STAT 582 FINAL EXAM

1. Let $T(F) = \mu_F^2 = (\int x dF(x))^2$. Find, using the asymptotic theory for statistical functionals, the limiting distribution of $T(F_n)$ when $\mu_F \neq 0$.

2. Let $(X_1, Y_1), \ldots, (X_n, Y_n)$ be independent random variables with distribution

$X \backslash\backslash Y$	0	1
0	λpq	$p(1-\lambda q)$
1	$q(1-\lambda p)$	λpq

where $q = 1 - p$ and $0 \leq \lambda \leq \min(1/p, 1/(1-p))$.

(a) If λ is unknown, derive a test of the hypothesis $p = \frac{1}{2}$.

(b) If λ is known to be 1, how would you test $p = \frac{1}{2}$.

(c) Compare the Fisher informtion about p in the data when λ is unknown and known, respectively.

3. Let $X_1, \ldots, X_n$ be iid nonnegative random variables with cdf $F(x) = (1/\mu) \int_0^x t dG(t)$, where G is a cdf on $[0,\infty)$ with mean $0 < \mu = \int_0^\infty t dG(t) < \infty$.

(a) Show that $\mathbf{E}(1/X) = 1/\mu$.

(b) Show that $G(x) = \int_0^x (\mu/y) dF(y)$.

(c) Use the results of (a) and (b) to suggest an estimator of $G(x)$ based on the X's. Outline how you would determine the asymptotic properties of the estimator.