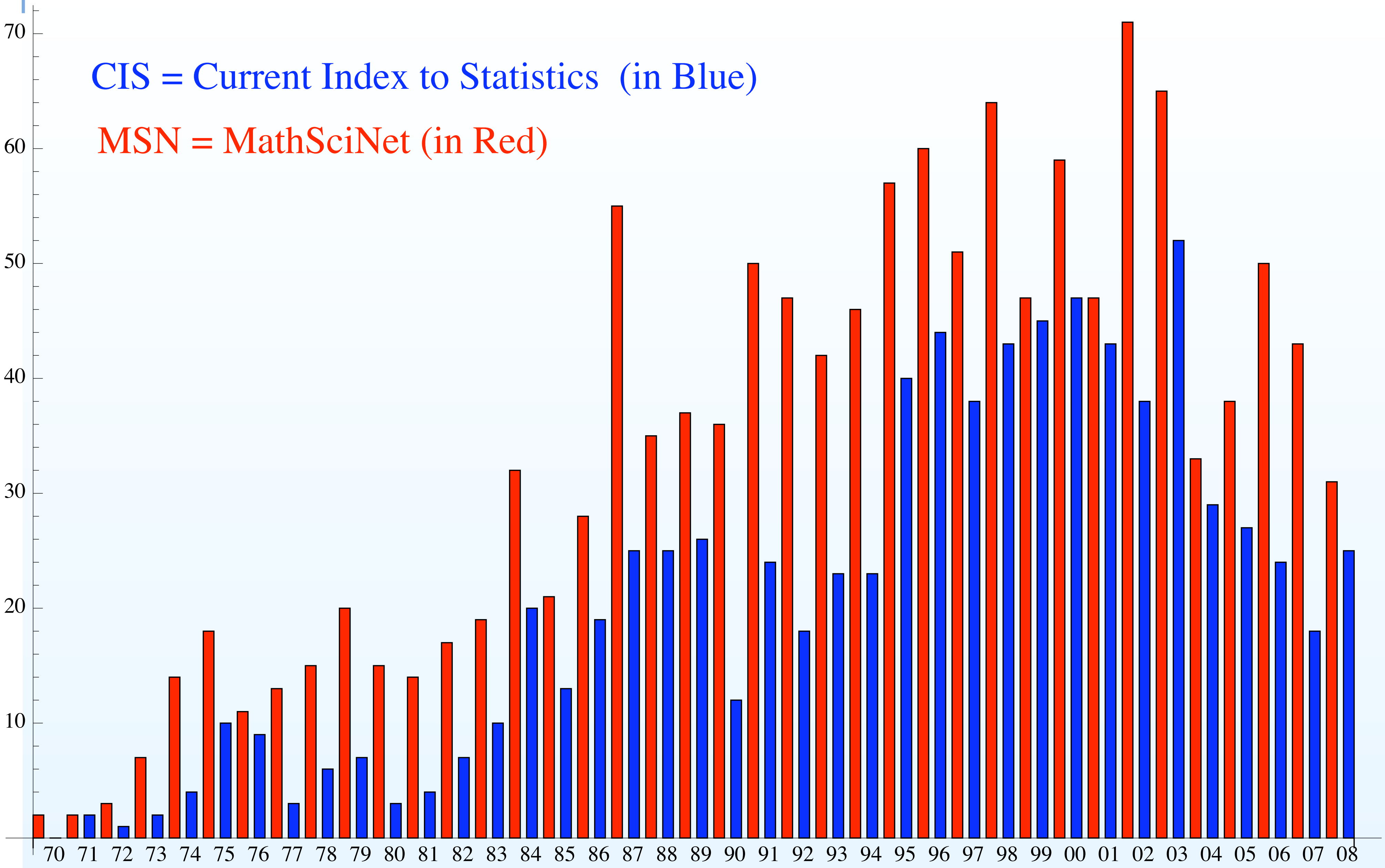
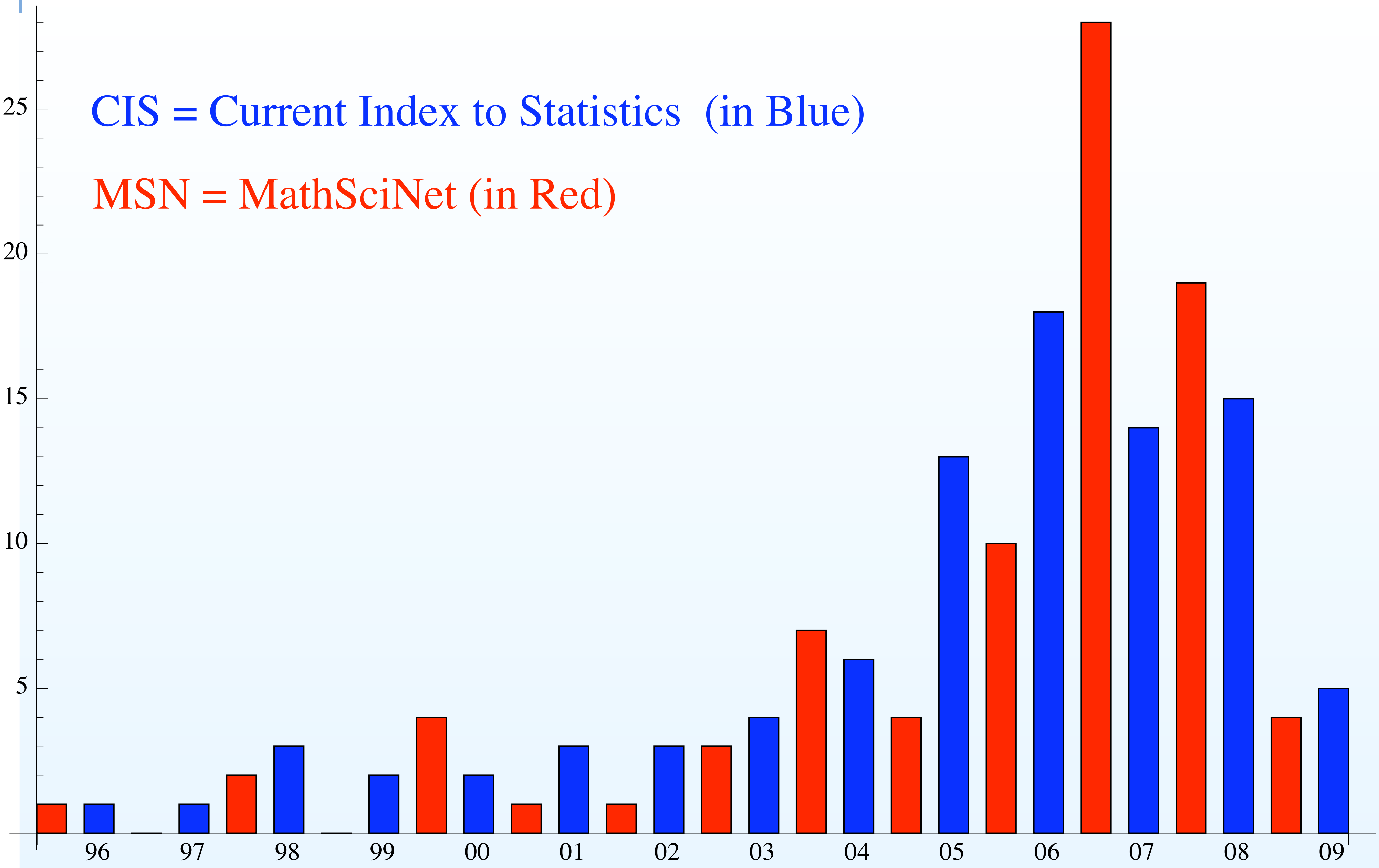


