1. Let X_n have the discrete uniform distribution on $\{1/n, \ldots, (n-1)/n, 1\}$, *i. e.*, $\mathbf{P}(X_n = k/n) = 1/n$, $k = 1, \ldots, n$. Determine whether or not X_n converges in distribution, and if it does, to what limiting distribution.

2. Let (X_k) be iid exponential random variables with mean θ, having cdf

$$F(x;\theta) = 1 - \exp(-x/\theta), \quad x > 0.$$

Let $Y_n = (1/n)\sum_1^n X_i$ and $Z_n = F(X_1; Y_n)$. Does Z_n converge in distribution as $n \to \infty$?

3. Let F be a cdf, and for $0 < p < 1$ define the *inverse cdf*

$$F^{-1}(p) = \inf\{x \colon F(x) \geq p\}.$$

Show that

(a) F^{-1} is non-decreasing and left-continuous;

(b) $F^{-1}(F(x)) \leq x$, $-\infty < x < \infty$;

(c) $F(F^{-1}(t)) \geq t$, $0 < t < 1$;

(d) $F(x) \geq t$ if and only if $x \geq F^{-1}(t)$.

4. (a) If $X_n \xrightarrow{P} X$, show that $X_n \xrightarrow{d} X$.

(b) If $X_n \xrightarrow{d} c$, where c is a constant, show that $X_n \xrightarrow{P} c$.

(c) Show that $X_n \xrightarrow{d} X$ does not imply that $X_n \xrightarrow{P} X$ even if the variables are defined on the same probability space.