Numbers of papers by year, Empirical Processes



Numbers of papers by year, Lasso + Regression

3. Possible future directions?

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  - Networks ubiquitous: internet, social networks, citation networks, ...

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  - Nested Laplace approximatons: Rue, Martino, Chopin (2008)

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  - Ingster (1993a,1993b, 1997)
  - Searching for sparse signals: Donoho and Jin (2004), Johnstone and Silverman (2004)
  - Estimation of the proportion of sparse signals: Meinshausen and Rice (2006), Cai, Jin and Low (2007), Jin and Cai (2007)

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- More on model selection
  - Enormous qualitative changes in past 10 years:
  - Changing perspectives: often no one “true” model.
  - Replace with specified goals: prediction or variable/feature selection
  - Often now based on “model averaging”, or “weighting”, or “aggregation” methods.
  - Need for much more work on inference following model selection (e.g. H. Leeb, B. Pötscher)

4. Some specific problems (of special interest to me)

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  - Interactions between (nonparametric) mixture models and Empirical Bayes methods: C-H Zhang (2009), Jiang and Zhang (2009).

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    - Kosorok, Lee, and Fine (2004), Newey (2004).

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- In 1967 Leonard J. Savage (1917-1971) was 50.
- In 1967 Lucien Le Cam (1924-2000) was 43.
- In 1967 Jack Kiefer (1924-1981) was 43.
- In 1967 David Freedman (1938-2008) was 29.
- In 1969 (time of the first moon landing) less than half of the current population of the U.S. had been born. (The national median age in the U.S. was 36.7 years in February 2009.)

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